

# Particle Swarm Optimization of PID Controller under Constraints on Performance and Robustness

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**Abstract**—This paper presents a design procedure of the PID controller where optimal parameters of controller ( $k_p^*$ ,  $k_i^*$ ,  $k_d^*$ ,  $T_i^*$ ) are obtained by solving the constrained optimization problem. The objective function is given in the form of the Integral of Absolute Error (IAE) under specifications to achieve predictable performance and robustness. The constraints within the optimization problem setup are desired maximum sensitivity, desired maximum complementary sensitivity and maximum sensitivity to measurement noise under high frequencies. The optimization problem is transformed to an unconstrained problem using penalty function approach. Solution to the optimization problem is obtained using Particle Swarm algorithm (PSO) which leads to an efficient suppression of disturbance as well as an adequate reference tracking performance of the closed-loop system with negligible overshoot. The suggested method is applicable to the large class of stable, integral and unstable processes, processes with oscillatory dynamics with and without dead-time. Effectiveness of the proposed design procedure is verified via numerical simulations on test batch consisting of processes typically encountered in industry. Paper also provides two another solutions of the defined optimization problem using genetic algorithm (GA) and fminunc trust region based approach (TR). Performance of the PSO, GA, and TR based control system is compared with those using recently proposed maximization of proportional gain denoted with  $\max(k_p)$  method. Although the present paper is focused to tune the PID controller, the same procedure may be used to design PI controller, lead and lag compensators, high-order controllers as well as fractional-order controllers.

**Keywords**- PID controller; Optimization; Particle Swarm; Disturbance; Performance; Robustness; Genetic algorithm;

## I. INTRODUCTION

PID controllers are used more than seventy years as an inevitable part of control loops in process industry. The study in [1] classifies the PID controller as the second greatest contribution of twentieth century in area of control, decision and communication right behind microprocessor. PI/PID controllers are used in more than 95% control loops in process industry [2], while the usage percent of PID controller in refining, chemical and paper industries is 97% according to Honeywell's survey [3]. Widespread application of these controllers in various fields of engineering arises from small number of adjustable parameters, simplicity of implementation and preprogramming in every control system [3]. Furthermore, PID controllers show adequate robust performance in large range of operating conditions in industry environment. Nevertheless 25% control loops with PID controllers use factory settings, which motivated *O'Dwyer* to collect large number analytical formulae in [4] with the goal of accessibility of tuning rules to apply as much as possible within process control in engineering industrial practice.

Most of tuning methods assume that process model is known in advance. Large number of techniques is developed to perform process dynamics characterization such as relay techniques [5,6], phase-locked loops [7-9] as well variety of other identification methods [10]. Permanent development of technology has led to the need to improve conventional control algorithms. Modern design methods of controller include solving optimization problems under specified requirements to achieve trade-off between robustness and performance [11,12]. These requirements are usually given in the form of four sensitivity functions (*Gang of Four*). However, alternative constraints within optimization problem may be also used such as: phase and gain margins, location of dominant poles, settling time, rise time, overshoot etc. [13,14]. Efficient optimization techniques are available in scientific and professional literature for tuning parameters of PID controller with empirical adopted filter time constant [15-28], while complex methods use filter time constant as an integral part of classic optimization procedure [29-33].

However, heuristic algorithms largely emerge in the process control design due to their ability to solve complex, high-dimensional and multimodal optimization problems. These algorithms can be classified into two large groups: stochastic and deterministic. Deterministic heuristics solve problems by making deterministic decisions, while stochastic

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heuristics apply random rules in search process. At deterministic heuristics, the same initial population always generates the same final solution, while at stochastic heuristics different final solutions can be generated using the same initial population. The most popular stochastic algorithms used for PID controller design are evolutionary algorithms [34] such as: genetic algorithms [35-37], swarm techniques (ant colony [37-39], particle swarm [33,40-43], bee colony [44,45]) etc.

This paper deals with design of PID controller where optimal parameters ( $k_p^*$ ,  $k_i^*$ ,  $k_d^*$ ,  $T_f^*$ ) are obtained with Particle Swarm Optimization (PSO). The employed objective function is Integral of Absolute Error while the constraints are of inequality type expressed in the form of maximum desired values:  $M_s^d$  of sensitivity function,  $M_p^d$  of complementary sensitivity function and  $M_n^d$  of sensitivity to measurement noise. An additional constraint  $Q^d$  is used to achieve negligible overshoot in step response. At first the constrained optimization problem is set up which is converted into unconstrained problem incorporating the constraints within the objective function. Two frequently used techniques to perform this conversion are barrier and penalty approaches [46]. In this paper to solve the optimization problem with inequality constraints the latter is used to penalize iterations outside the feasible region to apply the PSO algorithm from [47]. Solution to the optimization problem may be obtained using different optimization algorithms, but in this paper genetic algorithm (GA) and *fminunc* trust-region based optimization method are used to show that these algorithms are comparable with proposed PSO. At the end, the system performance and robustness obtained with PSO, GA and TR based optimization algorithm are analyzed and compared with recently proposed and widely accepted  $\max(k_p)$  from [21] to validate correctness of the optimization problem setup and effectiveness of optimization algorithms. A large test batch of stable, integral, unstable and processes with oscillatory dynamics including dead-time is used to demonstrate validity of the presented controller design procedure. It should be noted that same procedure may be used for optimization of PI controller, lag- and lead compensators, e.g. it is worth mentioning that similar PSO optimization approach is applied to DTC-PID controller in [48] which later may be parametrized as PID controller.

This paper is organized as follows. The design procedure of PID controller including analysis of optimization problem setup is presented in Section 2. Section 3 provides results of numerical simulations with obtained indices of performance and robustness. Concluding remarks are given in the Section 4.

## II. THE DESIGN PROCEDURE OF PID CONTROLLER

The block structure of the automatic control system is shown in Fig. 1, where the following notation is used:  $G_p(s)$  –

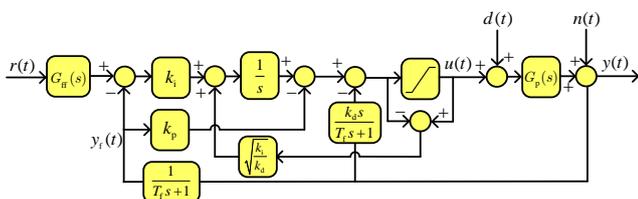


Figure 1. Block diagram of the control system structure with differential and proportional gain of PID controller moved to the feedback path including blocks used for integral anti-windup control using back-calculation [49, p.79]

process transfer function,  $r(t)$  – reference signal,  $y(t)$  – output signal,  $u(t)$  – control signal,  $d(t)$  – disturbance,  $n(t)$  – measurement noise.

