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(S,T)-NORMED INTUITIONISTIC FUZZY $\beta-$SUBALGEBRAS

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ABSTRACT. This paper deals about Intuitionistic Fuzzy $\beta-$subalgebras of $\beta-$algebras using (S,T) norms. Further the notion of product and level subset on Intuitionistic Fuzzy $\beta-$subalgebras of $\beta-$algebras using (S,T) norms are introduced. Some interesting and elegant related results are being discussed.

1. Introduction

In 2002, J.Neggers and Kim [4], [5] introduced new class of algebras: $\beta-$ algebras arising from the classical and non-classical propositional logic. In 1965, L.A.Zadeh [9] introduced a notion of fuzzy sets. The notion of fuzzy algebraic structures was initiated by A.Rosenfeld [7], K.T.Atanassov [2], introduced the notion of intuitionistic fuzzy set a generalization of fuzzy set.

Recently, in 2013 the authors introduced the concept of Fuzzy $\beta-$subalgebras of $\beta-$algebras [1]. Motivated by this the authors [8] introduced Intuitionistic Fuzzy $\beta-$subalgebras. T-norms were introduced by Schweizer and Sklar in 1961. In 2007, Kyong. Ho. Kim, introduced Intuitionistic (T, S) normed subalgebras of BCK-algebras [3]. In this paper, we introduce (S, T) normed Intuitionistic Fuzzy $\beta-$subalgebras and some properties and simple results.

The paper has been organised as follows: Section 2 provides the preliminaries. In section 3 (S, T) - Intuitionistic fuzzy $\beta-$ subalgebra is discussed, in section 4, Product of (S, T) - Intuitionistic fuzzy β - subalgebras is studied and section 5 gives the notion of Level subset of (S,T) Intuitionistic fuzzy β -subalgebras. Finally the section 6 ends with the conclusion.

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2. Preliminaries

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1. A $\beta-$algebra is a non-empty set X with a constant 0 and two binary operations $+$ and $-$ satisfying the following axioms:

(1) $x - 0 = x$
(2) $(0 - x) + x = 0$
(3) $(x - y) - z = x - (z + y) \ \forall \ x, y, z \ \in \ X$.

Example 2.2. From the following Caley's tables, $(X = \{0, 1, 2\}, +, -, 0)$ is a $\beta-$ algebra.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

−	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Definition 2.3. The function $S : [0, 1] \times [0, 1] \to [0, 1]$ is called a S-norm, if it satisfies the following conditions,

(1) $S(x, 1) = x$
(2) $S(x, y) = S(y, x)$
(3) $S(S(x, y), z) = S(x, S(y, z))$
(4) $S(x, y) \leqslant S(x, z)$ if $y \leqslant z \ \forall \ x, y, z \ \in \ [0, 1]$.

If norm S has the property,$S(x, y) = min(x, y)$, then

(1) $S(x, 0) = 0$
(2) $S(S(u, v), S(x, y)) = S(S(u, x), S(v, y)) \ \forall \ x, y, u, v \ \in \ [0, 1]$
(3) $S(x, x) = x$.

Definition 2.4. The function $T : [0, 1] \times [0, 1] \to [0, 1]$ is called a T-norm, if it satisfies the following conditions,

(1) $T(x, 0) = x$
(2) $T(x, y) = T(y, x)$
(3) $T(T(x, y), z) = T(x, T(y, z))$
(4) $T(x, y) \leqslant T(x, z)$ if $y \leqslant z \ \forall \ x, y, z \ \in \ [0, 1]$.

If norm T has the property,$T(x, y) = max(x, y)$, then

(1) $T(x, 0) = x$
(2) $T(T(u, v), T(x, y)) = T(T(u, x), T(v, y)) \ \forall \ x, y, u, v \ \in \ [0, 1]$
(3) $T(x, x) = x$.

Definition 2.5. Let $(X, *, 0)$ be any algebra. Let μ be a fuzzy set with respect to S-norm [T-norm] is said to be a $S - Fuzzy\ subalgebra[T - Fuzzy\ subalgebra]$ of X, if $\mu(x * y) \geqslant S(\mu(x), \mu(y))$ $[\mu(x * y) \leqslant T(\mu(x), \mu(y))]$, $\forall \ x, y \ \in \ X$.

