IJEEC

International Journal of Electrical Engineering and Computing
Vol. 1, No. 1 (2017)

Original research paper
UDC: **66.097:519.87**
DOI: **10.7251/IJEEC1701011S**

Modern methods of design, analysis, optimization and implementation of conventional control algorithms for processes with finite and infinite degrees of freedom

Tomislav B. Šekara

Signals & Systems Department, School of Electrical Engineering, University of Belgrade, Belgrade, Serbia

tomi@etf.rs

Abstract - **This paper presents characterization methods for a large class of industrial processes using a critical experiment as well as modern methods of design, analysis, optimization and implementation of conventional control algorithms. Special attention is set to the process characterization methods using relay techniques and phase-locked loops in order to form a general process model which serves as a base for adequate controller design. This general process model adequately approximates processes which behaviour can be described with linear mathematical models with finite and infinite degrees of freedom including conventional finite dimension systems, time-delay systems, systems whose behaviour is dominated by a wave and transport problems such as mass and energy transfer, systems described with fractional differential equations etc. Based on characterization, an important accent is also put on the design of PI/PID controller due to their large application in industry which exceeds 93% compared to all the other controllers according to Honeywell's surveys. In order to illustrate validity of characterization model and effectiveness of presented design method, the paper provides an example of optimal PID controller designed under constraints on robustness and sensitivity to the measurement noise. Digital implementation is considered for both the controllers with rational and those with non-rational transfer functions. At the end, controller analytical design methods are elaborated and analytical formulae for PI/PID controllers tuning are presented.**

Keywords - characterization; control; optimization; PI/PID.

I. INTRODUCTION

Characterization of the processes by application of the critical experiment for purpose of tuning a controller is originally suggested by Ziegler and Nichols (ZN) in 1942 [1]. As a result of critical experiment in frequency domain, critical (ultimate) gain k_u and critical frequency ω_u are obtained and used to form an approximate model of the process $G_m^{ZN1}(s)=\exp(-Ls)/(bs)$, and $G_m^{ZN2}(s)=\exp(-Ls)/(bs+a)$ including known $G_p(0)$. On the basis of these and more complex models, a large number of tuning controller parameters formulae, including ZN rules are developed with a special emphasis on PI/PID controllers. A permanent progress in technology has led to the need for improving control algorithms [2]. This implies better process characterization including optimization design methods of controllers under constraints on performance and robustness of the control system. Need for better process characterization using the original idea, i.e. method of the critical experiment suggested by ZN, is still actual and inspiring for process dynamics estimation regardless if it is performed in the time or in the frequency domain. These two directions in the process characterization through critical experiment under certain conditions imply analysis of the process response in the time domain (the first direction) and in the frequency domain (the second direction) for assumed form of the transfer function of the process model [3-14]. Many of developed methods are referred to uncontrolled process (open-loop system), so their application is limited. Furthermore, the presence of the measurement noise, immeasurable disturbance and limitation on amplitude of critical oscillations also present limited conditions to be faced within the process characterization. Frequency characteristic of the process which is of concern for the process characterization and controller design is in the range $0.5\omega_u \le \omega \le 2.5\omega_u$ including $G_p(0)$ [3]. In accordance with these conditions, for better process characterization and adequate controller design, an efficient method has been recently reported in [15-25]. In addition to the parameters k_u

This paper is a revised and expanded version of the paper presented at the XVI International Symposium INFOTEH-JAHORINA 2017 [63].

Corespondence to: T. Šekara (tomi@etf.rs)

and ω_u, this method proposes to determine the angle (critical angle) of the tangent to the Nyquist curve in critical point $(-1/k_u, i0)$ using critical experiment. On the basis of estimated parameters, one obtains a general model process

$$G_m^{SM}(s) = \frac{1}{k_u} \frac{A\omega_u \exp(-\tau s)}{s^2 + \omega_u^2 - A\omega_u \exp(-\tau s)}, \qquad (1)$$

where $\tau = \varphi/\omega_u$ and $A = \omega_u k_u G_p(0)/(1 + k_u G_p(0))$ or $A \approx \frac{2}{k_u} \left| \partial G_p(i\omega)/\partial\omega \right|_{\omega=\omega_u}^{-1}$ if $G_p(0)$ is unknown. By using this characterization approach, selection of the process model is reduced to the unified model for all processes which satisfy critical experiment. This general model adequately approximates processes which behaviour can be described with linear mathematical models with finite and infinite degrees of freedom, including conventional systems of finite dimension, time-delay systems, systems with dominant wave and transport problems, such as mass and energy transfer, systems described with fractional differential equations etc. A partial drawback of this model occurs for the processes with dominant zeros. Let us emphasize that methods for estimation of parameters k_u, ω_u, φ and A of the processes using modified relay critical experiment and modified phase-locked loops [15–24], as well as methods based on the time response to the step excitation [25], can be performed with controlled process (process in the closed-loop with controller) or with the uncontrolled process (process in the open-loop) under certain conditions. Besides, other general identification techniques may be used for process characterization such as ARX (*AutoRegressive with eXternal input*), ARMA (*AutoRegressive Moving Average*), ARMAX (*AutoRegressive Moving Average with eXogeneous inputs*), BJ (*Box and Jenkins*), OE (*Output Error*) [26–28], etc.

Modern requirements within controller design are usually given in the form of certain optimality criteria with constraints. High-quality automatic control system (ACS) is the one which efficiently suppresses a load disturbance and tracks a desired reference. This problem mostly has two different solutions in terms of control signals which is a disadvantage, because these two problems cannot be considered separately from each other, since they are part of the same system. However, this can be solved with control structures which enable separate design of controller for load disturbance and the controller for reference tracking. Both the subsystems may be formally reduced to the simplified block diagram of ACS shown in Fig. 1.

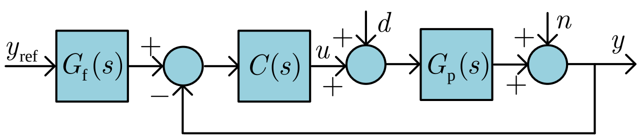

Figure 1. Simplified structure block diagram of ACS

In the rest of this paper the following notation is used: $G_p(s)$ is the process transfer function, $C(s)$ is controller transfer function, y_{ref} is reference signal, y is output signal, d is load disturbance, and n is measurement noise. Load disturbance is modelled at the input of the process which is emphasized in [29] by Shinskey citing that it is the most frequent case in the industry and $G_f(s)$ is feed forward filter.