In the present paper PID controller in series with first-order noise filter is given by

$$C(s) = \frac{k_p + k_i/s + k_d s}{(T_f s + 1)}, \quad (1)$$

where the proportional, integral, differential gain of PID controller and filter time constant are adjustable parameters of controller denoted with  $k_p$ ,  $k_i$ ,  $k_d$  and  $T_f$  respectively. Fig. 1 presents two-degrees of freedom implementation of control system where control law is defined by

$$U(s) = -k_p Y_f(s) + \frac{k_i}{s} (G_{ff}(s)R(s) - Y_f(s)) - k_d s Y_f(s), \quad (2)$$

where  $G_{ff}(s)$  is feed-forward filter from reference to control signal, and  $y_f$  is signal at the output of the low-pass filter generated with  $Y_f(s) = \frac{1}{T_f s + 1} Y(s)$ .

### A. Requirements within the design procedure of PID controller

Open-loop transfer function of the control structure in Fig. 1 is

$$L(s) = \gamma \frac{k_d s^2 + k_s s + k_i}{s(T_f s + 1)} G_p(s) \quad (3)$$

where for positive static gain of the process  $G_p(s)$  parameter  $\gamma=1$ , while for negative static gain  $\gamma=-1$ . Without loss of the generality the presented design procedure considers the case  $\gamma=1$ .

Main goal of controller design in the process industry is an efficient suppression of disturbance. Disturbance is modeled to be at the input of the process as stated in [50] it is the most usual case in the process control industry. Therefore, as performance measure it is used the Integral of Absolute Error defined by Eq. (4)

$$IAE = \int_0^{\infty} |y_d(t)| dt, \quad (4)$$

where  $y_d(t)$  is the response of the system to a unit step disturbance signal  $d(t)$ . If  $e(t)=r(t)-y(t)$  is positive then  $IAE = IE = \int_0^{\infty} y_d(t) dt$ , while for well damped systems  $IAE \approx IE$ .

Constraint to the robustness is expressed with maximum value  $M_s$  of sensitivity function of the closed-loop system given by

$$M_s = \max_{\omega} \left| \frac{1}{1+L(j\omega)} \right|, \quad (5)$$

while sensitivity to the modeling errors may be quantitatively described with maximum value  $M_p$  of complementary sensitivity function defined by

$$M_p = \max_{\omega} \left| \frac{L(j\omega)}{1+L(j\omega)} \right|. \quad (6)$$

For stable processes adequate values of  $M_s$  are in the range  $1.2 \leq M_s \leq 2$ , while  $M_p$  should be as less as it is possible to make system more robust.

According to the analysis from [29], it is suitable to use an additional constraint

$$Q = \max_{\omega} \left| \frac{\frac{k_i}{j\omega} G_p(j\omega)}{1+L(j\omega)} \right|, \quad (7)$$

which is actually introduced in order to minimize performance criterion  $\min_q J_v = 1.01/k_i$ , where

$$J_v = \max_{\omega} \left| \frac{G_p(j\omega)}{j\omega(1+L(j\omega))} \right| \quad (8)$$

as discussed in [6].

The feedback loop inevitably introduces noise which is especially large at high frequencies. As result undesired control signal can cause wear of actuators and deteriorate the system performance. Among different ways to characterize the measurement noise, in this paper it is used the maximum gain  $M_n$  of the transfer function from measurement noise  $n$  to the control signal  $u$  at high frequencies defined as

$$M_n \approx M_{n,\infty} = \lim_{\omega \rightarrow \infty} \left| \frac{C(j\omega)}{1+L(j\omega)} \right| = \frac{|k_d|}{T_f} \quad (9)$$

Inadequate sensitivity to measurement noise is the reason why derivative term is often excluded in process industry control and why it is used PI control.

Relative stability measures are characterized by the gain and phase margins  $g_m$  and  $\varphi_m$ , which are defined by relations (10) and (11), respectively

$$g_m = \frac{1}{|L(i\omega_\pi)|}, \quad \arg\{L(i\omega_\pi)\} = -\pi \quad (10)$$

where  $\omega_\pi$  is phase crossover frequency

$$\varphi_m = \pi + \arg\{L(i\omega_g)\}, \quad |L(i\omega_g)| = 1 \quad (11)$$

where  $\omega_g$  is gain crossover frequency. For unstable processes the Nyquist curve wraps from the left to the right around critical point, so it is adequate to define gain margin as minimum of left and right gain margin as  $g_m = \min(g_m^l, g_m^r)$ .

### B. The optimization problem setup of PID controller

The objective function of the optimization problem is given in the form of *Integral of Absolute Error – IAE*. Constraints are of inequality type expressed in terms of maximum desired values of sensitivity function  $M_s^d$ , complementary sensitivity functions  $M_p^d$  and value  $Q^d$  i.e. restriction used to limit the overshoot of the step response. Thus, unknown parameters of PID controller  $q = (k_p, k_i, k_d, T_f)$  are obtained by solving the optimization problem given in compact form as follow

$$\begin{aligned} \min_q \quad & IAE \\ & M_s \leq M_s^d \\ & M_p \leq M_p^d \\ & Q \leq Q^d \end{aligned} \quad (12)$$

The objective function is modified using penalty function approach to set up constraint-free optimization problem defined by

$$\min_q IAE + \lambda_{pf} \left[ (M_s > M_s^d) + (M_p > M_p^d) + (Q > Q^d) \right] \quad (13)$$

where  $\lambda_{pf}$  is penalty factor. Expressions of the form  $(X > X^d)$  in (13) are shorter notation of the functions defined with

$$(X > X^d) = \begin{cases} 1, & X > X^d \\ 0, & X < X^d \end{cases}, \quad (14)$$

where  $X \in \{M_s, M_p, Q\}$ . This is main idea of penalty approach: the more iteration points move outside the feasible region the more the penalty factor increases so in this paper is selected fixed factor  $\lambda_{pf} = 10^4$ .