Definition 2.6. Let $(X, *, 0)$ be any algebra. An Intuitionistic fuzzy set $A = \{x, \mu_A(x), \nu_A(x) \mid x \in \ X\}$ is called a (S,T) Intuitionistic Fuzzy subalgebra of X, if it satisfies the following conditions,

(1) $\mu_A(x * y) \geqslant S(\mu_A(x), \mu_A(y))$

(2) $\nu_A(x * y) \leqslant T(\nu_A(x), \nu_A(y))$, $\forall\ x, y\ \in\ X$, where $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$

DEFINITION 2.7. Let A be an Intuitionistic Fuzzy subaset of X, and $s, t \in\ [0, 1]$. Then $A_{s,t}$ $=\{x, \mu_A(x) \geqslant s, \nu_A(x) \leqslant t \mid x \in\ X\}$ where $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ is called a level intuitionistic fuzzy subset of X.

3. (S,T)- Intuitionistic fuzzy $\beta-$ subalgebras

In this section we introduce the notion of (S,T)- Intuitionistic fuzzy $\beta-$ subalgebra of a $\beta-$ algebra and prove some related results. Also, in the rest of the paper, X is a $\beta-$algebra unless, otherwise specified.

DEFINITION 3.1. Let $(X, +, -, 0)$ be a β algebra. An Intuitionistic fuzzy set $A = \{x, \mu_A(x), \nu_A(x) \mid x \in\ X\}$ is called a (S,T)- Intuitionistic fuzzy β subalgebra of X, if it satisfies the following conditions.

(1) $\mu_A(x + y) \geqslant S(\mu_A(x), \mu_A(y))$ and $\nu_A(x + y) \leqslant T(\nu_A(x), \nu_A(y))$
(2) $\mu_A(x-y) \geqslant S(\mu_A(x), \mu_A(y))$ and $\nu_A(x-y) \leqslant T(\nu_A(x), \nu_A(y))$,$\forall\ x, y \in\ X$, where $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$.

EXAMPLE 3.2. Let $X = \{0, 1, 2, 3\}$ be a $\beta-$algebra with constant 0 and two binary operations + and - are defined on X with the Cayley's table

$+$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	1	0
3	3	2	0	1

$-$	0	1	2	3
0	0	1	3	2
1	1	0	2	3
2	2	3	0	1
3	3	2	1	0

Now, A is defined as,

$$\mu_A(x) = \begin{cases} .7 & x = 0, 1 \\ .6 & otherwise \end{cases} \quad and \quad \nu_A(x) = \begin{cases} .1 & x = 0, 1 \\ .3 & otherwise \end{cases}$$

Then A is (S, T) Intuitionistic fuzzy $\beta-$subalgebra of X.

THEOREM 3.3. *Let A be (S, T) Intuitionistic fuzzy $\beta-$ subalgebra of X. Let $\chi_A = \{\mu(x) = \mu(0)$ and $\nu(x) = \nu(0)\}$. Then χ_A is a $\beta-$ subalgebra of X.*

PROOF. For any $x, y \in \chi_A$, we have $\mu(x) = \mu(0) = \mu(y)$ *and* $\nu(x) = \nu(0) = \nu(y)$. Now,

$$\mu_A(x + y) \geqslant S(\mu_A(x), \mu_A(y)) = S(\mu_A(0), \mu_A(0)) = \mu(0) \cdots (1)$$

and

$$\mu_A(x - y) \geqslant S(\mu_A(x), \mu_A(y)) = S(\mu_A(0), \mu_A(0)) = \mu(0) \cdots (2)$$
$$\mu_A(0) = \mu(0 + 0) \geqslant S(\mu_A(0), \mu_A(0)) = S(\mu_A(x), \mu_A(y)) = \mu_A(x + y) \cdots (3)$$
$$\mu_A(0) = \mu(0 - 0) \geqslant S(\mu_A(0), \mu_A(0)) = S(\mu_A(x), \mu_A(y)) = \mu_A(x - y) \cdots (4)$$

(1) and (3) implies $\mu_A(x+y) = \mu_A(0)$, and (2) and (4) implies $\mu_A(x-y) = \mu_A(0)$. Hence $\mu_A(x-y) = \mu_A(0) = \mu_A(x+y)$. Similarly, we can prove for non membership function, $\nu_A(x-y) = \nu_A(0) = \nu_A(x+y)$. Thus $x + y$ *and* $x - y \in \chi_A$ proving that χ_A is $\beta-$ subalgebra of X. □