The requirements within controller design are usually given in the form of four basic sensitivity functions: sensitivity function $S(s) = 1/(1 + C(s)G_p(s))$, complementary sensitivity function $T(s) = 1 - S(s)$, sensitivity function to the load disturbance $S_d(s) = G_p(s)/(1 + C(s)G_p(s))$ and sensitivity function to the measurement noise $S_n(s) = C(s)/(1 + C(s)G_p(s))$ [30–51]. A measure of robustness can be expressed through functions $S(s)$ and $T(s)$ with the form of their maximum values as $M_s = \max_\omega |S(i\omega)|$ and $M_p = \max_\omega |T(i\omega)|$. For stable processes of ACS, M_s should be in range $1.2 \le M_s \le 2$, while M_p should be as small as possible thus making system more robust. One of the performance measures is to maximize suppression of the load disturbance and is expressed in the form of function $S_d(s)$. This condition may be expressed equivalently by requiring that integral criterion (the most common choice is IAE (*Integral of Absolute Error*) [29]) applied to the time response of the sensitivity function $S_d(s)$ with respect to the disturbance d is minimal. Typical test signals of load disturbance are unit step and unit ramp. Besides, the measurement noise can cause large variations of control signal and thereby actuator damage. Therefore, design procedure should take into consideration sensitivity to the measurement noise which can be quantitatively defined as $M_n = \max_\omega |S_n(i\omega)|$ with aim to have acceptable values [33].

By applying optimization design procedures under constraints on robustness/performances one obtains a controller which may or may not be realizable in his original form. For example, it is common case for complex controllers with fractional differential and integral effect. Solution to this problem may be found in [52-54] where efficient techniques have been recently developed for rational approximations of complex transfer functions, which has made possible an adequate implementation of control algorithms and structures. Due to large application of PI/PID controllers, which exceeds 93% in industry according to Honeywell's surveys [55] compared to all the other regulators, many modern design methods have been developed in recent years. Thus, on the basis of general process model described with four parameters k_u, ω_u, φ and A, formulae for PI/PID controller under constraints to previously defined sensitivity functions in terms of adequate robustness/performance have been derived [17, 18].

II. The Application of Relay Techniques and Phase-Locked Loops for Process Characterization

Since the beginning of the harmonic linearization of nonlinear systems it has been concluded that replacement of the controller $C(s)$ with a conventional two-position relay with symmetric static characteristic in many control loops has led to a self-oscillation mode in the absence of an input signal

r regardless of the amplitude μ of the output signal from the relay. If it is assumed that the relay characteristic is without hysteresis, it turns out that $k_u \approx 4\mu/(\pi A_1)$ is valid, where established oscillations have amplitude A_1 of the first harmonic with angular frequency $\omega \approx \omega_u$. By using these principles a lot of methods for estimation of process parameters $k_u, \omega_u, G_p(0)$ and corresponding models have been developed [3–10]. Then, methods of the critical experiment based on the PLL (*Phase Locked Loop*) principle for process characterization have been developed [11, 12], etc. In accordance with these techniques for estimation of model parameters k_u, ω_u, φ and A and controller design (control adaptation), certain modifications of the relay experiment and PLLs have been developed [15-24]. The basic schemes of these structures, e.g. for process characterization with control are shown in Fig. 2 and Fig. 3. Block F is realized with cascade connection of adaptive band-pass filters defined with transfer function (2):

$$\mathrm{AF}_{p,q} = \prod_{m=p}^{q} \frac{\beta_m \omega_u s}{s^2 + \beta_m \omega_u s + \omega_u^2}, \beta_m = 2\cos(\pi(2m-1)/16) \quad (2)$$

Block denoted with F_{mod} in Fig. 2 presents a modification of AF_1 filter [19].

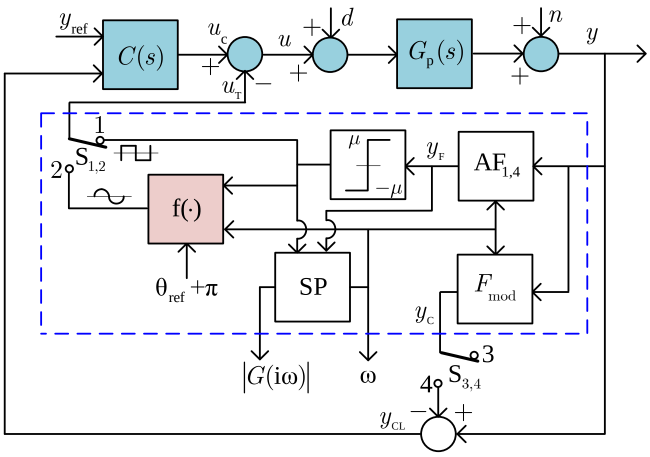

Figure 2. Simplified modified relay structure for estimation of Nyquist curve $G_p(i\omega)$ in several points for assigned values $\theta_{ref} = \arg\{G_p(i\omega)\}$

The same figure also shows block SP which executes an algorithm [19] for calculation of $k_u = 1/|G_p(i\omega_u)|$ and ω_u for $\theta_{ref} = -\pi$ which directly may be applied in the structure expanded with block $f(\cdot)$ shown in Fig. 2 for calculation of $|G(i\omega)|$ and ω in several points for assigned values θ_{ref} [5]. Start-up procedure and algorithm for estimation of parameters are applied in a similar way as in [19]. Thus, for assigned value θ_{ref}, switches are placed at first in positions S_1 and S_3. After several time periods until the permanent oscillations occur switches are set in positions S_2 and S_4, and after appearance of new oscillations, $|G_p(i\omega)|$ and ω are determined. Hence, after start-up procedure, algorithm executes dynamically in the time converging to values $|G_p(i\omega)|$ and ω for specified θ_{ref}.

Properties and the working principle of the structure shown in Fig. 3 for estimation of parameter $|G_p(i\omega)|$ and ω, for assigned values θ_{ref} are particularly analysed in [20].

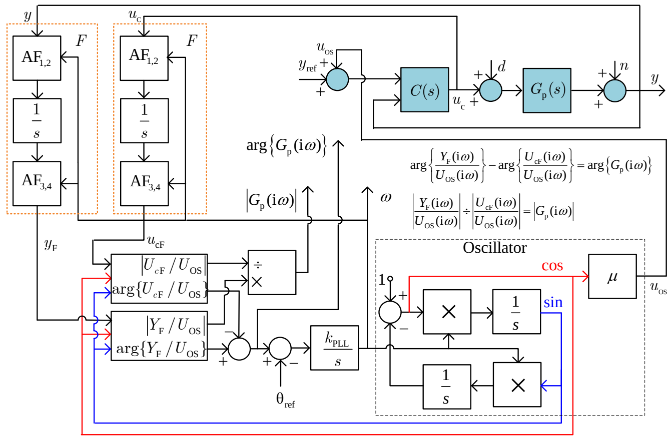

Figure 3. Simplified modified PLL structure for estimation of Nyquist curve $G_p(i\omega)$ in several points for assigned values $\theta_{ref} = \arg\{G_p(i\omega)\}$

The estimation of the angle $\varphi = \arg\left(\partial G_p(i\omega)/\partial\omega\big|_{\omega=\omega_u}\right)$ of the tangent to the Nyquist curve of the process $G_p(i\omega)$ in critical point $(-1/k_u, i0)$ is determined by the formula $\varphi \approx (\varphi^+ + \varphi^-)/2$ [15], where

$$\varphi^{\pm} = \arctan\left(\frac{\rho^{\pm}\sin\phi^{\pm}}{\rho^{\pm}\cos\phi^{\pm} - 1}\right), \; 0<\varphi^{\pm} < 2\pi, \; \rho^{\pm} = \frac{k_u}{k_u^{\pm}}, \quad (3)$$

for $\theta_{ref} = -\pi + \phi^{\pm}$, $\phi^+ = \phi$, $\phi^- = -\phi$, $\phi > 0$, $k_u^{\pm} = 1/|G_p(i\omega_u^{\pm})|$ as it is shown in Fig. 4.