The presented optimization procedure takes into account the relationship  $T_f = k_d/M_n$ , which is directly obtained from (9) for adequately tuned PID controller. Hence, initial requirements within the optimization procedure of PID controller design are given in the form  $M_s^d, M_p^d, M_n^d$  and  $Q^d$ . Desired values are specified according to those from Ref. [21] with aim to make relevant comparison with  $\max(k_p)$  method from [21]. Thereby desired values  $M_s^d, M_p^d, M_n^d$  are specified individually for each process from the class of test batch of typical industrial processes while the value  $Q^d = 1.01$  is the same for all processes since this value guarantees negligible overshoot in the step response.

### C. Particle Swarm optimization

The particle swarm optimization (PSO) is a global optimization technique originally developed by Kennedy and Eberhart in 1995 [51]. The PSO concept arises from studying behavior of bird individuals with aim to simulate their social behavior in swarm. The algorithm characterizes with a small number of parameters, possibility of simple implementation and computationally is inexpensive. PSO algorithm is efficient in solving complex multidimensional, non-differentiable, non-convex problems, problems with multiple optimums, i.e. multimodal problems, which is reason of its successful application in many branches of science and engineering [40-43,52,53].

Each particle of the population (swarm) is in each moment defined with its position  $x[k]$  and velocity  $v[k]$ . The current position of the particle is potential solution of considered multidimensional problem. If new position is better than previous one, the previous position is updated in each iteration. The best position of the particle is given with vector

$p[k]$ , while the best position of all individuals (global) in population is memorized in vector  $g[k]$ . The velocity of each particle in  $k$ -th iteration is determined with velocity in  $(k-1)$ -th iteration as

$$v[k] = w[k] \cdot v[k-1] + cp[k] \cdot rp[k] \cdot (p[k] - x[k]) + cg[k] \cdot rg[k] \cdot (g[k] - x[k]), \quad (15)$$

and position is defined by

$$x[k+1] = x[k] + v[k], \quad (16)$$

where following notation is used:  $w$  – inertial,  $cp$  – cognitive and  $cg$  – social factor, while  $rp$  and  $rg$  are random numbers from uniform distribution in the range  $[0,1]$ . Factor  $w$  is used to regulate impact of previous positions on the current position wherein the location of the particle for smaller values of  $w$  is limited to narrow search space (local search). For larger values of  $w$  search space is expanding (global search). Factor  $cp$  is used to simulate the impact of own experience from previous positions while  $cg$  simulates the impact of experience of swarm/surroundings on movement of individual particle. Table I presents some parameters of PSO algorithm from [47].

TABLE I. MAIN PARAMETERS OF THE PARTICLE-SWARM OPTIMIZATION ALGORITHM [47]

Parameters	Value	Explanation
npart	30	The number of particles
niter	60	The number of iterations.
cpi	2.5	Initial value of the individual-best factor.
cpf	0.5	Final value of the individual-best factor.
cgi	0.5	Initial value of the global-best factor.
cgf	2.5	Final value of the global-best factor.
wi	0.9	Initial value of the inertia factor.
wf	0.4	Final value of the inertia factor
vspaninit	1	The initial velocity span.
initoffset	$[k_p^0, k_i^0, k_d^0]$	Offset of the initial population.
initspan	1	Span of the initial population.

### III. SIMULATION ANALYSIS AND DISCUSSION

The effectiveness of the proposed PID controller design procedure is verified via numerical simulations on test batch consisting of large class of typical industrial processers: stable, integral, oscillatory and unstable including dead-time [9,15-19,22].

$$G_{p1}(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}, \quad G_{p2}(s) = \frac{1}{(s+1)^4}, \quad G_{p3}(s) = \frac{e^{-5s}}{(s+1)^3},$$

$$G_{p4}(s) = \frac{1-s}{(s+1)^3}, \quad G_{p5}(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)},$$

$$G_{p6}(s) = \frac{(2s+1)e^{-4s}}{(10s+1)(7s+1)(3s+1)}, \quad G_{p7}(s) = \frac{e^{-s}}{s^2+0.1s+1},$$

$$G_{p8}(s) = \frac{e^{-0.5s}}{s}, \quad G_{p9}(s) = \frac{4e^{-2s}}{4s-1},$$

$$G_{p10}(s) = \frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)}$$

In simulations the first order filter  $G_{ff}(s)=1/(T_{ff}s+1)$  is used to achieve fine shaping of reference signal. Filter time constant  $T_{ff}$  is selected as  $T_{ff} \in [1.5-2.5]/\omega_u$ , where  $\omega_u$  is the ultimate frequency of the process.

Figs. 2-11 show unit step reference responses with load disturbance  $D(s)=d_o/s$ , where  $d_o$  is the amplitude of the applied step signal in the closed-loop system in Fig. 1. Reference tracking and disturbance suppression performance of the closed-loop system with proposed PSO based tuning method of PID controller's parameters is compared with those obtained with  $\max(k_p)$  method from [21] where maximization of proportional gain is performed. Besides, the objective function in (13) is used to perform PID optimization with another two algorithms: genetic algorithm (GA) and *fminunc*-Trust Region based algorithm implemented in MATLAB. Thus Figs. 2-11 show comparison of responses of four PID optimization methods denoted with PSO,  $\max(k_p)$ , GA and TR.

Figs. 2-11 demonstrate effectiveness of the proposed PSO design procedure of PID controller. For all processes desired maximum values  $M_s$  of sensitivity function are obtained, except for oscillatory process  $G_{p7}$  where for constraint  $M_s^d = 2$  more robust system is obtained with  $M_s=1.7$ . Besides, responses in Figs. 2-11 show that algorithms GA and TR are equal with PSO from the point view of achieved adequate performance and robustness indices. Parameters of PID controller and obtained performance, stability and robustness indices are presented in Table 2. As it can be seen, PSO, GA and TR methods are fully comparable with widely accepted  $\max(k_p)$  optimization method. It should be noted that efficient suppression of load disturbance is obtained for all methods which is validated with approximately the same values of IAE.

Convergence analysis of presented methods is a beyond the scope of this paper, but it should be noted it is of great importance to know adequate range of unknown parameters of PID controller to initialize properly particle swarm as well as other algorithms. Hence, initial settings of PSO (initoffset in Table I etc.) and other heuristic algorithms affects their search ability, but it can be additionally improved with e.g. selection of inertial weighting factors in objective functions etc.