From above theorem we can obtain the following

Corollary 3.4. *Let A be (S,T) Intuitionistic fuzzy $\beta-$ subalgebra of X. Let $\chi_A = \{\mu(x) = \mu(0)$ and $\nu(x) = 1 - \nu(0)\}$. Then χ_A is a $\beta-$ subalgebra of X.*

One can easily prove the following

Theorem 3.5. *Let A and B be (S,T) Intuitionistic fuzzy $\beta-$ subalgebras of X. Then $A \cap B$ is also a (S,T) Intuitionistic fuzzy β- subalgebra of X. In general, the intersection of a family of (S,T) Intuitionistic fuzzy $\beta-$ subalgebras of X is also a (S,T) Intuitionistic fuzzy $\beta-$ subalgebra of X.*

Theorem 3.6. *If A is (S,T) Intuitionistic fuzzy $\beta-$ subalgebra of X, then $\mu(x) \leqslant \mu(x-0)$ and $\nu(x) \geqslant \nu(x-0)$.*

Proof.

$$\begin{aligned} \mu_A(x-0) &\geqslant S(\mu_A(x), \mu_A(0)) \\ &= S(\mu_A(x), \mu_A(x-x)) \\ &\geqslant S\{(\mu_A(x), S(\mu_A(x), \mu_A(x))\} \\ &= S(\mu_A(x), \mu_A(x)) \\ &= \mu_A(x). \end{aligned}$$

Corresponding to T-norm, we can prove that, $\nu_A(x-0) \leqslant \nu_A(x)$. □

Theorem 3.7. *If A is a (S,T) Intuitionistic fuzzy $\beta-$subalgebraof X, then μ_A and $\overline{\nu}_A$ are S-fuzzy $\beta-$subalgebras of X.*

Proof. Let $A = (\mu, \nu)$ be a (S,T) Intuitionistic fuzzy $\beta-$subalgebra of X. Clearly, μ_A is a S-fuzzy $\beta-$subalgebra of X. For every $x, y \in X$, we have

$$\begin{aligned} \overline{\nu}_A(x+y) &= 1 - \nu_A(x+y) \\ &\leqslant 1 - T\{\nu_A(x), \nu_A(y)\} \\ &\geqslant S\{1 - \nu_A(x), 1 - \nu_A(y)\} \\ &\geqslant S\{\overline{\nu}_A(x), \overline{\nu}_A(y)\} \end{aligned}$$

Similarly, we can prove that $\overline{\nu}_A(x-y) \geqslant min\{\overline{\nu}_A(x), \overline{\nu}_A(y)\}$. Hence $\overline{\nu}_A$ is a S-fuzzy $\beta-$subalgebra of X. □

Theorem 3.8. *If μ_A and $\overline{\nu}_A$ are S-fuzzy $\beta-$subalgebras of X, then $A = (\mu_A, \nu_A)$ is a (S,T) IF$\beta-$subalgebra of X.*

Proof. Let μ_A and $\overline{\nu}_A$ be S-fuzzy $\beta-$subalgebras of X. We get $\mu_A(x+y) \geqslant S\{\mu_A(x), \mu_A(y)\}$ *and* $\mu_A(x-y) \geqslant S\{\mu_A(x), \mu_A(y)\}$ Now, for every $x, y \in X$, we have

$$\begin{aligned} 1-\nu_A(x+y) & \\ &= \overline{\nu}_A(x+y) \\ &\geqslant S\{\overline{\nu}_A(x),\overline{\nu}_A(y)\} \\ &= S\{1-\nu_A(x),1-\nu_A(y)\} \\ &= 1-T\{\nu_A(x),\nu_A(y)\} \end{aligned}$$

That is, $\nu_A(x+y)\leqslant T\{\nu_A(x),\nu_A(y)\}$. Similarly, we can prove that, $\nu_A(x-y)\leqslant T\{\nu_A(x),\nu_A(y)\}$. Hence $A=(\mu_A,\nu_A)$ is a (S,T) Intuitionistic fuzzy $\beta-$subalgebras of X. □

DEFINITION 3.9. Let $f:X\to Y$ be a $\beta-$homomorphism. Let A *and* B be two (S,T) Intuitionistic fuzzy $\beta-$subalgebras in X and Y respectively. Then inverse image of B under f is defined by

$$f^{-1}(B)=\left\{f^{-1}(\mu_B(x)),f^{-1}(\nu_B(x))|x\in X\right\}$$

such that $f^{-1}(\mu_B(x))=(\mu_B(f(x)))$ and $f^{-1}(\nu_B(x))=(\nu_B(f(x)))$.

