BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874; ISSN (o) 2303-4955
www.imvibl.org / JOURNALS / BULLETIN
Vol. 7(2017), 1-9

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

ON WEAK AND STRONG FORMS OF β-OPEN SETS

Hakeem A. Othman

Abstract. In this paper, new classes of sets in general topology called a supra-β-open (closed) $\big($an infra-β-open (closed) $\big)$ set are introduced. Using new concepts, the fundamental properties and special results are highlighted. The relations between supra-β-open (closed) $\big($an infra-β-open (closed) $\big)$ set and other topological sets are investigated. Moreover, counter-examples are given to show that the converse of these relations in Diagram 1 need not be true, in general. Finally, some special theorems are introduced by adding condition to achieve the converse relations in Diagram 1.

1. Introduction

In 1983, Abd El-Monsef, El-Deeb and Mahmoud introduced new class of set in general topology called a β-open set and in 1982 Dunham defined new operators (Cl^* and Int^*). Our goal in this paper is to introduce weak and strong forms of β-open Sets in general topology, namely, supra-β-open (closed) $\big($an infra-β-open (closed) $\big)$set by using that new operator, and investigate special theorems related to these new concepts. The paper also discusses the relation and converse relation between these new sets and other set in general topology.

Throughout this paper if λ is a set and p is a point in X then $N(p)$, $Int\lambda$, $cl\ \lambda$ and λ^c denote respectively, the neighborhood system of p, the interior of λ, the closure of λ and complement of λ.

Now we recall some of the basic definitions and results in topology.

Definition 1.1. A set $\lambda \in X$ is called a $\alpha-$open [**11**] (resp. preopen [**10**], semi open [**9**] set if $\lambda \subseteq$ Int cl Int λ (resp. $\lambda \subseteq$ Int cl λ, $\lambda \subseteq$ cl Int λ). The family of all α-open (resp. preopen, semi open) sets of X is denoted as $\alpha O(X)$ (resp. $PO(X)$, $SO(X)$).

2010 *Mathematics Subject Classification.* 54A40, 54C08, 54C10.
Key words and phrases. infra-β-open set; infra-β-closed set; infra-β-closure; supra-β-closure; infra-β-Interior; supra-β-Interior.

Definition 1.2. A set $\mu \in X$ is called:

- semi*open (infra-semiopen) [**13**] if $\eta \subseteq \mu \subseteq Cl^*(\eta)$ where η is an open or equivalently $\mu \subseteq Cl^*(Int(\mu))$;

- semi*closed (infra-semiclosed) [**13**] if $Int^*(\eta) \subseteq \mu \subseteq \eta$ where where η is a closed or equivalently $Int^*(Cl(\mu)) \subseteq \mu$;

- pre*open (supra-preopen) [**14**] if $\mu \subseteq Int^*(Cl(\mu))$ and pre*closed (supra-preclosed) if $Cl^*(Int\mu))) \subseteq \mu$;

- α^*-open (supra-α-open) [**12**] if $\mu \subseteq Int^*(Cl(Int^*(\mu)))$ and α^*-closed (supra α -closed) if $Cl^*(Int(Cl^*(\mu))) \subseteq \mu$;

- infra α-open [**8**] if $\mu \subseteq Int(Cl^*(Int(\mu)))$;

- infra α-closed [**8**] if $Cl(Int^*(Cl(\mu))) \subseteq \mu$.

The family of all infra-semiopen, infra-semiclosed, supra-preopen, supra-preclosed, supra α-open, supra α-closed, infra α-open and infra α-closed sets in X will be denoted as $ISO(X)$, $ISC(X)$, $SPO(X)$, $SPC(X)$, $S\alpha{-}O(X)$, $S\alpha{-}C(X)$, $I\alpha{-}O(X)$ and $I\alpha - C(X)$ respectively.

Definition 1.3. ([**2**]) Let λ any set. Then

(i) $Cl^*\lambda = \cap\ \{\mu : \mu \supseteq\ \lambda,\ \mu$ is a generalized closed set of $X\}$ is called closure*.

