International Journal of Electrical Engineering and Computing
Vol. 8, No. 1 (2024)

Original research paper
UDC 621.313.333:681.532.5
DOI 10.7251/IJEEC2401017M

Disturbance Compensation in Discrete-Time Sliding Mode Control – A Comparative Study

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Abstract— **This work is intended to compare three methods for disturbances estimation and compensation in discrete-time variable structure systems with sliding modes. All methods detect disturbance, which appear in control channel, with time delay of one sampling period. The first method is based on nominal discrete-time plant model and can be used in any type of discrete-time control systems. The second and the third methods detect disturbance by measurement of sliding function, and it is applicable in discrete-time sliding mode control systems only. The main task of this work is to check efficiency of the given methods in the presence of unmodeled inertial dynamics in actuators and position and velocity sensors of a positional servo system. It is shown that all three methods give near identical results in the nominal case, while the second and the third methods are superior in the presence of unmodelled dynamics. The third method introduces zig-zag motion in the nominal case. Results of research are illustrated by computer simulation of an example of positional servo system.**

Keywords- electrical drives; sliding mode control; speed control; position control; discrete-time variable structure control

I. Introduction

Continuous-time variable structures systems with sliding mode (SM) are completely invariant (insensitive) to disturbances that enter through the control channel, provided they meet the matching conditions [1]. The design process for these systems begins with the selection of an appropriate function $\boldsymbol{s}(\boldsymbol{x})$=0 in the state space. Generally, this geometrically represents a sliding hypersurface, or a hyperplane in special cases. In second-order systems, this is a sliding line, and in first-order systems, it is an origin point. The objective is to bring the system state from any initial condition to the sliding manifold $\boldsymbol{s}(\boldsymbol{x})$=0 in finite time and maintain on it, regardless of disturbances. This hypersurface $\boldsymbol{s}(\boldsymbol{x})$=0 contains the system equilibrium state $\boldsymbol{x}$=0. The equation $\boldsymbol{s}(\boldsymbol{x})$=0 defines sliding mode dynamics. The control task is to bring the system state into the origin, while always remaining on s(x)=0. This type of motion is called sliding, and $\boldsymbol{s}(\boldsymbol{x})$ is the sliding function.

For linear systems, it is easy to find a control that will guide the nominal system (system with nominal parameters and without disturbances) along $\boldsymbol{s}(\boldsymbol{x})$=0 with the initial condition on $\boldsymbol{s}(\boldsymbol{x}(0))$=0. This control is called equivalent control. However, in linear continuous-time systems, it is not possible to bring the system state to $\boldsymbol{s}(\boldsymbol{x})$=0 from any initial state $\boldsymbol{s}(\boldsymbol{x}(0)) \neq 0$ in finite time. Therefore, additional nonlinear (discontinuous) control must be introduced – the reaching control. Thus, in continuous-time SM systems, the control contains two components: equivalent control and reaching control. Reaching control is of the form:

$$\boldsymbol{u}_r = \boldsymbol{\alpha}\mathrm{sign}(\boldsymbol{s}(\boldsymbol{x})), \quad (1)$$

where $\boldsymbol{\alpha} = \mathrm{dij}(\alpha_i)$ is a diagonal $m \times m$ matrix, α_i =constant>0, and its magnitude depends on the absolute value of the disturbance acting on the system. The function

$$\mathrm{sign}(\boldsymbol{s}(\boldsymbol{x})) = \begin{cases} -1 & \text{for } \boldsymbol{s} < 0 \\ 0 & \text{for } \boldsymbol{s} = 0 \\ 1 & \text{for } \boldsymbol{s} > 0 \end{cases} \quad (2)$$

is a switching function that changes its sign on $s(x)$=0. The complete control is of the form:

$$\boldsymbol{u}(t) = \boldsymbol{u}_{eq}(t) + \boldsymbol{u}_r(t), \quad (3)$$

where $\boldsymbol{u}_{eq}(t)$ is the equivalent control and $\boldsymbol{u}_r(t)$ is the reaching control.

The discontinuous control term drives the system to fulfill condition $\boldsymbol{s}(\boldsymbol{x})$=0, if there is no time delay in information processing, which is achievable in continuous-time systems. However, if the system has delays in information processing (pure or inertial delay, or hysteresis), the system trajectory cannot achieve exactly $\boldsymbol{s}(\boldsymbol{x})$=0 but will oscillate around $\boldsymbol{s}(\boldsymbol{x})$=0. Thus, according to (2), $\boldsymbol{s}(\boldsymbol{x})$ is also called a switching function, as the control continuously switches between positive and negative values. This leads to parasitic oscillations known as chattering. The amplitude of chattering creates a quasi-sliding domain around $\boldsymbol{s}(\boldsymbol{x})$=0. The narrower this zone, the higher the system quality. Chattering is the primary drawback and a problem for the broader application of systems with sliding

This paper is a revised and expanded version of the paper presented at the XXIII International Symposium INFOTEH-JAHORINA 2024

mode control (SMC), besides the requirement for knowledge of all the state variables of the control plant.

Modern control systems are designed using elements with discrete-time (DT) information processing. Since the advent of the first microcontrollers, significant progress has been made in terms of the speed and accuracy of data processing. However, regardless of the processor speed, there will always be a time delay of one sampling period. Moreover, most control plants are continuous, with disturbances that cannot be measured and discretized. This means that the digital implementation of analog SMC algorithms would lead to the occurrence of chattering [2], and invariance cannot be achieved in DT SMC systems. Fortunately, significant robustness is achieved, but it is often insufficient. Therefore, disturbance compensation is frequently utilized.

DT SMC for linear control plants is characterized by the fact that DT equivalent control provides both the reaching and the sliding components [3-8]. The DT equivalent control is sufficient to bring the system to the surface $\boldsymbol{s}(\boldsymbol{x})$=0 and maintains it on that surface at each sampling instant, under nominal conditions (without external or internal disturbances). In reference [8], the reaching and equivalent controls are separated using δ-discrete model of the system instead of the conventionally used shift model.