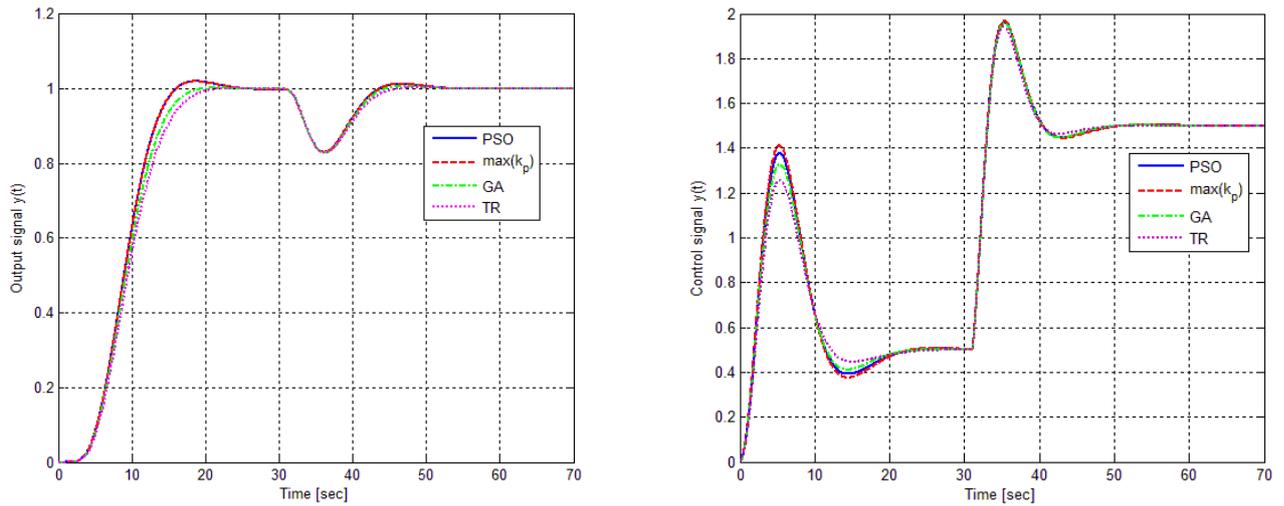


Figure 2. The reference step response of process  $G_{p1}(s)$  with step load disturbance of amplitude  $d_0=1$  starting from  $t=30$  sec (left), and control signal  $u(t)$  (right)

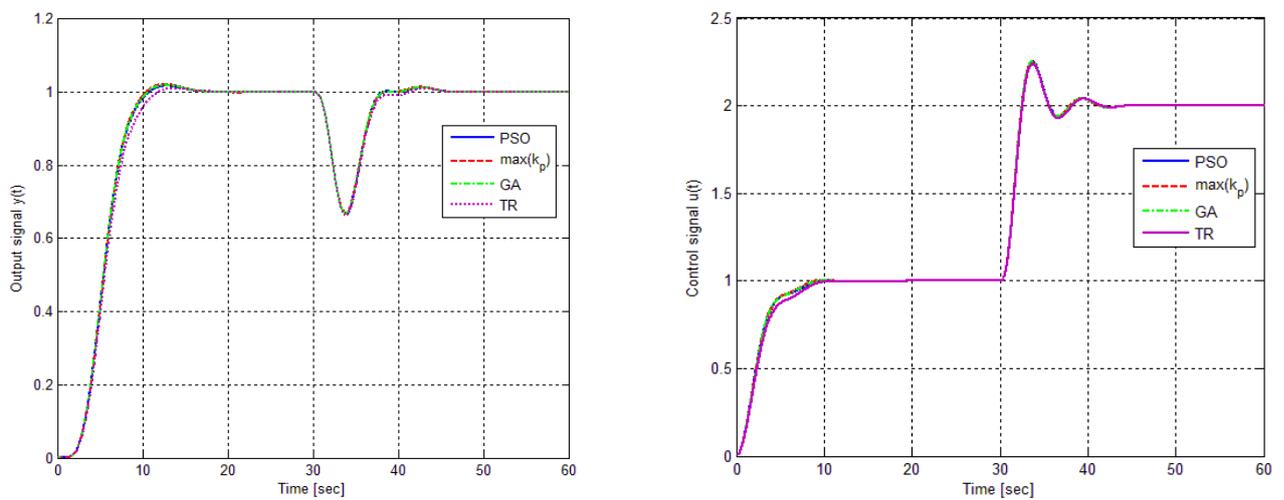


Figure 3. The reference step response of process  $G_{p2}(s)$  with step load disturbance of amplitude  $d_0=1$  starting from  $t=30$  sec (left), and control signal  $u(t)$  (right)

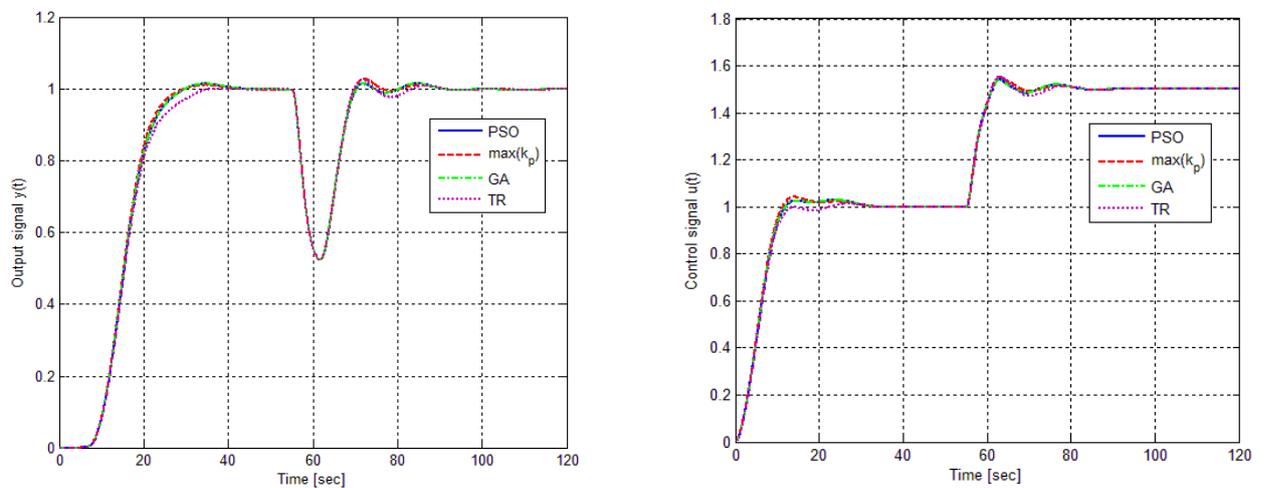


Figure 4. The reference step response of process  $G_{p3}(s)$  with step load disturbance of amplitude  $d_0=1$  starting from  $t=50$  sec (left), and control signal  $u(t)$  (right)