Let us mention that angle φ can be estimated from two close points, i.e. for small angle ϕ, but because of the small difference between measured parameters (k_u, k_u^+), and (k_u, k_u^-), measurement uncertainty can be rather large. In order to decrease the measurement uncertainty of parameters k_u, k_u^+ and k_u^-, greater angle ϕ should be used. However, this approach leads to large error in the estimation of the angle φ. Compromise found for adequate estimation of angle φ is using three points at Nyquist curve as shown in Fig. 4.

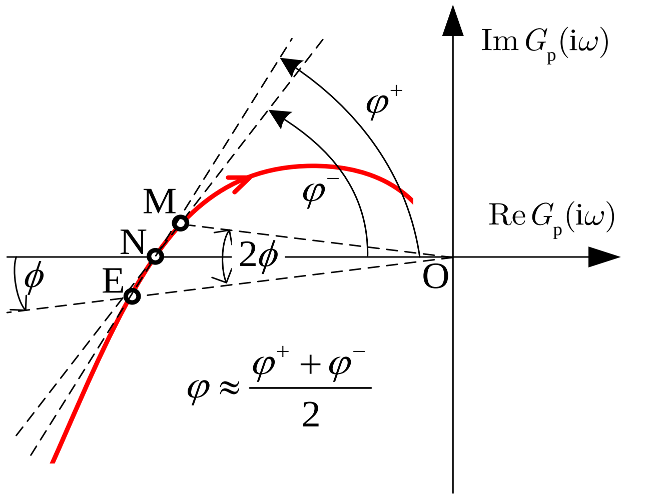

Figure 4. The estimation of parameter φ from three points at Nyquist curve [15]

Series of simulations show that suitable angle is $\phi \approx \pi/36$. Estimation of the process parameter $A=\omega_u k_u G_p(0)/(1+k_u G_p(0))$ in some working regime in the presence of measurement noise and load disturbance [16, 17] can be determined from formula $A \approx A_0$ where

$$A_0 = \frac{2}{k_u}\left|\frac{\partial G_p(\mathrm{i}\omega)}{\partial \omega}\right|^{-1}_{\omega=\omega_u} \approx \frac{A_0^+ + A_0^-}{2}, \qquad (4)$$

ie.

$$A_0^{\pm} = \frac{2\left|\omega_u - \omega_u^{\pm}\right|}{\sqrt{1\text{-}2\rho^{\pm}\cos\phi^{\pm} + (\rho^{\pm})^2}}, \; \rho^{\pm} = \frac{k_u}{k_u^{\pm}}, \qquad (5)$$

for $\phi^+ = \phi$, $\phi^- = -\phi$, $\phi > 0$, $k_u^{\pm} = 1/\left|G_p(\mathrm{i}\omega_u^{\pm})\right|$ as illustrated in Fig. 4. For integral processes equality $A=\omega_u$ holds true [15]. Since the general model does not require $G_p(0)$, but the estimated parameter A, such model is suitable for controller design and process classification [15–23].

Let us note that sensitivity of the parameter A with respect to $k_u G_p(0)$ is small, thus reverse estimation $k_u G_p(0)$ from A_0 is not practical (it is unacceptable) because it may cause large deviations. More accurate estimation of parameter A of the process in working regime in presence of measurement noise and load disturbance can be achieved by direct estimation of $k_u G_p(0)$ using another methods, either in the frequency or in the time domain [25]. For example, the estimation of $k_u G_p(0)$ from Nyquist curve $G_p(\mathrm{i}\omega)$ at several points for specified values $\theta_{\mathrm{ref}}=\arg\{G_p(\mathrm{i}\omega)\}$, etc. In the case of multiple solution of the equation $\theta_{\mathrm{ref}}=\arg\{G_p(\mathrm{i}\omega)\}$, as well as for those solutions with $\arg\{G_p(\mathrm{i}\omega)\}>2\pi$, PLL approach requires that frequency of interest is known a priori. Approaches using relay experiment have advantage in that sense, because they give unique solution and experimental procedures are shorter.

For example, Fig. 5 shows Nyquist curves of distributed process described with transfer function $G_{p13}(s) = 1/\cosh\sqrt{2s}$ and corresponding model $G_{m13}^{SM}(s)$, for $k_u = 11{,}5919$, $\omega_u = 9{,}8696$, $\tau = 0{,}0796$ and $A = 9{,}0858$. It is obvious from Fig. 5 that obtained Nyquist curves of process and its model are practically the same [15].

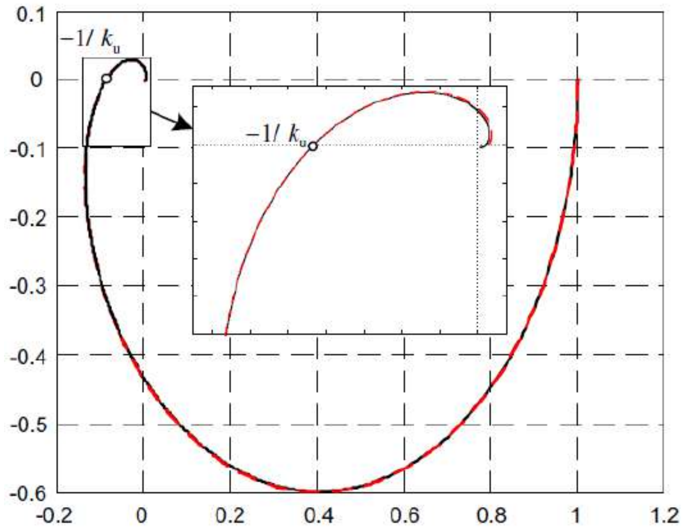

Figure 5. Nyquist curves for the process $G_p(\mathrm{i}\omega)$ (thick black line) and its corresponding model $G_m^{SM}(\mathrm{i}\omega)$ (dashed red line) [15]