A nonlinear component of the form (1) must be added to this unique control to compensate for disturbances. This component will also lead to chattering. To minimize chattering, it is necessary to estimate disturbance and compensate it to the greatest extent possible, so that the parameter $\boldsymbol{\alpha}$ in (1) should be grater only than the uncompensated part of the disturbance. This reduces the width of the quasi-sliding domain and improves the system quality.

In current theory and practice, Luenberger-type disturbance observers are commonly used. This paper focuses on three simple methods for disturbance estimation and compensation in DT systems. The first method in SMC was initially applied in [5] and subsequently in other works, e.g. [9]. It estimates the disturbance with a one-step delay. Therefore, in this paper, we will refer to it as One Step Delay (OSD) or the *First estimation method*.

The Second and Third estimation methods are based on detecting disturbances by measuring the sliding function $\boldsymbol{s}(\boldsymbol{x})$. The disturbance is observed with a one-step delay, as with the OSD method, but it cannot be directly used for compensation and requires further processing, which involves integrating the sliding function $\boldsymbol{s}(\boldsymbol{x})$. Therefore, we will refer to it as the integration (INT) method. This method was firstly applied and compared with OSD in [10], experimentally tested on a positioning servo system in [11], theoretically explained in [12], and further verified for speed or position control of DC motors or asynchronous motors in [13] and [14]. Similar research is presented in [15], [16]. A variant of this method, applied to systems designed using Gao's reaching law [17], [18] is provided in [19].

The objective of this paper is to compare the three methods when applied to the positional servo system from [9] under various conditions: a nominal system with external disturbances without unmodeled dynamics, and the case when unmodeled dynamics are present. Unmodeled dynamics can be in the power amplifier (actuator), position and speed sensors. Based on this analysis, the paper will provide comprehensive conclusions and recommendations.

The paper is organized as follows. After the introduction, the second section provides the theoretical foundations, summarized from the cited literature [5]-[9]. The third section presents an example of a servo system, and the comparative simulation results of the mentioned compensation methods are given. The paper concludes with a summary and a list of references.

II. Description of the System and Fundamentals of the Sliding Mode Control Theory

Consider a linear time-invariant control plant, described by a dynamical model in the CT domain with

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{D}\boldsymbol{f}(t), \tag{4}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the measurable state vector, $\boldsymbol{u}(t) \in \mathbb{R}^m$ and $\boldsymbol{f}(t) \in \mathbb{R}^l$ are the control and disturbance inputs, respectively. Matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{D}$ are constant and have appropriate dimensions. It is assumed that the disturbance acts through the control channel, so the following matching conditions [1] holds:

$$\mathrm{rank}([\boldsymbol{B}\boldsymbol{D}]) = \mathrm{rank}(\boldsymbol{B}). \tag{5}$$

It is also assumed that $|\boldsymbol{f}(t)| \le \boldsymbol{f}_0 < \infty; |\dot{\boldsymbol{f}}(t)| \le \boldsymbol{f}_1 < \infty$.

The DT realization of (4) with a sampling period (T) leads to the model ($\boldsymbol{x}(kT) \equiv \boldsymbol{x}_k$, $k \in N_0$)

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_d\boldsymbol{x}_k + \boldsymbol{B}_d\boldsymbol{u}_k + \boldsymbol{v}_k, \tag{6}$$

where $\boldsymbol{A}_d = e^{\boldsymbol{A}T}$; $\boldsymbol{B}_d = \int_0^T e^{\boldsymbol{A}\tau} d\tau\, \boldsymbol{B}$,

$$\boldsymbol{v}_k = \int_0^T e^{\boldsymbol{A}\tau} \boldsymbol{D}\boldsymbol{f}(k+1)T - \tau) d\tau.$$

A. Discrete-time Sliding Mode

If the system satisfies conditions (5) and the disturbance is slowly varying and constant during the sampling period, then the following holds:

$$\boldsymbol{v}_k = \boldsymbol{B}_d\boldsymbol{d}_k. \tag{7}$$

Then (6) becomes

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_d\boldsymbol{x}_k + \boldsymbol{B}_d(\boldsymbol{u}_k + \boldsymbol{d}_k). \tag{8}$$

The task is to achieve DT SM along the surface

$$\boldsymbol{s}_k = \boldsymbol{C}_d\boldsymbol{x}_k = 0, \boldsymbol{C}_d \in \mathbb{R}^{m\times n} \tag{9}$$

regardless of the initial system state, which may or may not satisfy relation (9). The condition for the DT SM to occur [3]-[5] is

$$\boldsymbol{s}_{k+1} = \boldsymbol{C}_d\boldsymbol{x}_{k+1} = \boldsymbol{0} \text{ for } \forall \boldsymbol{x}_k. \tag{10}$$

This means that the system is required to reach the specified sliding surface (9) in one sampling period.

Substituting (8) into (10) yields

$$\boldsymbol{C}_d\boldsymbol{x}_{k+1} = \boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{x}_k + \boldsymbol{C}_d\boldsymbol{B}_d(\boldsymbol{u}_k + \boldsymbol{d}_k) = 0. \tag{11}$$

Now, the control that satisfies (10) is determined as

$$\boldsymbol{u}_k = -(\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{x}_k - \boldsymbol{d}_k,\ \boldsymbol{C}_d\boldsymbol{B}_d \neq \boldsymbol{0}. \tag{12}$$

The control (12), in general, is not feasible since disturbance $\boldsymbol{d}_k$ cannot be measured. Therefore, an equivalent control is defined for the nominal system ($\boldsymbol{d}_k$=0)

$$\boldsymbol{u}_{eq,k} = -(\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{x}_k. \quad (13)$$

To achieve the required DT SM, the disturbance should be estimated and compensated. This approach to organizing DT SM is referred to in the literature as Equivalent Control-Based SM (ECBSM) [9].