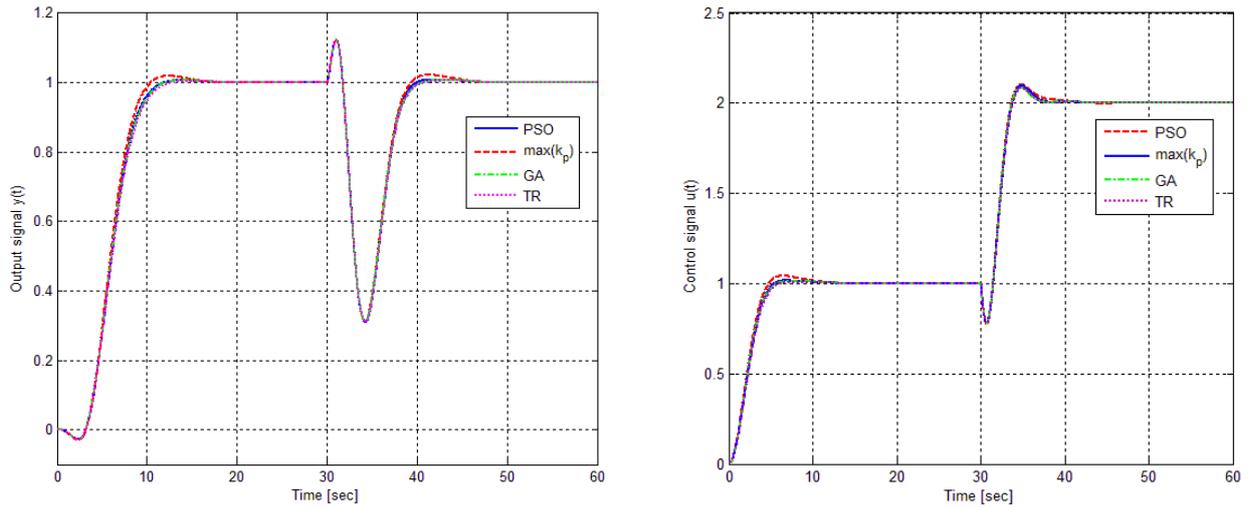


Figure 5. The reference step response of process  $G_{pd}(s)$  with step load disturbance of amplitude  $d_0=1$  starting from  $t=30$  sec (left), and control signal  $u(t)$  (right)

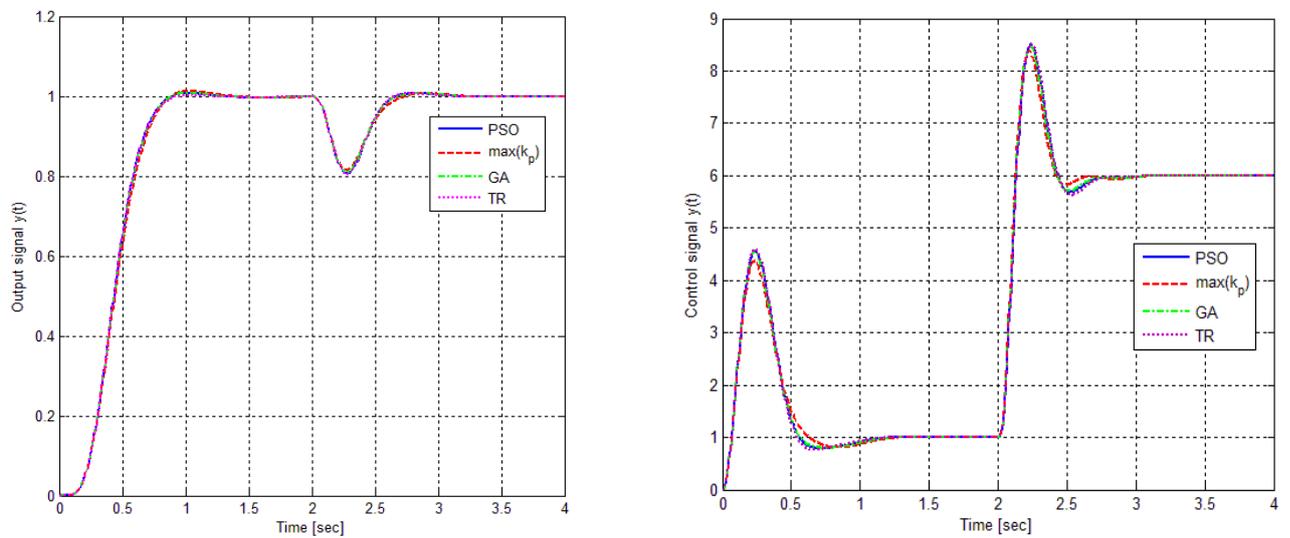


Figure 6. The reference step response of process  $G_{ps}(s)$  with step load disturbance of amplitude  $d_0=5$  starting from  $t=2$  sec (left), and control signal  $u(t)$  (right)