III. Classification of Large Class of Processes in Two-Parameter Plane

Normalization of general process model $G_m^{SM}(s)$ by time $\omega_u t$ (i.e. in complex domain as s/ω_u) and by amplitude with $k_u G_m^{SM}(s)$, gives a general normalized model

$$G_m^{SM}(s_n) = \frac{\rho\exp(-\varphi s_n)}{s_n^2 + 1 - \rho\exp(-\varphi s_n)}, \qquad (6)$$

as the function of two parameters $\rho = A/\omega_u$ and φ [18]. Fig. 6 shows a stability area defined with $0<\varphi<\pi/\sqrt{\rho+1}$, $0<\rho<1$ and classification of the large class of processes $G_{pi}(s), i=\overline{1,14}$ obtained by characterization in parameter $\rho-\varphi$ parameter plane [18]:

$$G_{p1}(s) = \frac{e^{-s}}{Ts+1}, \; 0{,}02 \le T \le 1000, \; G_{p2}(s) = \frac{e^{-s}}{(Ts+1)^2}, \; 0{,}01 \le T \le 500$$

$$G_{p3}(s) = \frac{1}{(s+1)(Ts+1)^2}, \; 0{,}005 \le T \le 10, \; G_{p4}(s) = \frac{1}{(s+1)^n}, \; 3 \le n \le 8,$$

$$G_{p5}(s) = \frac{1}{(s+1)(\alpha s+1)(\alpha^2 s+1)(\alpha^3 s+1)}, \; 0.1 \le \alpha \le 0.9,$$

$$G_{p6}(s) = \frac{e^{-sL_1}}{s(s(1-L_1)+1)}, \; 0.01 \le L_1 \le 1,$$

$$G_{p7}(s) = \frac{T}{(Ts+1)((1-L_1)s+1)}e^{-sL_1}, \; T=1,2,5,10, \; 0{,}01 \le L_1 \le 1,$$

$$G_{p8}(s) = \frac{1-\alpha s}{(s+1)^3}, \; 0.1 \le \alpha \le 1.1,$$

$$G_{p9}(s) = \frac{1}{(s+1)(T^2 s^2 + 1.4Ts + 1)}, \; 0{,}1 \le T \le 1,$$

$$G_{\mathrm{p10}}^{\triangle}(s)=\frac{e^{-Ls}}{(s^2+0.1s+1)},\ L=0,5;1;2,$$

$$G_{\mathrm{p11}}^{\square}(s)=\frac{e^{-Ls}}{(5s-1)(2s+1)(0.5s+1)},\ L=0,5;1,$$

$$G_{\mathrm{p12}}^{\square}(s)=\frac{4e^{-Ls}}{4s-1},\ L=0,5;1;2,\ G_{\mathrm{p13}}(s)=1/\cosh\sqrt{2s},\ G_{\mathrm{p14}}(s)=e^{-\sqrt{s}}.$$

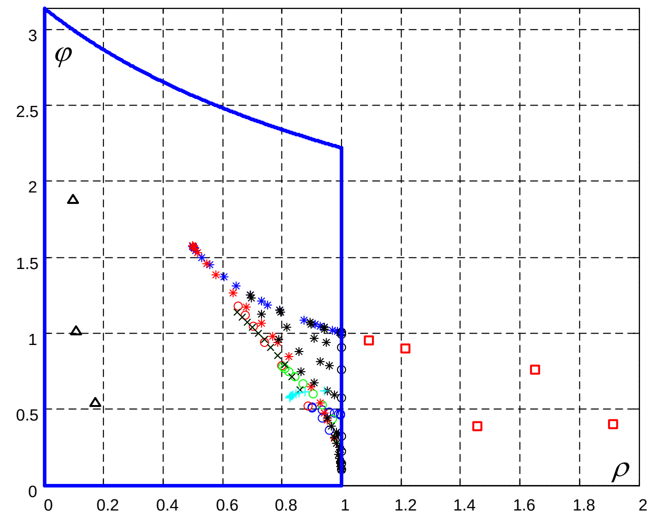

Figure 6. Classification of the large class of the processes $G_{\mathrm{p}i}(s), i=\overline{1,14}$ in the parameter ρ–φ parameter plane [18]

The classification of processes enables a designer to develop a priori look-up tables and controller structure which can satisfy assigned performance/robustness as well as an ability for adequate adaptation of the control system [15–25].

IV. Modern Controller Design Methods

In order to achieve high-quality ACS with adequate performance and robustness indices it is necessary to efficiently supress the load disturbance (min(IAE) [29]) and achieve the desired reference tracking under specified conditions on the sensitivity functions S, T, S_{d} and [30–51]. Besides these conditions, alternative constraints such as: gain and phase margins, location of dominant poles, settling time, overshoot etc. may be used. In this sense, criterion functions for optimization of controller parameters under specified constraints on sensitivity functions or by using alternative conditions are formed. To do this, specific types of controller should be assumed, and adequate control chosen, whether the design procedure is performed in the time or in the frequency domain. For example, a fractional PID controller (PI$^{\alpha}$D$^{\beta}$) described with transfer function

$$C(s)=\frac{k+k_{\mathrm{i}}/s^{\alpha}+k_{\mathrm{d}}s^{\beta}}{(T_{\mathrm{f}}s+1)^{\beta}} \tag{7}$$

can be used, where $k, k_{\mathrm{i}}, k_{\mathrm{d}}, T_{\mathrm{f}}, \alpha$ and β are real parameters which are determined using some of design methods to satisfy desired requirements of ACS. Fig. 7 shows PI$^{\alpha}$D$^{\beta}$ in α,β parameter parameter plane, which for certain values of parameters α and β reduces to the conventional controllers: P, PI, PD, PID, etc.

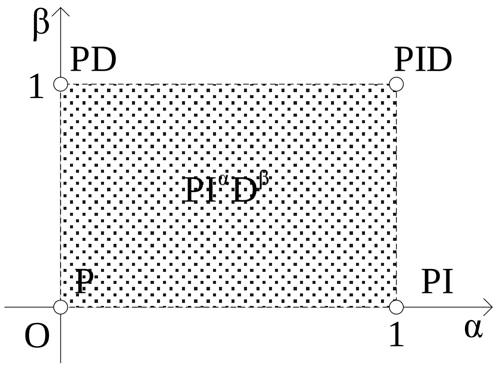

Figure 7. Parameter plane of PI$^{\alpha}$D$^{\beta}$ controller [56]

In a similar way, fractional PIDC controller (PI$^{\alpha}$D$^{\beta}$C$^{\gamma}$) described with transfer function

$$C(s)=\frac{k+k_{\mathrm{i}}/s^{\alpha}+k_{\mathrm{d}}s^{\beta}}{(T_{\mathrm{f}}s+1)^{\beta}}\left(\frac{bs+1}{as+1}\right)^{\gamma}, \tag{8}$$

may be used, corresponding to the cascade connection of PI$^{\alpha}$D$^{\beta}$ controller with fractional compensator $((bs+1)/(as+1))^{\gamma}$.

As it can be seen from (7) and (8), the degree of the controller complexity and number of unknown parameters could make the design of ACS more difficult. This has motivated many researchers to develop new design methods of control algorithms including, at the first place, different optimization algorithms [30-51].

For example, for distributed process $G_{\mathrm{p13}}(s)$, with approximate model $G_{\mathrm{m13}}^{\mathrm{SM}}(s)$ obtained by some of the characterization methods, optimal PID controller is designed under constraint on $M_{\mathrm{s}}=2$ and $M_{\mathrm{n}}=2k_{\mathrm{u}}$ with control structure shown in Fig. 8.