The second approach to synthesizing DT SM is based on Gao's reaching-law method, developed for CT systems in [17], and then adapted for DT SMC in [18]. The method relies on the prior definition of the sliding function dynamics, given by the expression [18]

$$\begin{gathered}\boldsymbol{s}_{k+1} = (\boldsymbol{I}-\boldsymbol{q}T)\boldsymbol{s}_k - \boldsymbol{\varepsilon}T sign(\boldsymbol{s}_k) + \boldsymbol{C}_d\boldsymbol{d}_k,\\ \boldsymbol{\varepsilon},\boldsymbol{q} > 0, 0 < (\boldsymbol{I}-\boldsymbol{q}T) < \boldsymbol{I}.\end{gathered} \quad (14)$$

By substituting $\boldsymbol{s}_{k+1}$ in (14) with $\boldsymbol{C}_d\boldsymbol{x}_{k+1}$ for $\boldsymbol{x}_{k+1}$, defined in (8), solving this equation for the control input yields

$$\begin{aligned}\boldsymbol{u}_k = -(\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\{\boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{x}_k + (\boldsymbol{I}-\boldsymbol{q}T)\boldsymbol{s}_k\\ - \boldsymbol{\varepsilon}T sign(\boldsymbol{s}_k)\}\end{aligned} \quad (15)$$

The control input (15) is derived under the assumption that the disturbance $\boldsymbol{d}_k$ in (14) is known. If it is not known, the disturbance should be estimated. Then, instead of $\boldsymbol{d}_k$, the estimated value $\widehat{\boldsymbol{d}}_k$ will appear in (14).

B. Disturbance Estimation Methods

1) The First Estimation Method

It is based on the DT model of the nominal control plan (6) and was first applied for DTSMC in [5]. The system (6) can be rewritten as

$$\boldsymbol{x}_k = \boldsymbol{A}_d\boldsymbol{x}_{k-1} + \boldsymbol{B}_d\boldsymbol{u}_{k-1} + \boldsymbol{v}_{k-1}, \quad (16)$$

from which it follows

$$\boldsymbol{v}_{k-1} = \boldsymbol{x}_k - \boldsymbol{A}_d\boldsymbol{x}_{k-1} - \boldsymbol{B}_d\boldsymbol{u}_{k-1}. \quad (17)$$

Notice that $\boldsymbol{v}_k$, in the general case, does not satisfy (5).

However, if the sampling period T is sufficiently small, the component of the disturbance that does not satisfy (5) is negligible compared to the one that does [8]. This will be assumed hereafter. In that case, $\boldsymbol{v}_k = \boldsymbol{B}_d\boldsymbol{d}_k$, and (3) can be written in the form (8).

By introducing in (6) the compensating control $\boldsymbol{u}_{c,k} = -\boldsymbol{v}_{k-1} = -\boldsymbol{B}_d\boldsymbol{d}_{k-1}$ with the opposite sign to $\boldsymbol{d}_k$, it is obtained

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_d\boldsymbol{x}_k + \boldsymbol{B}_d(\boldsymbol{u}_k + \boldsymbol{d}_k - \boldsymbol{d}_{k-1}). \quad (18)$$

If the disturbance is continuous and bounded, $\boldsymbol{d}_k - \boldsymbol{d}_{k-1}$, as the uncompensated equivalent disturbance, is of the order $O(T^2)$ [5].

Therefore, instead of $\boldsymbol{d}_k$, there will now be a disturbance

$$\boldsymbol{d}_n = \boldsymbol{d}_k - \boldsymbol{d}_{k-1}, \quad (19)$$

so, the magnitude $\boldsymbol{\alpha}$ in (1) should take a significantly smaller value ($\boldsymbol{\alpha} \geq |\boldsymbol{d}_n|$), which drastically reduces the amplitude of the switching control of the form (1), and consequently, the level of chattering.

2) The Second Estimation Method

This method is based on the fact that the disturbance, which satisfies (5), is directly reflected in the sliding function (9) with one sampling period delay. The sliding function value can always be calculated using the measured state variables, since it is a linear combination of the system state coordinates. However, unlike the previous method, the determined value cannot be directly added to the control for disturbance compensation. Instead, the disturbance signal must be reconstructed using this value. This leads to processing of the sliding function using integrators. This compensation is reminiscent of classical proportional-integral (PI) control, where the sliding function signal is used instead of the error signal.

Multiply both sides of equation (8) by $\boldsymbol{C}_d$. Now, substitution of the equivalent control (13) into the resulting equation will give

$$\boldsymbol{C}_d\boldsymbol{x}_{k+1} = \boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{x}_k + \boldsymbol{C}_d\boldsymbol{B}_d(-(\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{x}_k + \boldsymbol{d}_k) \quad (20)$$

or equivalently:

$$\boldsymbol{s}_{k+1} = \boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{d}_k \leftrightarrow \boldsymbol{d}_{k-1} = (\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\boldsymbol{s}_k. \quad (21)$$

In this way, a disturbance estimate is obtained, which is simpler than (17). However, the estimated value $\boldsymbol{d}_{k-1} = (\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\boldsymbol{s}_k$ from (21) cannot be directly applied as in the previous case. An explanation follows.

Multiplying both sides of (18) by $\boldsymbol{C}_d$ and substituting $\boldsymbol{d}_{k-1}$ from (21), gives

$$\boldsymbol{s}_{k+1} = -\boldsymbol{s}_k + \boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{d}_k = -\boldsymbol{s}_k + \widetilde{\boldsymbol{d}}_k \quad (22)$$

which leads to oscillations. For example, in the case of a constant disturbance, this can be easily verified. Let the system, until time t=t_k, be without disturbance and $\boldsymbol{s}_k = 0$. At t=t_{k+1}, $\boldsymbol{s}_{k+1} = \widetilde{\boldsymbol{d}}_k$ and in the next sampling period t=t_{k+2}, it will be $\boldsymbol{s}_{k+2} = 0$, and so on. Therefore, the system oscillates.

Another way to compensate the disturbance using the estimated value $\boldsymbol{d}_{k-1}$ from (21) should be found.