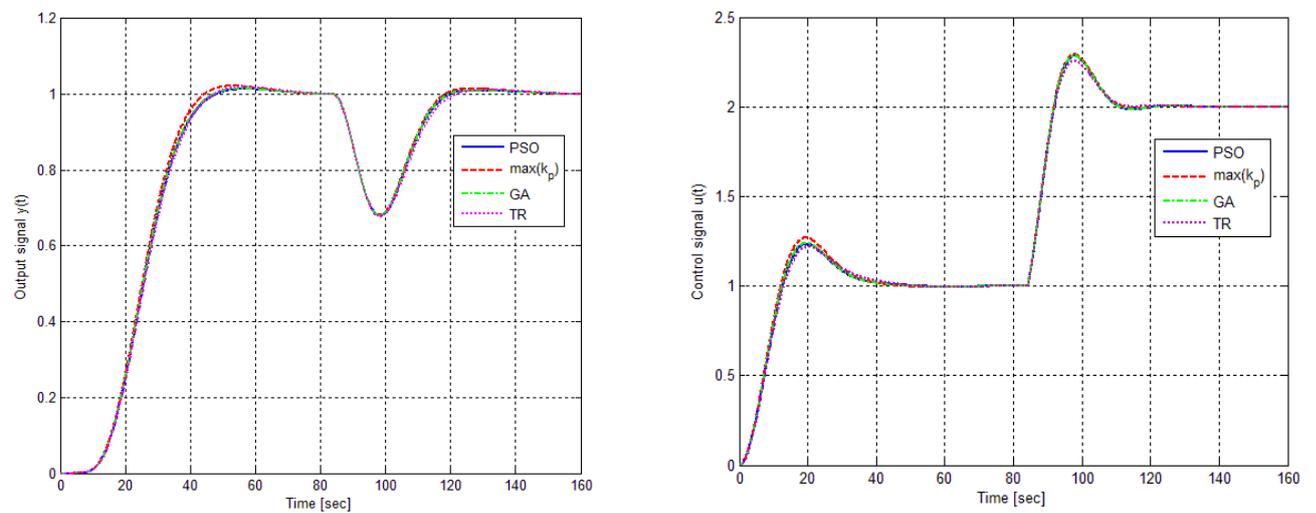


Figure 7. The reference step response of process  $G_{po}(s)$  with step load disturbance of amplitude  $d_0=1$  starting from  $t=80$  sec (left), and control signal  $u(t)$  (right)

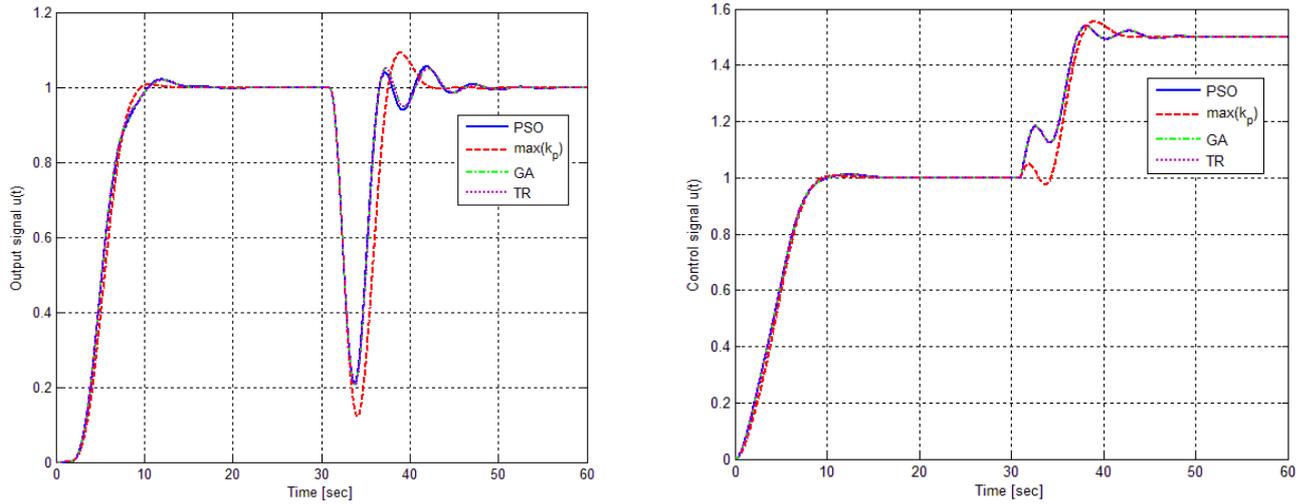


Figure 8. The reference step response of process  $G_{p7}(s)$  with step load disturbance of amplitude  $d_0=0.5$  starting from  $t=30$  sec (left), and control signal  $u(t)$  (right)

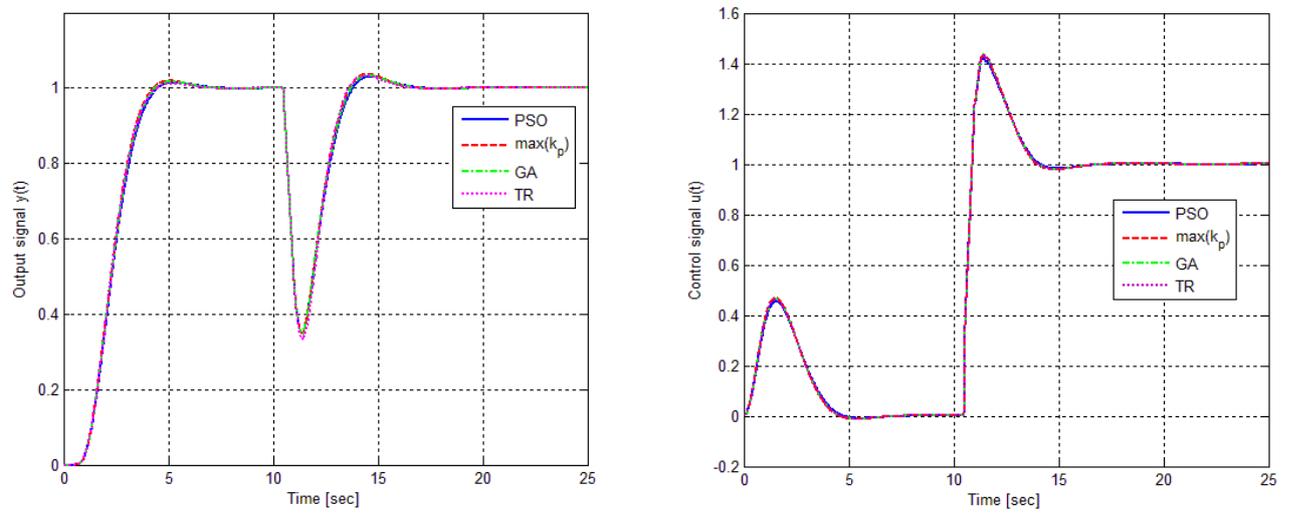


Figure 9. The reference step response of process  $G_{p8}(s)$  with step load disturbance of amplitude  $d_0=1$  starting from  $t=10$  sec (left), and control signal  $u(t)$  (right)