THEOREM 3.10. *Let $f:X\to Y$ be a $\beta-$homomorphism. If A is a (S,T) IF$\beta-$subalgebra of Y, then $f^{-1}(A)$ is a (S,T) IF$\beta-$subalgebra of X.*

PROOF. Let A be a (S,T) IF$\beta-$subalgebra of Y, $x,y\in Y$.

$$\begin{aligned} f^{-1}(\mu_A(x+y)) &= \mu_A(f(x+y)) \\ &= \mu_A(f(x)+f(y)) \\ &\geqslant S\{\mu_A(f(x)),\mu_A(f(y))\} \\ &= S\left\{f^{-1}(\mu_A(x)),f^{-1}(\mu_A(y))\right\} \end{aligned}$$

and $f^{-1}(\mu_A(x-y))\geqslant S\left\{f^{-1}(\mu_A(x)),f^{-1}(\mu_A(y))\right\}$. Similarly, we can prove,

$$\begin{aligned} f^{-1}(\nu_A(x+y)) &= \nu_A(f(x+y)) \\ &= \nu_A(f(x)+f(y)) \\ &\leqslant T\{\nu_A(f(x)),\nu_A(f(y))\} \\ &= T\left\{f^{-1}(\nu_A(x)),f^{-1}(\nu_A(y))\right\} \end{aligned}$$

and $f^{-1}(\nu_A(x-y))\leqslant T\left\{f^{-1}(\nu_A(x)),f^{-1}(\nu_A(y))\right\}$. Hence $f^{-1}(A)$ is a (S,T) IF$\beta-$subalgebra of X. □

THEOREM 3.11. *Let X and Y be two $\beta-$subalgebras. Let $f:X\to Y$ be an endomorphism of $\beta-$algebra. If A is (S,T) IF$\beta-$subalgebra of X, then $f(A)$ is a (S,T) IF$\beta-$subalgebra of Y.*

PROOF. Let A be a (S,T) IF$\beta-$subalgebra of Y, $x,y\in X$.

$$\begin{aligned} \mu_f(x+y) &= \mu(f(x+y)) \\ &= \mu(f(x))+\mu(f(y)) \\ &\geqslant S\{\mu(f(x)),\mu(f(y))\} \\ &= S\{\mu_f(x),\mu_f(y)\} \end{aligned}$$

and $(\mu_f(x-y)) \geqslant S\{\mu_f(x)), \mu_f(y))\}$ Similarly, we can prove that

$$\begin{aligned}\nu_f(x+y) &= \nu(f(x+y)) \\ &= \nu(f(x)) + \nu(f(y)) \\ &\leqslant T\{\nu(f(x)), \nu(f(y))\} \\ &= T\{\nu_f(x), \nu_f(y)\}\end{aligned}$$

and $\nu_f(x-y) \leqslant T\{\nu_f(x), \nu_f(y)\}$. Hence $f(A)$ is a (S,T) IF$\beta-$subalgebra of Y. □

4. Product of (S,T)- Intuitionistic fuzzy $\beta-$ subalgebras

In this section, we discuss the product of (S,T)- Intuitionistic fuzzy $\beta-$ subalgebras

Definition 4.1. Let $A = \{x, \mu_A(x), \nu_A(x) \mid x \in X\}$ and $B = \{x, \mu_B(x), \nu_B(x) \mid x \in Y\}$ be two (S,T)- Intuitionistic fuzzy $\beta-$ subalgebras of X and Y respectively. Then we define

$$A \times B = \{(\mu_A \times \mu_B)(x,y) \; and \; (\nu_A \times \nu_B)(x,y) \mid x, y \in X \times Y\}$$

where

$$(\mu_A \times \mu_B)(x,y) = S(\mu_A(x), \mu_B(y)) \text{ and } (\nu_A \times \nu_B)(x,y) = T(\nu_A(x), \nu_B(y)).$$

Theorem 4.2. *Let A and B be (S,T) Intuitionistic fuzzy $\beta-$ subalgebras of X and Y respectively. Then $A \times B$ is a (S,T)- Intuitionistic fuzzy $\beta-$ subalgebra of $X \times Y$.*

Proof. Take $x = (x_1, x_2), y = (y_1, y_2) \in X \times Y$ and $\mu = \mu_A \times \mu_B$ and $\nu = \nu_A \times \nu_B$.