Let compensating control $\boldsymbol{u}_{c,k}$ be introduced into the system. Then, it is obtained

$$\begin{gathered}\boldsymbol{s}_{k+1} = \boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{u}_{c,k} + \boldsymbol{d}_k \leftrightarrow \boldsymbol{s}_k = \boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{u}_{c,k-1} + \boldsymbol{d}_{k-1}\\ \boldsymbol{d}_{k-1} = \boldsymbol{s}_k - \boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{u}_{c,k-1}.\end{gathered} \quad (23)$$

Adopting that $\boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{u}_{c,k} = -\boldsymbol{d}_{k-1}$, the recursive relation is obtained

$$\boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{u}_{c,k} = \boldsymbol{C}_d\boldsymbol{B}_d\boldsymbol{u}_{c,k-1} - \boldsymbol{s}_k. \quad (24)$$

Equation (24) in the complex domain becomes [12]

$$\boldsymbol{u}_c(z) = -(\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\frac{z}{z-1}\boldsymbol{s}(z). \quad (25)$$

Remark 1: To suppress the chattering in practice due to unmodeled dynamics, the following control is used instead of (25)

$$\boldsymbol{u}_c(z) = -\boldsymbol{k}_{int}\frac{z}{z-1}\boldsymbol{d}_n(z), 0 < \boldsymbol{k}_{int} \leq (\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}. \quad (26)$$

This has a significant advantage over the first estimation method, as will be illustrated in section III on a system with unmodeled dynamics.

3) The Third Estimation Method

This method is also based on using the sliding function. It was proposed in the paper [19], where the synthesis of sliding control is based on Gao's Reaching Law (RL) method [18]. The control that achieves zig-zag motion around the selected sliding surface and compensates for disturbance is defined by the expression

$$\boldsymbol{u}_k = -(\boldsymbol{C}_d\boldsymbol{B}_d)^{-1}\{\boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{x}_k - (1-qT)\boldsymbol{s}_k + \varepsilon T sign(\boldsymbol{s}_k) + \boldsymbol{u}_{c,k}\}, \quad (27a)$$

where $\boldsymbol{u}_{c,k}$ is the compensating control:

$$\boldsymbol{u}_{c,k} = \sum_{i=2}^{k} [\boldsymbol{s}_i - (1-\boldsymbol{q}T)\boldsymbol{s}_{i-1} + \boldsymbol{\varepsilon} T sign(\boldsymbol{s}_{i-1})]. \quad (27b)$$

It is shown in [19] that expression (27b) is equal to $\boldsymbol{C}_d\boldsymbol{d}_{k-1}$.

As seen from (27b), the compensating control is of integral type, acting from the second sampling instant after the system starts (to eliminate overshoot) and contains three components. The first component is the integral of the sliding function, as in the previous estimator. The second component is the delayed signal of the first component with a weighting factor of $(1-qT)$. In this way, these two components together in the complex domain are identified as a lead-lag filter. Besides integral action, these components also introduce differential action, which can be useful for compensating unmodeled inertial dynamics. The third component forces the system motion around the sliding surface in a zig-zag manner.

III. Behavior of Systems with Unmodeled Dynamics

In this section, behavior of the systems with disturbance estimators will be examined in the presence of unmodeled inertial dynamics in the actuator and/or position and velocity sensors, using an example of a positional servo system. Its model (1) is a second-order system with scalar control

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ a_1 & a_2\end{bmatrix}\cdot\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ b\end{bmatrix}\left(u(t)+f(t)\right), \quad (28)$$

where x_1 and x_2 is the position and velocity, respectively, as directly measurable state coordinates. Systems of this type have been studied in various papers. In the study presented in [9], the system is depicted with disturbance compensation based on the first method (15), while in [11], INT disturbance compensation (24) is applied. In the paper [19], the third type of compensator (27) is employed.

The control parameters for the control plant (28), as discussed in [9], are: a_1=0, $a_2=-144$, b=6. The sampling period is T=1 ms, and $\boldsymbol{c}_d = [0.5 \quad 0.5]$. $\boldsymbol{c}_d\boldsymbol{A}_d = [15.4 \quad 9.7]$, $\boldsymbol{b}_d = [0.0047; \quad 0.09139]$. In this example, a more complex disturbance compared to the one given in [9] is used

$$f(t) = \frac{1}{6}\big(h(t-20) - 2h(t-30) + \left(h(t-40) - h(t-60)\right)5\sin 0.5\pi t + 4h(t-60) + 20h(t-70)\sin(0.125\pi t)\big). \quad (29)$$

which represents an external disturbance due to a change in mechanical load.

Additionally, a control constraint $|u_k| \le 20$ will be imposed.

The comparison of compensators will be conducted under nominal conditions and in the presence of unmodeled first-order inertial dynamics present in:

1. The actuator (power amplifier)
2. Position sensor
3. Velocity sensor
4. All the mentioned elements.

Fig. 1 shows the MATLAB/Simulink model of the system with controllers and disturbance compensators of all three types. The simulation results are depicted iin Figs. 2 - 12.

Fig. 2 illustrates the external disturbance used during the testing. It consists of constant components and sinusoidal components with specified frequencies and amplitudes.