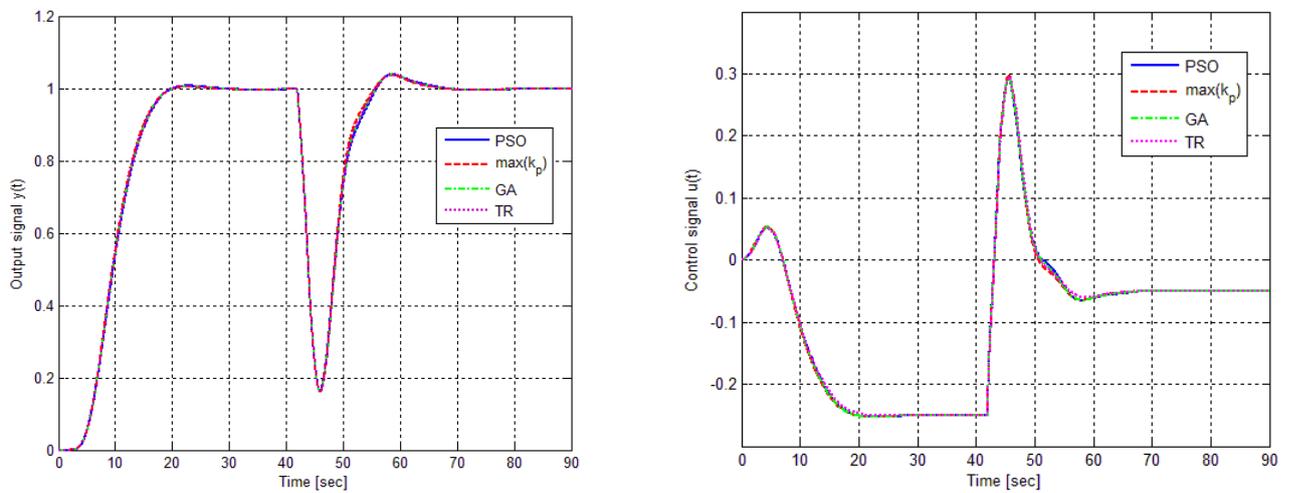


Figure 10. The reference step response of process  $G_{p9}(s)$  with step load disturbance of amplitude  $d_0=0.2$  starting from  $t=40$  sec (left), and control signal  $u(t)$  (right)

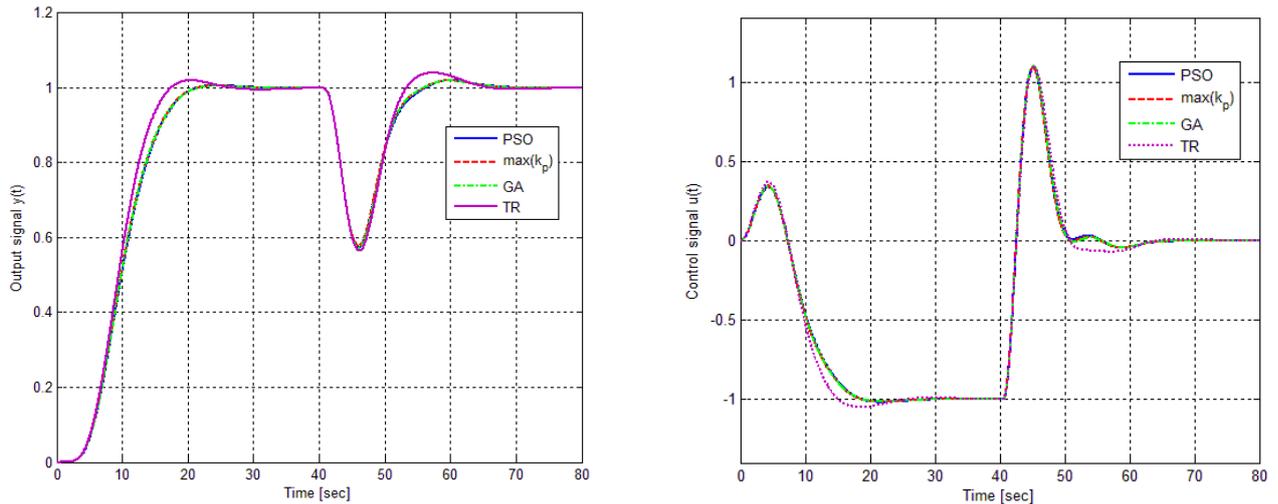


Figure 11. The reference step response of process  $G_{p10}(s)$  with step load disturbance of amplitude  $d_0=0.2$  starting from  $t=40$  sec (left), and control signal  $u(t)$  (right)

TABLE II. COMPARISON OF PERFORMANCE AND ROBUSTNESS INDICES OF THE PSO BASED PID CONTROLLER WITH THOSE OBTAINED USING MAX(KP), GENETIC ALGORITHM (GA) AND TRUST REGION (TR) BASED OPTIMIZATION METHODS