$$\begin{aligned}\mu(x+y) &= \mu((x_1,x_2)+(y_1,y_2)) \\ &= (\mu_A \times \mu_B)(x_1+y_1),(x_2+y_2)) \\ &= min\{\mu_A(x_1+y_1), \mu_B(x_2+y_2)\} \\ &\geqslant min\{S(\mu_A(x_1), \mu_A(y_1)), S(\mu_B(x_2), \mu_B(y_2))\} \\ &= min\{S(\mu_A(x_1), \mu_B(x_2)), S(\mu_A(y_1), \mu_B(y_2))\} \\ &= S\{(\mu_A \times \mu_B)(x_1,x_2), (\mu_A \times \mu_B)(y_1,y_2)\} \\ &= S\{(\mu_A \times \mu_B)(x), (\mu_A \times \mu_B)(y)\}\end{aligned}$$

Similarly, $\mu(x-y) \geqslant S\{(\mu_A \times \mu_B)(x), (\mu_A \times \mu_B)(y)\}$. Analogously, we can prove for the non-membership function,

$$\nu(x+y) \leqslant T\{(\nu_A \times \nu_B)(x), (\mu_A \times \nu_B)(y)\}$$

and $\nu(x-y) \leqslant T\{(\nu_A \times \nu_B)(x), (\mu_A \times \nu_B)(y)\}$ proving the theorem. □

Corollary 4.3. *Let $A_1, \cdots, A_n$ be (S,T) Intuitionistic fuzzy $\beta-$ subalgebras of $X_1, \cdots, X_n$ respectively. Then $\prod_{i=1}^{n} A_i$ is also a (S,T) Intuitionistic fuzzy $\beta-$ subalgebra of $\prod_{i=1}^{n} X_i$.*

5. Level subset of (S,T) Intuitionistic fuzzy $\beta-$subalgebras

In this section we intend to apply the notion level intuitionistic fuzzy subset on (S,T) Intuitionistic fuzzy $\beta-$subalgebras.

DEFINITION 5.1. Let A be (S,T) Intuitionistic fuzzy$\beta-$subalgebras of X. Let $\alpha, \beta \in [0,1]$. Then

$$A_{\alpha,\beta} = \{x, \mu_A(x) \geqslant \alpha, \nu_A(x) \leqslant \beta \mid x \in X\},$$

where $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$, is called a level subset of (S,T) IF-β subalgebra A. The level subset of (S,T) IF$\beta-$subalgebra $A \times B$ of $X \times X$ is defined as,

$$(A \times B)_{\alpha,\beta} = \{(\mu_A \times \mu_B)(x,y) \geqslant \alpha \text{ and } (\nu_A \times \nu_B)(x,y) \leqslant \beta \mid x, y \in X \times Y\}$$

where $(\mu_A \times \mu_B)(x,y) = S(\mu_A(x), \mu_B(y))$ and $(\nu_A \times \nu_B)(x,y) = T(\nu_A(x), \nu_B(y)$.

THEOREM 5.2. *If $A = \{x, \mu_A(x), \nu_A(x) \mid x \in X\}$ is a (S,T) IF$\beta-$subalgebra of X, then the set $A_{\alpha,\beta}$ is $\beta-$subalgebra of X, for every $\alpha, \beta \in [0,1]$.*

PROOF. Let $x, y \in A_{\alpha,\beta}$. It is implies $\mu_A(x) \geqslant \alpha$, $\mu_A(y) \geqslant \alpha$ and $\nu_A(x) \leqslant \beta$, $\nu_A(y) \leqslant \beta$. Further on, have

$$\mu_A(x+y) \geqslant S\{\mu_A(x), \mu_A(y)\} \geqslant S\{\alpha, \alpha\} = \alpha \cdots (1),$$
$$\nu_A(x+y) \leqslant T\{\nu_A(x), \nu_A(y)\} \leqslant T\{\beta, \beta\} = \beta \cdots (2).$$

From (1) and (2) we get $x + y \in A_{\alpha,\beta}$. In a similar way one can prove that $x - y \in A_{\alpha,\beta}$, proving that $A_{\alpha,\beta}$ is $\beta-$subalgebra of X. □