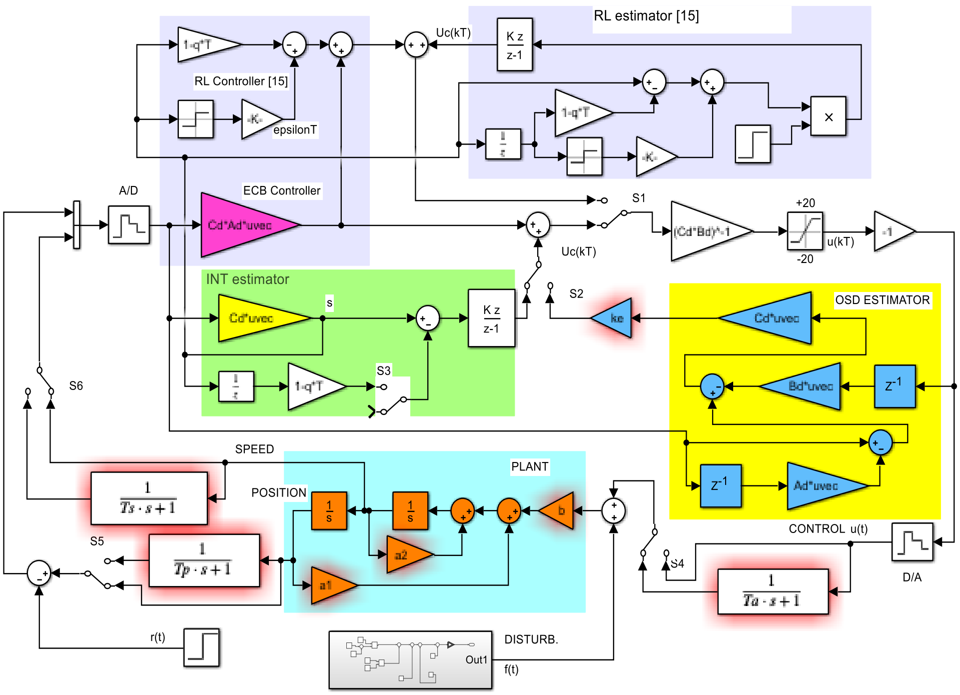

Figure 1. The MATLAB/Simulink model of the system with disturbance compensators: switches S1-S6 are in position with activated the equivalent control - based SMC (ECBSMC) with the integral disturbance compensation in nominal plant with disturbance. Switch S2 can activate integral estimator or OSD estimator. If switch S1 is in the second position, then RL based control system [19] is active. Switch S3 activates modification of the integral compensator

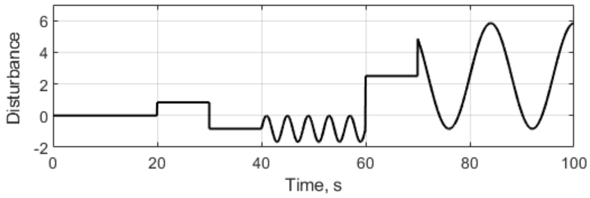

Figure 2. Disturbance $f(t)$ which acts on plant input.

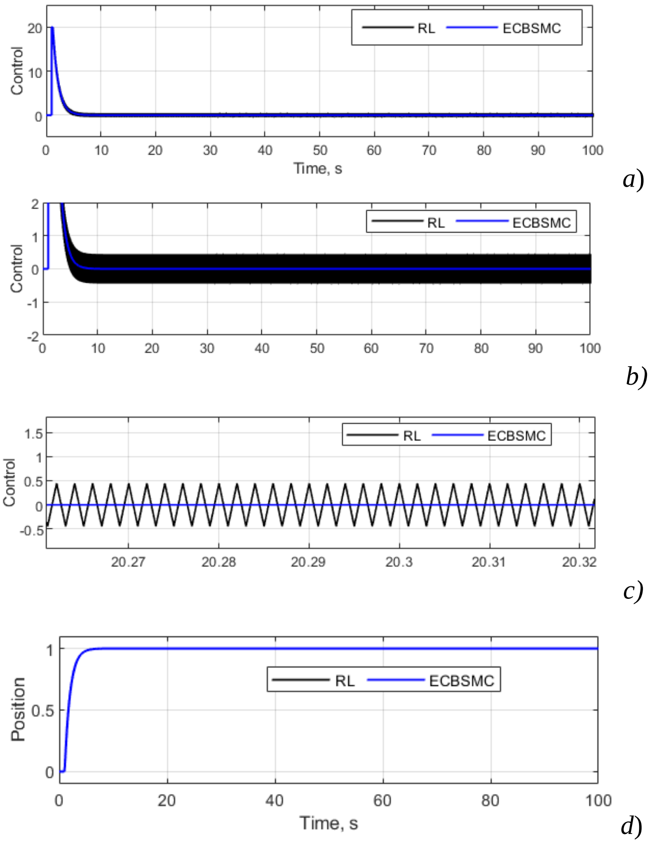

Figure 3. Control (*a*, *b*,*c*) and position response (*d*) of the nominal system *f*(*t*)=0, *q*=5, $\varepsilon = 3$.

First, Fig. 3 presents the following: control signals (*a*, *b*, *c*) and system response (*d*) without disturbance for the reference value *r*(*t*)=*h*(*t*-1). The system has no steady-state error with any type of controller. The system with ECBSMC has a smooth control signal, while the system based on RL (15) has a discontinuous control character, which is more clearly presented in Figs. 3b and 3c.

Next, the system without compensation is subjected to the given disturbance. The results are shown in Fig. 4. It can be seen from this figure that the system has a good response (Fig. 4c) but does not have high accuracy under the given disturbance, as shown in the magnified detail, Fig. 4c. The control does not exhibit chattering.

Remark 2: In this study, additional control of type (2) for systems with ECBSMC (Equivalent Control Based Sliding Mode Control) will not be applied in comparing compensation methods. In many cases, it is not necessary to apply (2) as it introduces chattering and zigzag motion around the sliding surface, which is a fundamental characteristic of the control algorithm (15).

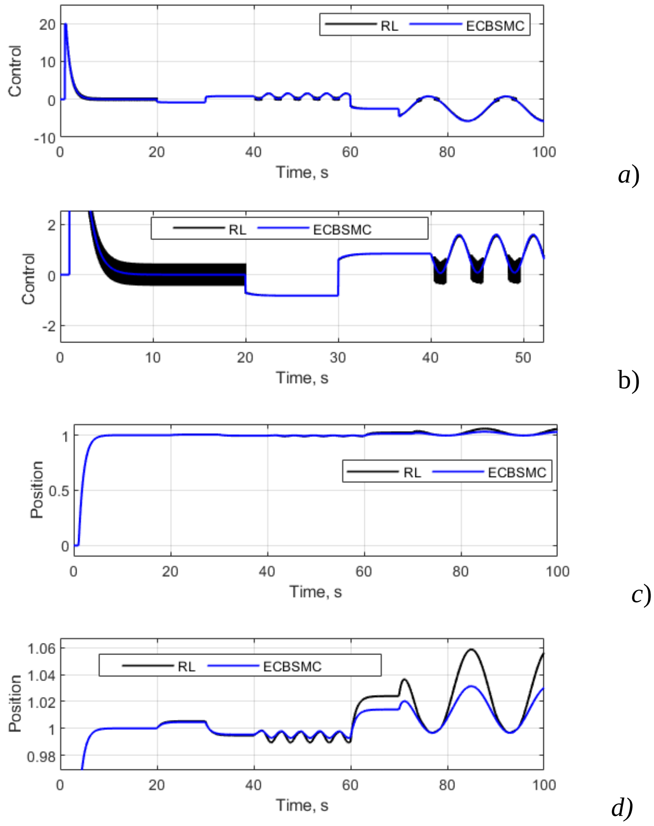

Figure 4. Response of the system with disturbance and without disturbance compensation: (*a*, b) control, (*c*, *d*) position response. The system has satisfactory response (*c*) but with steady state error even if constant type of disturbance is applied (*d*). *f*(*t*)=(29),*q*=5, $\varepsilon = 3$.