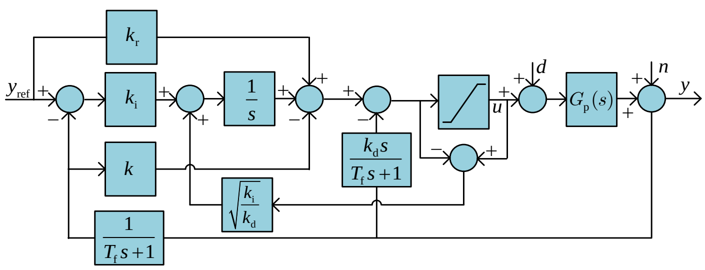

Figure 8. Structure block diagram of ACS with separated differential and proportional gain of PID controller and included elements with integral antiwindup [30-32]

As a result, an efficient suppression of load disturbance and desired reference tracking is achieved shown in Fig. 9, [15, 17, 18, 40], and corresponding control signal is shown in Fig. 10.

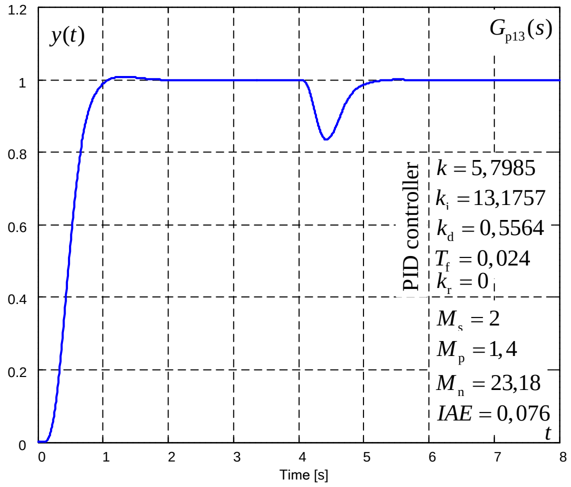

Figure 9. Output signal $y(t)$ of ACS for reference signal $Y_{\text{ref}}(s)=1/s$ and load disturbance $D(s)=\exp(-4s)/s$

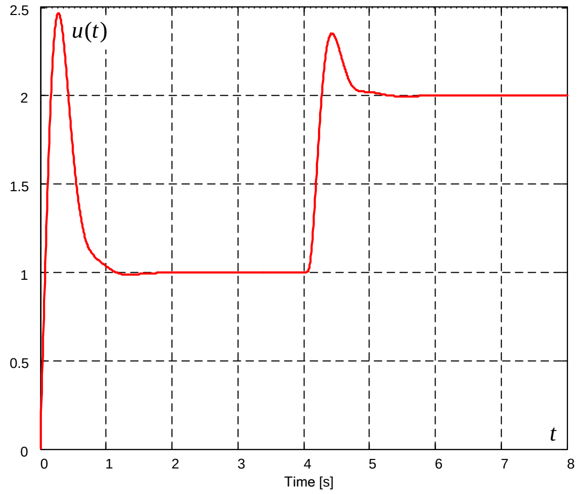

Figure 10. Control signal $u(t)$ of ACS for reference signal $Y_{\text{ref}}(s)=1/s$ and load disturbance $D(s)=\exp(-4s)/s$

For criterion function in the optimization procedure for PID controller under constraints in frequency domain [18], the proportional gain of controller (max k) which gives the best compromise in performance/robustness is used [40]. In literature, the integral gain for criterion function (max k_i) is usually used [31–39]. In general case, optimization procedures in the time domain are more demanding because of process with controller transfer function complexity in ACS and corresponding mapping to the time domain. Integral part of such procedures consists of numerical calculations of inverse Laplace transform and other numerical techniques within optimization procedure. The most common used criterions are IAE or ITAE (*Integral of Time-weighted Absolute Error*) with respect to the load disturbance input [29]. Also, depending on complexity of controller and optimization procedure, multi-criterion functions under certain constraints are often used.

A. *Digital implementation of control algorithms*

Implementation of control systems is nowadays performed in digital domain while controller design may be performed in both, continuous and digital domain. If process and controller are described with rational transfer functions, both of these approaches are equal because of effective transformations from continuous to digital domain without disturbing the quality of the control. If the controller (6), (7) or process is non-rational transfer function, the design is performed in continuous domain and then controller is approximated in continuous or digital domain [52-57]. It is important that rational approximation should include amplitude and phase frequency characteristic of complex controller with minimal deviation in order to preserve quality control. In general, continuous to digital domain transformations are actual whether the design has been performed in the time or in the frequency domain.

TABLE I. APPROXIMATION FORMULAE FOR SOME KNOWN DISCRETIZATION RULES GIVEN IN [52]

α	$f(z,\alpha)$	Name of approximation
0	$\frac{z-1}{T}$	Forward-Euler
0.5	$\frac{2}{T}\frac{z-1}{z+1}$	Tustin
1	$\frac{1}{T}\frac{z-1}{z}$	Backward-Euler
α	$\frac{1}{T}\frac{z-1}{1+\alpha(z-1)}$	Fractional approximation of the first order

Without loss of generality, let us consider the controller design performed in continuous domain. If controller transfer function is rational then different numerical approximations such as: Forward-Euler, Backward Euler, Tustin approximation may be used. These approximation methods are indirect methods since they approximate the operator s^{-1}, and may be generalized with fractional approximation of the first order as elaborated in [52]. Approximation formulae for these common discretization rules are given in Table I.

Another possibility to discretize rational transfer function is to use transformation polynomials for $1/s^k$, $k=0,\ldots,n$ where n is the order of the considered continuous transfer function. In this way, discrete transfer function is obtained by replacing s^{-k} with $f_k(z)$ from Table II. This approximation method is elaborated in detail in [52].

However, when transfer function is non-rational, then some of rational approximation must be used. Different proposed methods for rational approximation can be found in literature such as: interpolation of frequency characteristic (IFC) [54], ARX-based methods [57], expansions in Taylor series, use of Padé approximation [56] etc. In this way continuous transfer function becomes rational and we can apply some of the abovementioned discretization rules.

TABLE II. APPROXIMATION EQUIVALENTS OF TRANFORMATION POLYNOMIALS OF S^-K FOR K=1,2,3,4,5 GIVEN IN [52]

k	s^{-k}	$f_n(z)$
1	s^{-1}	$\frac{T}{2}\frac{z+1}{z-1}$
2	s^{-2}	$\frac{T^2}{12}\frac{z^2+10z+1}{(z-1)^2}$
3	s^{-3}	$\frac{T^3}{2}\frac{z(z+1)}{(z-1)^3}$
4	s^{-4}	$\frac{T^4}{720}\frac{-z^4+124z^3+474z^2+124z-1}{(z-1)^4}$
5	s^{-5}	$\frac{T^5}{24}\frac{z(z+1)(z^2+10z+1)}{(z-1)^5}$

B. Analytical design methods of PI/PID controller

The main idea for analytical design methods of PI/PID controllers comes from Ziegler-Nichols in 1942. [1], who were the first to give analytical tuning formulae. From then until now, great number of analytical formulae for tuning PI/PID controller has been developed [2]. Drawback of numerical methods for controller design are unknown initial values on which the final solution depends. Modern controller design requirements have led to need for efficient analytical formulae of PI/PID controller due to their large application in industry [58]. Thus, novel general analytical tuning formulae of PI/PID controllers have been developed recently for all processes which satisfy a critical oscillation requirement [17, 18, 22–24].

