LIGHT MANIPULATION BY QUANTUM METAMATERIALS

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**Abstract:** We propose a novel mechanism of light manipulation by using the quantum metamaterial which consists of a large number of linearly arranged superconducting charge qubits. The experimental confirmation of this idea may open up a new way to potentially powerful quantum computing.

**Keywords:** light slowing down, superconducting quantum metamaterials, quantum information and computing.

1. **INTRODUCTION**

The concept of metamaterials dates back to 1967, when Victor Veselago predicted the materials with simultaneously negative effective dielectric constant (\(\varepsilon\)) and magnetic permeability (\(\mu\)) which achieve negative values of refractive index [1]. However, due to the lack of the natural materials with predicted features, this subject was neglected in the next thirty years until 2000, when the first materials with negative relative magnetic permeability was made [2]. These media, metamaterials, are artificial; composite structures built up of multiple, usually periodically arranged, identical unit elements (cells) with size substantially smaller than the wavelength of the incident electromagnetic radiation. Originally, the metamaterials were conceived with an aim to achieve new and unusual passive electromagnetic properties such as negative refractive index and artificial magnetism. Such metamaterials are made of the conventional substances such as metals or dielectrics. The whole concept may be substantially advanced introducing the quantum degrees of freedom. It may be achieved by embedding the controllable, essentially quantum systems (e.g. molecules, quantum dots, cold atoms, Josephson junctions (JJ)), into larger structures in order to tailor their effective properties [3–7]. Owing to their quantum nature, these new media-quantum metamaterials (QMM), possess a number of novel features which do not appear in ordinary ones. Thus, the notion of QMM in narrow sense concerns the artificial optical media that [3–7]:

- are composed of quantum coherent unit elements with engineered parameters;
- exhibit controllable quantum states of these elements;
- maintain quantum coherence for times far exceeding the traversal time of an electromagnetic signal.

Superconducting circuits with Josephson junctions exhibit relatively high coherence times, extremely low dissipation and behave as artificial atoms [4] which make them good candidates as building blocks of QMMs. These circuits are fabricated at micrometer scale and operate at mK temperatures, which, in final instance, ensures that the energy spectrum of such quantum-mechanical system comprises only of the two lowest levels. For that reason the JJs based superconducting circuits may be viewed as artificial two-level atoms. Accordingly, these structures may be used as qubits - controllable quantum two-level systems for quantum computing [4,8]. Specifically, several variations of Josephson qubits that utilize either charge, flux or phase degrees of freedom have been used to implement a working quantum computer that could have enormous potential across a wide range of fields [1–9].

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In this paper we propose a novel aspect of the use of qubits in the quantum information processing. It consists of manipulating the light, its slowing down in particular, by employing the self-induced transparency (SIT). There are many ways of achieving slow light, among them SIT is one of the earliest [10–13]. Currently, the techniques based on electromagnetically induced transparency (EIT) in natural atoms [14] and metamaterials [15] are among the most widely used. These approaches turn out to be a powerful tool; nevertheless, the experiment of quantum EIT requires a special setup with extreme constraints, which cannot be easily achieved. For this reason, an efficient and simple way of light slowing is still a challenging problem. We propose a conceptually very simple approach which relies on SIT in superconducting QMM.

\[ H = \sum_{n,p=0,1} E_{p,n} a_{p,n}^+ a_{p,n} + \sum_n \left( \beta^2 (a_{n+1} - a_n)^2 + V_{60}(n) + V_{12}(n) \right) + \sum_{p,p',n} V_{p,p',n}^0 a_{p,n}^+ a_{p,n} a_{p',n}^+ a_{p',n}. \]  

(1)

In order to simplify further consideration we use Josephson energy \( E_J = \Phi_0 e/(2 \pi C) \), as an energy unit.

The first two sums stand for the SCQ and free light in the medium, while the third one corresponds to their interaction. Here we skip the details of the quantization of the qubit system and express its Hamiltonian in terms of the creation and annihilation operators. Since the qubit may be only in the ground (0) and excited (1) states with corresponding energies \(-E_p\), normalization condition: \( \sum_{p=0,1} a_{p}^+ a_{p} = 1 \), holds for each qubit unit. The electromagnetic field is represented by means of the dimensionless vector potential \( a_n = 2 \pi D A_{x,n} / \Phi_0 \), while the meaning of parameters is: \( \beta^2 = (8 \pi D E_j)^4 / (\Phi_0 / 2 \pi)^2 \) - dimensionless speed of light in QMM, \( \Phi_0 \) - the magnetic flux quantum, \( L \) and \( C \) are JJs critical current and capacitance, \( h \) and \( e \) being the Planck's constant and electron charge, respectively. Finally, the values of the qubit parameters and its interaction with EM field are defined by means of the eigenstates and eigenvalues of the Mathieu equation [6,7].