If disturbance compensators are applied, the result is shown in Fig. 5, where high positioning accuracy is achieved (Fig. 5d). It is important to note that the system with the RL controller has a larger error under sinusoidal disturbances, which is due to the slightly lower controller gain compared to ECBSMC.

When unmodeled inertial dynamics are introduced into the actuator, having a time constant $T_a = 1$ ms, chattering occurs in the control, as shown in Fig. 6a. Chattering is more pronounced with the ECBSMC controller due to its high gain and control limitations. The system based on the RL controller has an increased amplitude but does not enter extreme regimes. The system with the OSD compensator exits the sliding mode and does not fully suppress the disturbance. Systems with the INT compensator and RL compensator remain in a quasi-sliding mode and are unaffected by the disturbance. However, oscillations of this amplitude in control are undesirable.

Therefore, a compromise should be made to allow for less disturbance suppression while eliminating chattering.

If the gain of the INT and RL type compensators is reduced by half, chattering disappears, as seen in Fig. 7a, and the system very effectively suppresses the disturbance (Fig. 7b, red and black lines). If the gain of the OSD compensator is set to the same value, the system is free of chattering but exhibits significant deviation from the reference value (Fig. 7b, blue line).

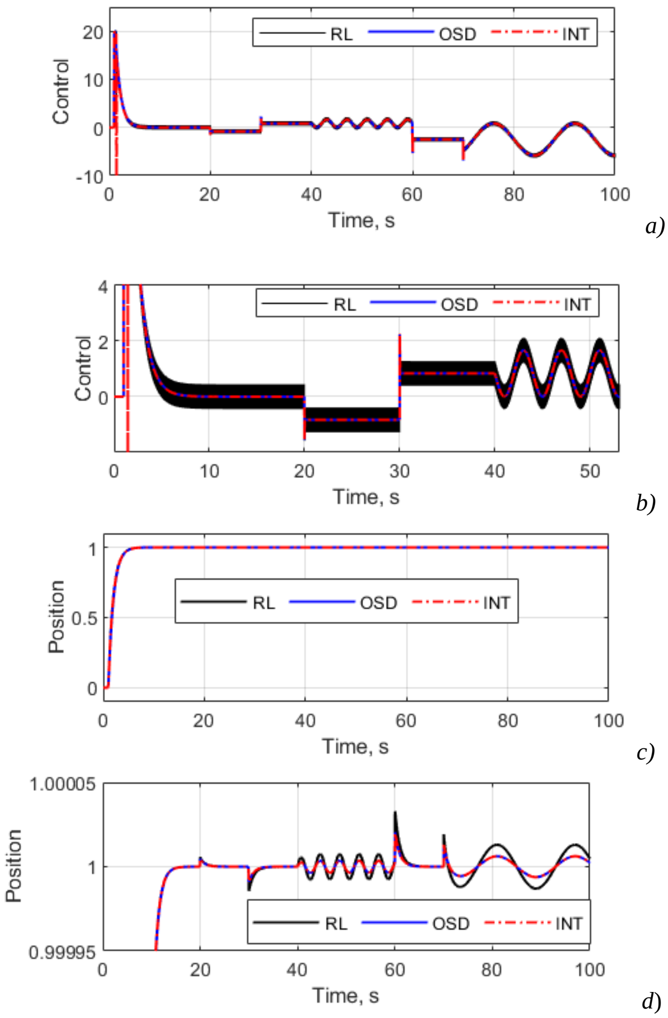

Figure 5. Reponses of the nominal system with disturbance and different types of disturbance compensators. The system with OSD and INT compensator has identical response, while the system with RL compensator has slightly worse response. This is a consequence of using different type of controller. *f*(*t*)=(29),. *q*=*5*, $\varepsilon = 3$.

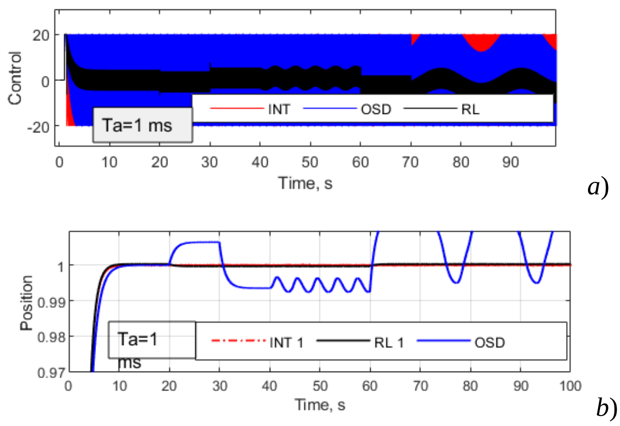

Figure 6. Responses of the system with unmodeled dynamics of the actuator $Ta = 1\ ms$. The control introduces chattering (*a*) for any type of disturbance compensator with nominal gain *k*=1. As can be seen, amplitude of chattering is the smallest for the RL method. Systems with RL or INT compensation have satisfactory position responses while the system with OSD disturbance compensator has bad response (*b*). *f*(*t*)=(29),. *q*=*5*, $\varepsilon = 3$.

Fig. 8 shows the case when unmodeled inertial dynamics with a time constant of 1 ms is introduced into the position sensor. It is observed that the system is not sensitive to these unmodeled dynamics. The simulation was repeated for unmodeled dynamics with increased time constants to 5 ms and 10 ms, yielding the same results.