The basic rule for design of any controller is the efficient suppression of the load disturbance. Considering that $Y_d(s)=S_d(s)D(s)$ is valid for the structure in Fig. 1 which is equivalent with $Y_d(s)=(1-T(s))G_p(s)D(s)$. Ideal case would be if sensitivity function $S_d(s)$ supressed load disturbance for all frequencies to zero, i.e. $T(s)=1$, which is not possible. The idea is to choose $T_z(s)$ on the basis of known process transfer function [17, 51]. In order to design a controller $C(s)$, the control structure with the controller $C^*(s)$ is introduced in [51] for the equivalent model

$$G_p^*(s)=\frac{A\omega_u\exp(-\tau s)}{s^2+\omega_u^2}\approx\frac{k_uG_p(s)}{1+k_uG_p(s)}. \tag{9}$$

Thereby, dynamics of suppression of the load disturbance at the input of $G_p^*(s)$ and at the input of $G_p(s)$ is equivalent [51]. For model $G_p^*(s)$ complementary sensitivity function is selected in the form:

$$T_z(s)=N(s)\exp(-\tau s)/P(s), \tag{9}$$

where $N(s)=(\eta_2s^2+\eta_1s+1)$ and $P(s)=(\lambda^2s^2+2\zeta\lambda s+1)^2$, with $\lambda>0$ is the time constant, $\zeta>0$ is relative damping factor, and η_1 i η_2 are free parameters determined to efficiently suppress the load disturbance.

TABLE III. ANALYTICAL TUNING FORMULAE FOR PARAMETERS OF PI/PID CONTROLLERS FOR GENERAL PROCESS MODEL DEFINED WITH FOUR PARAMETERS k_u,ω_u,φ AND A [17]

General process model	PI controller	PID controller
$G_m^{SM}(s)=\frac{1}{k_u}\frac{A\omega_u\exp(-\tau s)}{s^2+\omega_u^2-A\omega_u\exp(-\tau s)}$	$C_{PI}(s)=k+\frac{k_i}{s}$ $k_i=a_0,\quad k=a_1$	$C_{PID}(s)=\frac{k_i+ks+k_ds^2}{s(T_fs+1)}$ $T_f=a_2/(M_n-a_1)$, or $T_f=\lambda/N,\ N>2$ $k_i=a_0,\quad k=a_1+a_0T_f,\quad k_d=a_2+a_1T_f$

$$a_0=k_u\beta_1,\quad a_1=a_0(\eta_1-\beta_2)+k_u,\quad a_2=a_0(\eta_2-\beta_3+1/\omega_u^2)-(a_1-k_u)\beta_2$$

$$\beta_1=\frac{\omega_u}{A(-\eta_1+\tau+4\zeta\lambda)},\quad \beta_2=\frac{-\eta_2+\tau\eta_1-\tau^2/2+2\lambda^2+4\zeta^2\lambda^2}{-\eta_1+\tau+4\zeta\lambda},\quad \beta_3=\frac{\tau\eta_2-\tau^2\eta_1/2+\tau^3/6+4\zeta\lambda^3}{-\eta_1+\tau+4\zeta\lambda}$$

$$\eta_1=\frac{\alpha_1\sin(\omega_u\tau)+\alpha_2\cos(\omega_u\tau)}{\omega_u},\quad \eta_2=\frac{\alpha_2\sin(\omega_u\tau)-\alpha_1\cos(\omega_u\tau)+1}{\omega_u^2}$$

$$\alpha_1=\lambda^4\omega_u^4-2\lambda^2\omega_u^2(1+2\zeta^2)+1,\quad \alpha_2=4\zeta\lambda\omega_u(1-\lambda^2\omega_u^2)$$

Controller $C^*(s)$ is defined with relation

$$C^*(s)=\frac{T_{\mathrm{z}}(s)}{1-T_{\mathrm{z}}(s)}\frac{1}{G_{\mathrm{p}}^*(s)}, \tag{11}$$

from where

$$C^*(s)=\frac{1}{A\omega_{\mathrm{u}}}\frac{(s^2+\omega_{\mathrm{u}}^2)(\eta_2 s^2+\eta_1 s+1)}{(\lambda^2 s^2+2\zeta\lambda s+1)^2-e^{-\tau s}(\eta_2 s^2+\eta_1 s+1)}. \tag{12}$$

The next step is shaping of dynamic error which can be done by cancelling poles $s_{\mathrm{p}1,2}=\pm \mathrm{i}\omega_{\mathrm{u}}$ of model $G_{\mathrm{p}}^*(s)$ with zeros of the following polynomial

$$((\lambda^2 s^2+2\zeta\lambda s+1)^2-e^{-\tau s}(\eta_2 s^2+\eta_1 s+1))\Big|_{s=\pm \mathrm{i}\omega_{\mathrm{u}}}=0. \tag{13}$$

Parameters η_1 and η_2 are determined from equation (13). In this way by selecting free parameter λ, a designer can adequately suppress the load disturbance without violating other restrictions of ACS.

Parameters of a conventional PID controller in form (7) for $\alpha=\beta=1$, are obtained by expanding the function

$$f(s)=s(T_{\mathrm{f}}s+1)(C^*(s)+1)k_{\mathrm{u}}, \tag{14}$$

in Maclaurin series in variable s using the first three members of expansion, i.e. $f(s)\approx f(0)+f'(0)s+0.5f''(0)s^2$. Parameters of PID controller are connected with coefficients of Maclaurin expansion through relations

$$k_{\mathrm{i}}=f(0),\;\; k=f'(0),\;\; k_{\mathrm{d}}=f''(0)/2. \tag{15}$$

Filter time constant is defined with $T_{\mathrm{f}}=\dfrac{\mathrm{sign}(k_{\mathrm{u}})k_{\mathrm{d}}}{M_{\mathrm{n}}}$, which is determined from equation

$$2T_{\mathrm{f}}M_{\mathrm{n}}-\mathrm{sign}(k_{\mathrm{u}})f''(0)=0 \tag{16}$$

Values of T_{f} and other parameters of PID controllers for all processes which satisfy condition of critical oscillations are given in Table III. Analogous approach is used to obtain parameters of PI controller which analytical formulae are also given in Table III. This methodology is successfully applicable and up-to-date for design of complex controllers [21, 51, 59, 60] and control structures [61, 62].