The converse of the above theorem is also true as seen from the following

THEOREM 5.3. *Let $A = \{x, \mu_A(x), \nu_A(x) \mid x \in X\}$ is an IF set in X such that $A_{\alpha,\beta}$ is subalgebra of X for every $\alpha, \beta \in [0,1]$. Then A is (S,T) Intuitionistic fuzzy$\beta-$subalgebra of X.*

Combining the two results above we obtain

THEOREM 5.4. *Any $\beta-$subalgebra of X can be realized as a level of $\beta-$subalgebra for some (S,T) Intuitionistic fuzzy $\beta-$subalgebra of X.*

THEOREM 5.5. *Let $A_{s,t}$ and $B_{s_1,t1}$ two level set of (S,T) Intuitionistic fuzzy$\beta-$subalgebras A and B where $s \leqslant s_1$ and $t \geqslant t_1$of X. If$\mu_A(x) \leqslant \mu_B(x)$ and $\nu_A(x) \geqslant \nu_B(x)$, then $A \subseteq B$.*

PROOF. Now,

$$A_{s,t} = \{x, \mu_A(x) \geqslant s \text{ and } \nu_A(x) \leqslant t \mid x \in A\}$$

and

$$B_{s_1,t_1} = \{x, \mu_B(x) \geqslant s_1 \text{ and } \nu_B(x) \leqslant t_1 \mid x \in B\}.$$

If $x \in \mu_B(s_1)$, then $\mu_B(x) \geqslant s_1 \geqslant s \implies x \in \mu_A(s)$. Therefore $\mu_B(x) \geqslant \mu_A(x)$. And if $x \in \nu_B(t_1)$, then $\nu_B(x) \leqslant t_1 \leqslant t \implies x \in \nu_A(t)$. Therefore $\nu_B(x) \leqslant \nu_A(x)$. Hence $A \subseteq B$. □

One can easily prove the following

THEOREM 5.6. *Let A be a (S,T) Intuitionistic fuzzy$\beta-$subalgebra of $X, \alpha \in [0,1]$. Then*

1. *if $\alpha = 1$,then upper-level set $U(\mu_A, \alpha)$ is either empty or $\beta-$subalgebra of X.*
2. *if $\beta = 0$,then lower-level set $L(\nu_A, \beta)$ is either empty or $\beta-$subalgebra of X.*
3. *if S= min, then upper-level set $U(\mu_A, \alpha)$ is either empty or $\beta-$subalgebra of X.*
4. *if T= max,then lower-level set $L(\nu_A, \beta)$ is either empty or $\beta-$subalgebra of X.*

THEOREM 5.7. *Let $A_{\alpha,\beta}$ and $B_{\alpha,\beta}$ be two level (S,T) Intuitionistic fuzzy$\beta-$subalgebras of X and Y respectively. Then the level of $(A \times B)_{\alpha,\beta}$ is also a level (S,T) Intuitionistic fuzzy $\beta-$subalgebra of $X \times Y$.*

PROOF. Take $X = (x_1, x_2), y = (y_1, y_2) \in X \times X$ and $\mu = \mu_A \times \mu_B$

$$\begin{aligned}
\mu(x+y) &= \mu((x_1,x_2)+(y_1,y_2)) \\
&= (\mu_A \times \mu_B)(x_1+y_1),(x_2+y_2)) \\
&= S\{\mu_A(x_1+y_1), \mu_B(x_2+y_2)\} \\
&\geqslant S\{S(\mu_A(x_1), \mu_A(y_1)), S(\mu_B(x_2), \mu_B(y_2))\} \\
&= S\{S(\mu_A(x_1), \mu_B(x_2)), S(\mu_A(y_1), \mu_B(y_2))\} \\
&= S\{(\mu_A \times \mu_B)(x_1,x_2), (\mu_A \times \mu_B)(y_1,y_2))\} \\
&= S\{(\mu_A \times \mu_B)(x), (\mu_A \times \mu_B)(y)\} \\
&= S\{\alpha, \alpha\} \\
&= \alpha
\end{aligned}$$

Similarly, $\mu(x-y) \geqslant \alpha$ and also, we can prove that, $\nu(x-y) \leqslant \beta$. Hence the Cartesian product of $A \times B$ is also level (S,T) Intuitionistic fuzzy $\beta-$ subalgebra of$X \times Y$. □

6. Conclusion

An investigation on (S,T) Intuitionistic fuzzy $\beta-$ subalgebra of $\beta-$algebrs is done and several interesting results are observed. One can extend this concept for various substructures of a $\beta-$algebra.

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