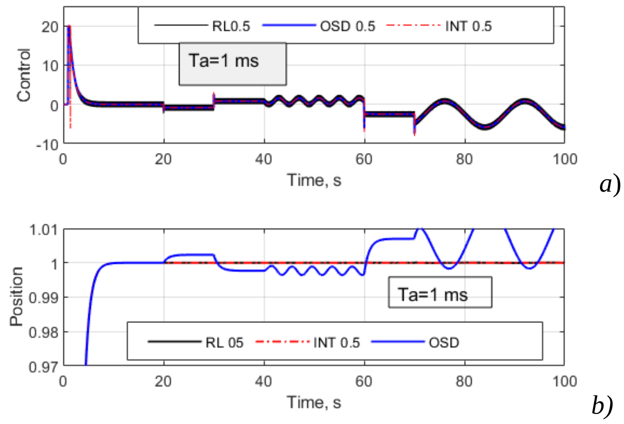

Figure 7. Responses of the system with unmodeled dynamics T_a =1 ms and reduced disturbance estimator gains at ½ their nominal values (OSD 0.5, INT 0.5 and RL 0.5). It can be seen that RL and INT compensator give high quality disturbance rejection and OSD does not.

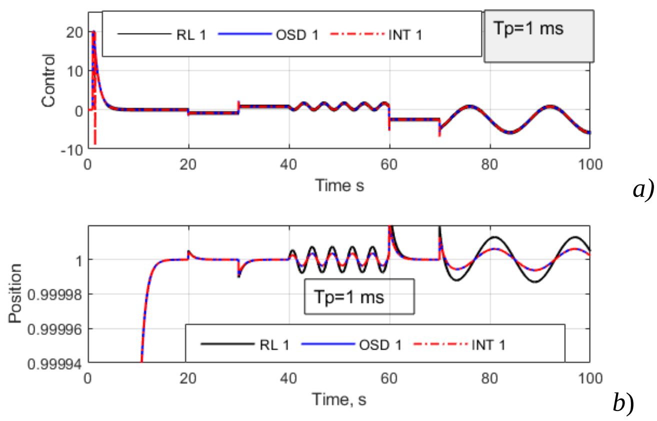

Figure 8. Reponses of the system with disturbance and unmodeled dynamics in positional sensor with T_p =1 ms and nominal gains of the disturbance compensators. *f*(*t*)=(29),. *q*=*5*, $\varepsilon = 3$.

Conversely, introduction of unmodeled dynamics with a time constant of 1 ms into the speed sensor leads to chattering (Fig. 9*a*), like the case with unmodeled dynamics in the actuator. Chattering is eliminated if the compensator gains are reduced to 50% of their nominal values. As before, the second and third type of compensators are much more effective, as shown in Fig. 10.

The same conclusions are reached when unmodeled dynamics with a time constant of 1 ms are introduced into both the actuator and sensors. Chattering occurs. However, a system with compensatory gains reduced to 50% of their nominal values still has chattering. By reducing the gains to 0.2, 0.1 and 0.2, respectively for the first, second, and third type of compensators, the chattering is eliminated. The system with the second and third type of compensators is significantly more efficient, as shown in Fig. 11.

In conclusion, it should be noted that in practice, an encoder is used as a position sensor, generating a digital signal without inertial delay. Therefore, the results regarding the unmodeled dynamics of the position sensor have no practical significance, except to demonstrate the system low sensitivity to unmodeled dynamics in the measurement of the controlled variable. On the other hand, the presence of inertial unmodeled dynamics in the actuator (power amplifier) is very realistic. In the case of DC motors, the rotor current dynamics in the positional system is usually neglected during the design, which can be equated to actuator unmodeled dynamics. Regarding the speed measurement, the speed is most often estimated in practice, and this signal may also have unmodeled dynamics, which must be considered in the design and tuning of high-precision positional servo systems.

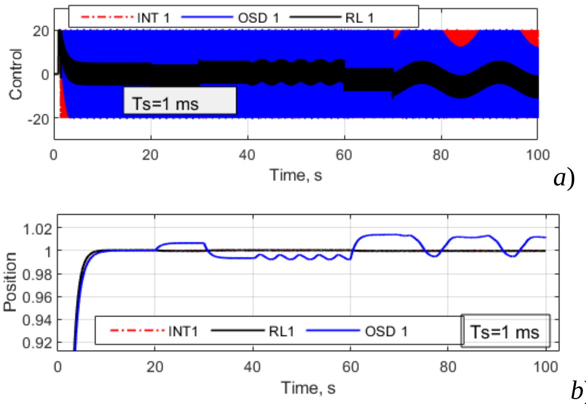

Figure 9. Fig. 9. Responses of the system with unmodeled dynamics in the speed sensor $T_s = 1$ ms. The control induces chattering (*a*) for any type of disturbance compensator with nominal gain *k*=1. As can be seen, amplitude of chattering is smallest for the RL method. Systems with RL or INT compensation have satisfactory position responses while the system with OSD disturbance compensator has bad response (*b*), *f*(*t*)=(29),. *q=5*, $\varepsilon = 3$.

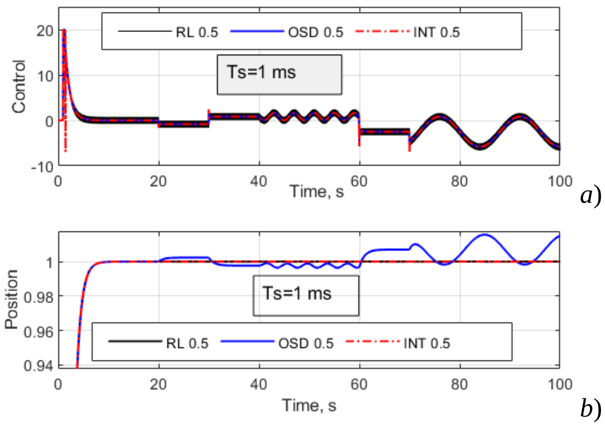

Figure 10. Fig. 10. Responses of the system with unmodeled dynamics T_s =1 ms and decreased disturbance estimator gains at ½ of their nominal values (OSD 0.5, INT 0.5 and RL 0.5). RL and INT compensator give high quality disturbance rejection and OSD not. *f*(*t*)=(29),. *q=5*, $\varepsilon = 3$.