Process	$\omega_n$	$k_p$	$k_i$	$k_d$	$T_f$	IAE	$M_n$	$M_s$	$M_p$	$g_m$	$\varphi_m$
$G_{p1}(s)$ – PSO	0.5389	5.6586	0.9627	11.9741	0.1859	1.1622	64.42	2.00	1.70	2.56	34.72
$G_{p1}(s)$ – max( $k_p$ )		5.6539	0.9894	12.0512	0.1871	1.1633	64.41	2.00	1.71	2.55	34.72
$G_{p1}(s)$ – GA		5.6616	0.9183	11.7059	0.1817	1.1644	64.42	2.00	1.70	2.59	34.55
$G_{p1}(s)$ – TR		5.6687	0.8754	12.0079	0.1864	1.1847	64.42	2.00	1.64	2.55	35.66
$G_{p2}(s)$ – PSO	1.0000	2.3052	0.8585	2.5697	0.0982	1.2489	26.16	2.00	1.34	2.75	47.31
$G_{p2}(s)$ – max( $k_p$ )		2.3057	0.8682	2.6478	0.1012	1.2499	26.16	2.00	1.34	2.73	47.92
$G_{p2}(s)$ – GA		2.3060	0.8624	2.6144	0.0999	1.2492	26.16	2.00	1.34	2.74	47.68
$G_{p2}(s)$ – TR		2.3005	0.8103	2.5052	0.0958	1.2945	26.16	2.00	1.34	2.77	47.79
$G_{p3}(s)$ – PSO	0.4000	0.6260	0.1341	1.3397	0.8373	8.0894	1.60	1.97	1.01	1.89	65.51
$G_{p3}(s)$ – max( $k_p$ )		0.6302	0.1363	1.2675	0.8178	8.1519	1.55	2.00	1.02	1.93	64.31
$G_{p3}(s)$ – GA		0.6264	0.1342	1.3421	0.8388	8.0992	1.60	2.00	1.01	1.89	65.49
$G_{p3}(s)$ – TR		0.6519	0.1314	1.2015	0.7509	8.1145	1.60	2.00	1.01	1.89	66.37
$G_{p4}(s)$ – PSO	1.0000	1.0141	0.4088	0.8056	0.1007	2.7533	8.00	2.00	1.15	2.14	53.82
$G_{p4}(s)$ – max( $k_p$ )		1.0145	0.4268	0.8446	0.1021	2.7964	8.27	2.00	1.15	2.13	52.38
$G_{p4}(s)$ – GA		1.0193	0.4075	0.8271	0.1000	2.7586	8.27	2.00	1.14	2.12	54.53
$G_{p4}(s)$ – TR		1.0093	0.4010	0.7999	0.0967	2.7892	8.27	1.99	1.14	2.16	54.83
$G_{p5}(s)$ – PSO	11.1803	25.1076	91.9705	2.7106	0.0108	0.0124	250.00	1.99	1.67	4.11	33.94
$G_{p5}(s)$ – max( $k_p$ )		25.6860	92.2890	2.9116	0.0105	0.0121	265.60	2.00	1.68	3.81	34.61
$G_{p5}(s)$ – GA		25.4409	93.3127	2.7605	0.0106	0.0120	260.00	2.00	1.74	4.02	33.81
$G_{p5}(s)$ – TR		25.1522	92.0173	2.6323	0.0101	0.0121	260.00	2.00	1.68	4.21	33.16
$G_{p6}(s)$ – PSO	0.2144	2.5879	0.1894	11.9613	0.4430	5.7030	27.00	2.00	1.35	2.34	44.44
$G_{p6}(s)$ – max( $k_p$ )		2.5929	0.1969	12.1891	0.4367	5.6914	27.91	2.00	1.35	2.33	43.92
$G_{p6}(s)$ – GA		2.5947	0.1910	12.0001	0.4300	5.6817	27.91	2.00	1.34	2.34	44.22
$G_{p6}(s)$ – TR		2.5133	0.1844	12.0001	0.4300	5.9157	27.91	1.95	1.27	2.39	46.34
$G_{p7}(s)$ – PSO	1.0304	-0.1490	0.2258	0.4605	0.1439	5.1116	3.20	1.75	1.09	2.81	56.15
$G_{p7}(s)$ – max( $k_p$ )		-0.2986	0.1795	0.2536	0.0798	6.5257	3.18	2.00	1.01	1.99	59.39
$G_{p7}(s)$ – GA		-0.1541	0.2255	0.4429	0.1393	5.1270	3.18	1.67	1.02	2.99	60.55
$G_{p7}(s)$ – TR		-0.1541	0.2255	0.4429	0.1393	5.4680	3.18	1.61	1.00	2.99	60.55
$G_{p8}(s)$ – PSO	3.1416	1.6237	1.0188	0.3393	0.0283	1.0787	12.00	2.00	1.60	2.09	38.15
$G_{p8}(s)$ – max( $k_p$ )		1.6232	1.0499	0.3391	0.0284	1.0696	11.91	2.00	1.62	2.09	37.47
$G_{p8}(s)$ – GA		1.6244	1.0370	0.3358	0.0280	1.0706	12.00	2.00	1.62	2.09	37.58
$G_{p8}(s)$ – TR		1.6023	1.0208	0.2950	0.0246	1.0798	12.00	2.00	1.67	2.17	35.95
$G_{p9}(s)$ – PSO	0.5828	0.5319	0.0437	0.5281	0.2200	25.3966	2.40	3.97	3.20	1.38	18.29
$G_{p9}(s)$ – max( $k_p$ )		0.5340	0.0446	0.5179	0.2149	24.7500	2.41	4.00	3.27	1.39	17.67
$G_{p9}(s)$ – GA		0.5311	0.0437	0.5198	0.2157	25.3175	2.41	3.94	3.20	1.39	18.05
$G_{p9}(s)$ – TR		0.5311	0.0421	0.4998	0.2074	25.4432	2.41	3.94	3.27	1.40	17.66
$G_{p10}(s)$ – PSO	0.4287	3.4891	0.3836	7.0282	0.3514	2.8860	20.00	3.00	2.61	1.92	22.14
$G_{p10}(s)$ – max( $k_p$ )		3.5178	0.3921	7.0284	0.3417	2.8203	20.57	3.00	2.61	1.91	21.75
$G_{p10}(s)$ – GA		3.4877	0.3848	6.9436	0.3472	2.8632	20.00	3.00	2.61	1.93	21.94
$G_{p10}(s)$ – TR		3.3688	0.4141	6.7973	0.3399	3.0068	20.00	2.87	2.61	2.01	22.28

## IV. CONCLUSION

The suggested particle swarm method for tuning PID controller effectively solves the optimization problem of PID controller parameters under constraints on performance and robustness. Solution to the problem gives the minimum of Integral of Absolute Error which leads to efficient suppression of load disturbance as well as adequate robustness indices of the closed loop system. The effectiveness of the presented design method is verified through numerical simulations on large class of industrial process including stable, integral and unstable processes with and without transport delay. Paper also provides solution to the optimization problem using genetic algorithm and trust region based search optimization algorithms which both give comparable results with PSO algorithm. Efficiency of the proposed algorithms is validated through comparison of presented optimization algorithms with recently proposed and widely accepted  $\max(k_p)$  method. As a result, it can be concluded that optimization problem is adequately set up and the presented solutions of the optimization problem are adequate since desired performance/robustness indices are obtained. The same approach can be used to optimize closed-loop system performance with PI controller, lead and lag compensators, or even high-order controllers as well as fractional-order controllers.

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