3. LIGHT PROPAGATION IN QMM: MAXWELL-BLOCH EQUATIONS FOR QMM

The description of light propagation in these media involves Schrödinger equation (SE) for QMM and Maxwell equation for EM wave. We took that qubit is in the superposition state \( |\psi\rangle = \sum_p |p\rangle a_p^+ |0\rangle \) and derived evolution equations for qubit amplitudes \( \Psi_p \) from the SE. In order to describe the propagation of EM radiation in this system we introduce the components of the qubit Bloch vector–\( R_x = |\Psi_1|^2 - |\Psi_0|^2 \), \( R_x = \Psi_1^* \Psi_0 + \Psi_0^* \Psi_1 \) and \( R_y = i(\Psi_0^* \Psi_1 - \Psi_1^* \Psi_0) \). In this way we obtain a set of nonlinearly coupled Maxwell-Bloch equations. It may be solved by passing to the continuum limit \( (a_n \rightarrow a(x,t)) \) and under the resonance condition \( \Delta - 2 \omega_0 = 0 \) and \( V(z) = 0 \) employing the slowly varying envelope approximation.

It turns out that the character of the solutions strongly depends on the choice of initial and boundary conditions. Thus, \( R_x(-\infty) = -1 \), \( \varepsilon(\infty) = 0 \) and \( R_x(-\infty) = 1 \), \( \varepsilon(\infty) = 0 \), we found that the electromagnetic radiation propagates through the transmission line in form of the Lorentzian pulse:

\[ \varepsilon(\tau) = \varepsilon_0 \frac{1}{1 + \frac{\tau^2}{\tau_p^2}} \]

(2)

whose amplitude \( \varepsilon_0 = 4 \rho / \mu \) and duration time \( \tau_p = \sqrt{\frac{\varepsilon_0}{\varepsilon_0}} \) are related through \( \varepsilon_0 \), with \( k = \beta = 2 \omega_0 / \mu \) and \( \tau_p = 4 \sigma / \mu \). In terms of these solutions, inversion population attains \( R_x = -1 \) \( \varepsilon(\infty) = 2 \), \( \varepsilon(\infty) = 2 \), with \( k = \beta = 2 \omega_0 / \mu \) being the wavevector of the EM radiation in the waveguide. The plus (minus) sign in the earlier equations corresponds to absorbing (amplifying) QMMs.
This is a well known result obtained in the studies of two photon SIT [11], and, exhibits quite similar features. The most interesting result is the dependence of the pulse velocity on input parameters $\varepsilon_0$ and duration time $\tau_p$.

$$v = \beta \{ 1 - \left[ \left( \frac{\hbar \omega J}{E J} \right) \left( \frac{\omega J}{\omega_0 J} \right) \right]^2 \left\{ 1 \pm \left[ \frac{\tau_p \mu \sigma \hbar \omega J}{(\omega J)^2} \right] \right\}^{-1}. \tag{3}$$

Apparantly, the enhanced light slowing down, even stopping, is possible for conveniently adjusted qubit parameters.

Thus, for absorbing initial conditions, more intense pulses propagate faster, and there is an upper limit on the pulse velocity $v_0 = \beta \{ 1 - 2[V_{11} + V_{00}] / [(E_1 - E_0)^2] \left[ (\hbar \omega J / E J)^2 \right] \}^{1/2},$ along with a SCQ-parameter-dependent condition $2[V_{00} + V_{11}] / [(E_1 - E_0)^2] \left[ (\hbar \omega J / E J)^2 \right] < 1$. For amplifying initial conditions, the superradiant pulse has an inverted Lorentzian form. In this case, however, more intense pulses propagate slower, while they exist only when their velocity exceeds the threshold value $v_0$. The latter implies that the duration of superradiant pulses cannot exceed $\tau_M = \frac{\omega E J}{(\mu \sigma \hbar \omega J)^2}$. The normalized velocity-amplitude curves for both absorbing and amplifying media are shown in Fig. 1. The results presented in Fig. 1 correspond to the choice: $V_{11} + V_{00} = 1.6(E_1 - E_0)^2$.

4. CONCLUDING REMARKS

The propagation velocity of the SIT pulse in an absorbing QMM could be significantly less than the speed ($\beta$) of „free“ pulse in the waveguide, which could in principle be controlled coherently by an external field [14] or by real time tuning of the qubit parameters. Superradiant pulses, on the other hand, could travel through the transmission line with their speed exceeding $\beta$. An ability to control the flow of optical, in the broad sense, information may have a technological relevance for quantum computing [16]. Note that the total inversion, i.e. excitation or de-excitation of all qubits during the pulse propagation is possible only if $\gamma = 0$, due to the matrix elements $V_{11}$ and $V_{00};$ the latter shift the energies of the qubit states (Stark shift) violating thus the resonance condition.

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6. REFERENCES


МАНИПУЛАЦИЈА СВЕТЛОШЊУ ПОМОЋУ КВАНТНИХ МЕТАМАТЕРИЈАЛА

Сажете: Предложили смо нови механизам манипулисања светлошћу коришћењем квантних метаматеријала грађених од великог броја линеарно распоређених суперпроводних „Q”-битова. Експериментална потврда ове идеје могла би отворити нове путеве у развоју квантних рачунара.

Кључне речи: Успоравање светлости, суперпроводни квантни метаматеријали, квантна информатика и рачунари.