The conducted research highlights the limitations of applying the compensators based on the DT model of the control plant due to unmodeled dynamics and the practical impossibility of changing its parameters. This limitation is not observed with the second and third type of compensators, which have integral action and the capability to adjust gains independently of changes in the model of the control plant. It should be noted that the variations in model parameters *a* and *b* in the observed example has a practically negligible impact on disturbance compensation, as both uncertainties are reflected as disturbances in the control channel. Furthermore, it is observed that the second type compensator is a special case of the third type compensator. For *q*=1/T and $\varepsilon = 0$, the third type compensator becomes the second type compensator, and for $q \neq 1/T$ and $\varepsilon = 0$, the third type compensator becomes a lead-lag compensator.

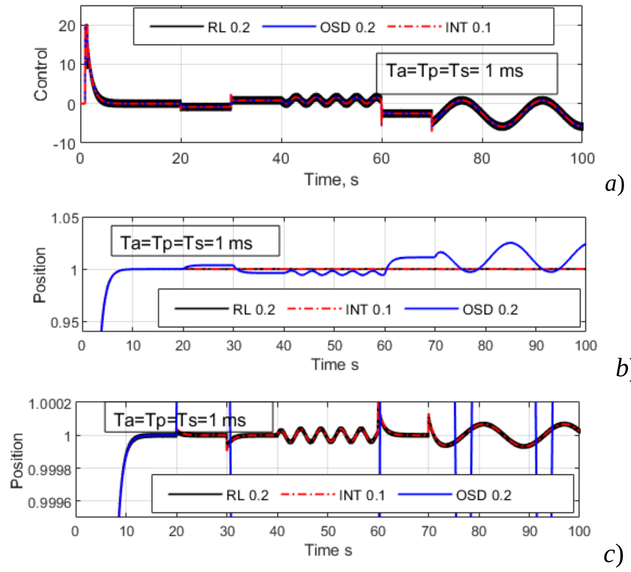

Figure 11. Responses of the system with disturbance and unmodeled dynamics in the actuator, position and speed sensors $T_a = T_p = T_s$ =1 ms1, with adjusted compensator gains to avoid chattering. RL compensator with gain of 0.2 and INT compensator with gain 0.1 give very good disturbance rejection without chattering, while OSD compensator has bad disturbance rejection capability. *f*(*t*)=(29),. *q=5*, $\varepsilon = 3$.

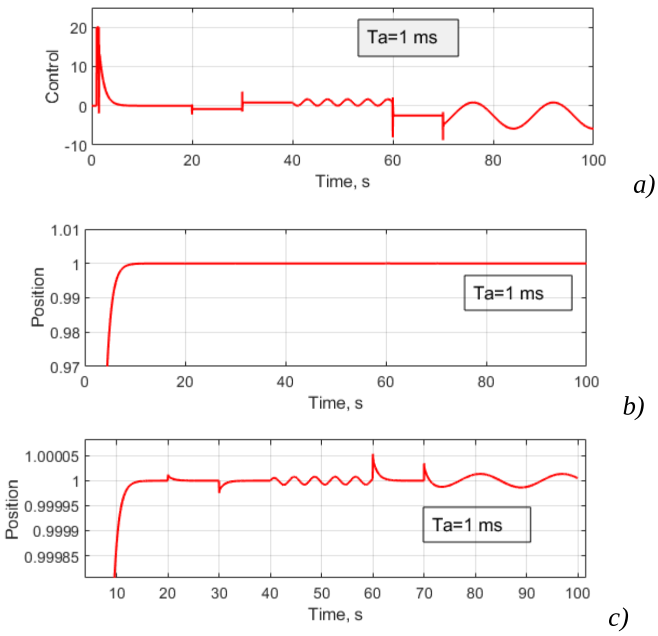

Figure 12. Response of the system with unmodeled dynamics T_a =1 ms, with control (13) and disturbance compensator (27*b*) with *q*=0.5/T and $\varepsilon = 0$.

When such a compensator is applied to a system with an ECBSM controller and a plant with unmodeled inertial dynamics in the actuator (switch S3 in Fig. 2 is on), the result is shown in Fig. 12. These results indicate the significant potential of such a system, which, under the given conditions, has very high accuracy and no chattering. This compensation capability should be thoroughly analyzed in the presence of measurement and other noises in practical systems due to the differential action in the compensator.

CONCLUSIONS

In this paper, control of time invariant linear CT plants by using DT sliding modes with disturbance estimators/ compensators are analyzed. Three simple types of disturbance estimators/compensators are inspected. All three types of estimators estimate disturbance with one sampling period delay. One of them is based on nominal DT plant model while the other two are based on the fact that the matched disturbance is directly contained in the sliding function with one sampling time delay. By adequately processing, the disturbance can be extracted and used for compensation. In the paper is shown that all three estimators/compensators give very close results for the nominal plant (plant without unmodeled dynamics). In the case of plants with unmodelled inertial dynamics of the first order, present in the actuator or/and position or speed sensors, there are significant differences. The system is less sensitive to unmodeled dynamics in the position sensor and is much more sensitive for unmodeled dynamics in the actuator and the speed sensor. Compensators based on the integration of the sliding function can be easily adapted by decreasing integral gain to avoid chattering in control system with unmodeled dynamics. Compensator based on nominal DT plant model has possibility to avoid chattering as well, but with very bad rejection capability of disturbances. In this way, beside simple realization, compensators based on the sliding function fully reject disturbances of constant type and significantly suppress slow-varying disturbances. By adjusting their gain, chattering problem can be reduced.

Further investigation will be dedicated to adaptation of the compensator based on DT plant model for possible use in the system with unmodelled dynamics. Barring that, it is necessary to compare the above considered compensators with compensators based on another principles, preferably with disturbance observers of Luenberger type.

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