V. Conclusion

Synthesis of an automatic control system (ACS) starts from the process model, thus this paper presents a general model $G_{\mathrm{m}}^{\mathrm{SM}}(s)$ for all processes which satisfy critical experiment. The general model is defined with four parameters $k_{\mathrm{u}},\omega_{\mathrm{u}},\varphi$ and A, which are determined using one of characterization methods e.g. relay techniques and phase-locked loops as the most popular ones. Besides, large class of processes can be classified in $\rho-\varphi$ parameter plane, from where parameters of PID controller are directly obtained for specified M_{s} and M_{n} from look-up tables on the basis of previously defined design procedure. The aim of ACS is to suitably select criterion function for optimization of controller under specified constraints on sensitivity functions S, T, S_{d} i S_{n} and achieve efficient suppression of load disturbance and adequate indices of robustness and performance. At the end of the paper, analytical tuning formulae of PI/PID controllers are presented because of their dominant use in process industry.

Acknowledgment

The author Tomislav B. Šekara gratefully acknowledges the support of Ministry of Education, Science and Technological Development of the Republic of Serbia under project TR33020. The author is particularly grateful to his colleagues Prof. dr Slobodan Milojković, Prof. dr Milan Rapaić, Prof. dr Milica Naumović and M.Sc. Marko Bošković for the support and valuable comments in preparing this paper.

References

[1] J. G. Ziegler, N. B. Nichols, “Optimum settings for automatic controllers”, Trans. ASME, vol. 64, pp. 759–768, 1942.

[2] A. O'Dwyer, “Handbook of PI and PID controller tuning rules”, Imperial College Press, 3rd edition, 2009.

[3] Ja. V. Rotač, “Design and Tuning of Industrial Control Systems” (in Czech), SNTL, Prague, 1964.

[4] K. J. Åström, T. Hägglund, “Automatic tuning of simple regulators with specifications on phase and amplitude margins”, Automatica, vol. 20, pp. 645-651, 1984.

[5] H. Lee, Q. G. Wang, K. K. Tan, “A modified relay-based technique for improved critical point estimation in process control”, IEEE Transactions on Control System Technology, vol. 3, pp. 330-337, 1995.

[6] I. Kaya, D. P. Atherton, “Parameter Estimation from Relay Autotuning with Asymmetric Limit Cycle Data”, Journal of Process Control, vol. 11, pp. 429-439, 2001.

[7] C.C. Hang, K. J. Åström, Q. G. Wang, “Relay feedback auto-tuning of process controllers – a tutorial review”, Journal of Process Control, vol. 12, pp. 143-162, 2002.

[8] R. C. Panda, C. C. Yu, “Shape Factor of Relay Response Curves and Its Use in Autotuning”, Journal of Process Control, vol. 15, pp. 893-906, 2005.

[9] S. Majhi, “Relay Based Identification of Processes with Time Delay”. Journal of Process Control, vol. 17, pp. 93-101, 2007.

[10] T. Liu, F. Gao, “A Generalized Relay Identification Method for Time Delay and Non-minimum Phase Processes”, Automatica, vol. 45, pp. 1072-1079, 2009.

[11] J. Crowe, M. A. Johnson, “Process identifier and its application to industrial control”, IEE Proceedings Control Theory and Applications, Vol. 147, pp. 196-204, 2000.

[12] D. W. Clarke, J. W. Park, “Phase-locked loops for plant tuning and monitoring”, IEE Proceedings on Control Theory and Applications, vol. 150, pp. 155-169, 2003.

[13] C. C. Hang, K. J. Åström, W. K. Ho, “Refinements of the Ziegler-Nichols tuning formula”, IEE Proceedings of Control Theory and Applications, vol. 138, pp. 111-118, 1991.

[14] K. J. Åström, T. Hägglund, “Revisiting the Ziegler-Nichols step response method for PID control”, Journal of Process Control, vol. 14, pp. 635-650, 2004.

[15] T.B. Šekara, M.R. Mataušek, “Revisiting the Ziegler–Nichols process dynamics characterization”, Journal of Process Control, vol. 20, pp. 360–363, 2010.

[16] T. B. Šekara, M. R. Mataušek, “Comparative analysis of the relay and phase-locked loop experiment used to determine ultimate frequency and ultimate gain”, ELECTRONICS, vol. 14, pp. 77-81, 2010.

[17] M.R. Mataušek, T.B. Šekara, "PID controller frequency-domain tuning for stable, integrating and unstable processes, including dead-time". Journal of Process Control, vol. 21, pp. 17–27, 2011.

[18] T. B. Šekara, M. R. Mataušek, "Classification of dynamic processes and PID controller tuning in a parameter plane", Journal of Process Control, vol. 21, pp. 620-626, 2011.

[19] T. B. Šekara, M. R. Mataušek, "Relay-based critical point estimation of a process with the PID controller in the loop", Automatica, vol. 47, pp. 1084-1088, 2011.

[20] T.B. Šekara, M. R. Mataušek, "Robust process identification by using Phase-Locked-Loop", (in Serbian), Proceedings of Conference Infoteh-Jahorina, Bosnia and Herzegovina, vol. 10, pp. 18-21, 2011.

[21] T. B. Trifunović, T.B. Šekara, "Tuning formulae for PID/PIDC controllers of processes with the ultimate gain and ultimate frequency" (in Serbian), Proceedings of Conference Infoteh-Jahorina, Bosnia and Herzegovina, vol. 10, pp. 12-17, 2011.

[22] T.B. Šekara, M. R. Mataušek, "PID controller tuning based on the classification of stable, integrating and unstable processes in a parameter plane", In: Serra G.L.O., editor. Frontiers in advanced control systems. InTechOpen, pp.117–142, 2012.

[23] T.B. Šekara, M. R. Mataušek, "New tuning formulae for the PI controller applied to processes with integral action", (in Serbian), Vojnotehnički glasnik, vol. 61, pp. 7-20, 2013.

[24] T.B. Šekara, M.R. Mataušek, "Nove formule podešavanja PID regulatora sa dva stepena slobode u frekvencijskom domenu", (in Serbian), INFOTEH-JAHORINA, vol. 11, pp. 976-981, March 2012.

[25] M.R. Mataušek, T.B. Šekara, "A fast closed-loop process dynamics characterization", ISA Transactions, vol. 53, pp. 489-496, 2014.

[26] L. Ljung, "System Identification, Theory for the User", 2nd ed., Prentice-Hall, Inc., 1999.

[27] Karel J. Keesman, "System Identification, An Introduction", Springer, 2011.

[28] R. Iserman, M. Münchhof, "Identification of Dynamic Systems, An Introduction with Applications", Springer, 2011.

[29] F. G. Shinskey, "Process Control Systems: Application, Design, and Tuning", 4th Edition, McGraw-Hill Inc., New York, USA, 1996.

[30] K. J. Åström, R. M. Murray, Feedback systems, An Introduction for scientists and engineers, Princeton University, 2008.

[31] K.J. Åström, T. Hägglund, "PID Controllers: Theory, Design and Tuning", Instrument Society of America, N. Carolina, USA, 1995.

[32] K. J. Åström, R. M. Murray, Advanced PID Control, ISA Instrumentation, Systems and Automation Society, 2006.

[33] O. Garpinger, "Analysis and Design of Software-Based Optimal PID Controllers", Lund University, Department of Automatic Control, Sweden, 2015.

[34] H. Panagopoulos, K.J. Åström and T. Hägglund, "Design of PID controllers based on constrained optimization", IEE Proceedings-Control Theory and Applications, vol. 149, pp. 32-40 January 2002.

[35] A. Wallen, K. J. Åström, and T. Hägglund, Loop-shaping design of PID controllers with constant T_i/T_d ratio", Asian Journal of Control, vol. 4, pp. 403-409, December 2002.

[36] A.J. Isaksson and S.F. Graebe, "Derivative filter is an integral part of PID design", IEE Proceedings-Control Theory and Applications, vol.149, pp. 41-45, January 2002.

[37] C. Hwang and C-Y. Hsiao, "Solution of non-convex optimization arising in PI/PID control design", Automatica, vol. 38, pp. 1895-1904, November 2002.

[38] B. Kristiansson and B. Lennartson, "Evaluation and simple tuning of PID controllers with high-frequency robustness", Journal of Process Control, vol. 16, pp. 91-102, February 2006.

[39] B. Kristiansson and B. Lennartson, "Robust tuning of PI and PID controllers: using derivative action despite sensor noise", IEEE Control Systems Magazine, vol. 26, pp. 55-69, February 2006.

[40] T.B. Šekara and M.R. Mataušek, "Optimization of PID controller based on maximization of the proportional gain under constraints on robustness and sensitivity to measurement noise ", IEEE Trans. Automatic Control, vol. 54, no.1, pp.184-189, Jan. 2009.

[41] P.O. Larsson, T. Hägglund, "Control signal constraints and filter order selectionfor PI and PID controllers", in: American Control Conference, San Francisco, CA, USA, pp. 4994–4999, 2011.

[42] O. Garpinger, T. Hägglund, K. J. Åström, "Criteria and Trade-offs in PID Design", IFAC Proc. Volumes, vol. 45, Issue 3, pp. 47-52, 2012.

[43] T. Hägglund, "A unified discussion on signal filtering in PID control", Control Engineering Practice, vol. 21, pp. 994-1006, Aug. 2013.

[44] V.R. Segovia, T. Hägglund, K.J. Åström, "Measurement noise filtering for PID controllers", Journal of Process Control, Vol. 24, pp. 299-313, 2014.

[45] O. Garpinger, T. Hägglund, "Software-based optimal PID design with robustness and noise sensitivity constraints", Joural of Process Control, vol 33, pp. 90-101, 2015.

[46] K. Soltesz, C. Grimholt, S. Skogestad, "Simultaneous design of proportional-integral-derivative controller and measurement filter by optimisation", IET Control Theory&Applications, pp.1-16, Nov. 2016.

[47] T.B. Šekara, M.R. Rapaić, M.P. Lazarević, "Optimal tuning of fractional $PIDC^{\alpha}$ controller in the frequency domain", in Proc. International Conference on Fractional Differentiation and Its Applications (ICFDA)-Catania, pp. 1-4, June 2014.

[48] Q-G Wang, Z. Zhang, K.J. Åström, „Guaranteed dominant pole placement with PID controllers", Journal of Process Control, vol. 19, pp. 349-352, 2009.

[49] P.D. Mandić, T.B. Šekara, M.P. Lazarević, M. Bošković, „Dominant pole placement with fractional order PID controller: D-decomposition approach", ISA Transactions, vol. 67, pp. 76-86, March 2017.

[50] T.B. Šekara, M.R. Rapaić, „A revision of root locus method with applications", Journal of Process Control, vol. 34, pp. 26-34, 2015.

[51] T.B. Šekara, M.B. Trifunović, V. Govedarica, "Frequency domain design of a complex controller under constraints on robustness and sensitivity to measurement noise", ELECTRONICS, vol. 15, pp. 40-44, 2011.

[52] T.B. Šekara, "New transformation polynomials for discretization of analogue systems", Electrical Engineering, vol. 89, pp. 137-147, 2006.

[53] M.R. Rapaić, T.B. Šekara, "Novel direct optimal and indirect method for discretization of linear fractional systems", Electrical Engineering, vol. 93, pp. 91-102, June 2011.

[54] T. B. Šekara, M. R. Rapaić, M. P. Lazarević, "An Efficient Method for Approximation of Non-Rational Transfer Functions", ELECTRONICS, Vol. 17, pp. 40-44, 2013.

[55] M. R. Rapaić**,** T. B. Šekara, M. P. Lazarević, "On Discrete, Finite-Dimensional Approximation of Linear, Infinite Dimensional System", Chapter 11 in "Fractional Calculus: Theory", Roy Abi Zeid Daou and Xavier Moreau (Eds.), Hauppauge, New York : Nova Science Publishers, Inc., pp. 257-274., 2014.

[56] M. Bošković, T.B. Šekara, M.R. Rapaić, B.B. Jakovljević, "Rational approximations to design controllers for unstable processes, including dead-time", International Symposium on Industrial Electronics INDEL, pp. 200-204, Banja Luka 2014.

[57] M. Č. Bošković, T. B. Šekara, M. R. Rapaić, M. Lazarević, P. Mandić, "A novel ARX-based discretization method for linear non-rational system", International Conference on Fractional Differentiation and its Applications, Novi Sad, 2016.

[58] L. Desbourough, R. Miller, "Increasing customer value of industrial control performance monitoring—Honeywell's experience", in: Sixth International Conference Chemical Process Control, AIChE Symposium Series Number 326, vol. 98, 2002.

[59] T. B. Šekara, "Fractional control systems", (in Serbian), Faculty of Electrical Engineering East Sarajevo, 2011.

[60] M. Č. Bošković, T. B. Šekara, M. R. Rapaić, V. Govedarica, "Novel tuning rules for stable dead-time processes with dominant left half-plane zero" XI International Symposium on Industrial Electronics – INDEL, Banja Luka, 2016.

[61] J.E. Normey-Rico, E.F. Camacho, "Unified approach for robust dead-time compensator design", J Process Control, vol. 19, pp. 38-47, 2009.

[62] M. Shamsuzzoha, M. Lee, "Enhanced disturbance rejection for open-loop unstable process with time delay", ISA Transactions, pp. 237-244, 2009.

[63] T.B. Šekara, "Savremeni postupci projektovanja, analize, optimizacije i implementacije konvencionalnih upravljačkih algoritama za procese sa konačno i beskonačno stepeni slobode", INFOTEH-JAHORINA, vol. 16, pp. 652-659, 2017.

Tomislav B. Šekara was born in Mjedenik (Gacko), Bosnia and Herzegovina, in 1965. He received the Dipl. Eng. degree in electrical engineering from the University of Sarajevo, Bosnia and Herzegovina, in 1991, and the M.Sc. and Ph.D. degrees in electrical engineering from the University of Belgrade, Belgrade, Serbia, in 1997 and 2006, respectively. From 1991 to 1992, he was with the Energoinvest Company, Sarajevo. In 1993, he joined the Faculty of Electrical Engineering at the University of Belgrade. Currently, he is a Full professor at the University of Belgrade, School of Electrical Engineering. His research interests are in control systems, fractional order control systems, physical and technical measurement sensors and power measurements.