(ii) $Int^*\lambda = \cup\ \{\mu : \mu \subseteq\ \lambda,\ \mu$ is a generalized open set of $X\}$ is called Interior*.

Lemma 1.1. ([**2**]) *Let λ any set. Then:*

$\lambda\ \subseteq Cl^*\ \lambda\ \subseteq\ Cl\lambda.$

$Int\lambda\ \subseteq Int^*\ \lambda\ \subseteq\ \lambda.$

2. Weak and Strong Forms of β-open Sets

Definition 2.1. A set $\eta \in X$ is called:

- infra -β-open if $\eta \subseteq Cl^*\ Int\ Cl^*\ \eta$.
- supra -β-open if $\eta \subseteq Cl\ Int^*\ Cl\ \eta$.
- infra-β-closed if $Int^*\ Cl\ Int^*\ \eta \subseteq \eta$.
- supra -β-closed if $Int\ Cl^*\ Int\ \eta \subseteq \eta$.

The family of all supra -β-open (infra -β-open) and supra- β-closed (infra -β-closed) sets in X will be denoted by Sβ-O(X) (Iβ-O(X))and Sβ-C(X) (Iβ-C(X)), respectively.

Definition 2.2. A set $\eta \in X$ is called:

- infra -preopen if $\eta \subseteq Int\ Cl^*\ \eta$.
- infra -preclosed if $Cl\ Int^*\eta \subseteq \eta$.
- supra -semiopen if $\eta \subseteq Cl\ Int^*\ \eta$.
- supra -semiclosed if $Int\ Cl^*\ \eta \subseteq \eta$.

The family of all infra -preopen, infra -preclosed, supra -semiopen and supra -semiclosed sets in X will be denoted by $I - PO(X)$, $I - PC(X)$, $S - SO(X)$ and $S - SC(X)$, respectively.

Theorem 2.1. *Let η any subset of X, then the next properties are equivalent:*

(i): $\eta \in I\beta - O(X)$.

(ii): $Cl^*\eta = Cl^*\ Int\ Cl^*\ \eta$.

(iii): $\delta \subseteq Cl^*\ \eta \subseteq Cl^*\ \delta$, *where* δ *is an open set.*
(iv): η^c *is infra-*β*-closed set.*
(v): $Int^*\ Cl\ Int^*\ \eta^c = Int^*\eta^c$.
(vi): $Int^*\ \delta \subseteq Int^*\ \eta^c \subseteq \delta$, *where* δ *is closed set.*

Proof. (i) $\Rightarrow$ (ii). We have $\eta \subseteq Cl^*\ Int\, Cl^*\ \eta$ and we know that $Cl^*\ Int\, Cl^*\ \eta \subseteq Cl^*\eta$, then $Cl^*\eta = Cl^*\ Int\ Cl^*\ \eta$.

(ii) $\Rightarrow$ (iii). We have $Cl^*\eta = Cl^*\ Int\ Cl^*\ \eta$. Therefore, $Int\ Cl^*\ \eta \subseteq Cl^*\eta \subseteq Cl^*\ Int\ Cl^*\ \eta$. If we take $\delta = Int\ Cl^*\eta$ for some $\eta \in \tau$, then $\delta \subseteq Cl^*\ \eta \subseteq Cl^*\ \delta$.

(iii) $\Rightarrow$ (i). We have $Int\ Cl^*\ \eta \subseteq Cl^*\eta \subseteq Cl^*\ Int\ Cl^*\ \eta$. This implies that $Cl^*\ Int\ Cl^*\ \eta \subseteq Cl^*\eta \subseteq Cl^*\ Int\ Cl^*\ \eta$. Then $\eta \subseteq Cl^*\ Int\ Cl^*\ \eta$.

(i) $\Rightarrow$ (iv). We have $\eta \subseteq Cl^*\ Int\ Cl^*\ \eta$. Therefore $(\eta)^c \subseteq (Cl^*\ Int\ Cl^*\ \eta)^c$ and we get $Int^*\ Cl\ Int^*\ \eta^c \subseteq \eta^c$.

(iv) $\Rightarrow$ (i). We have $Int^*\ Cl\ Int^*\ \eta^c \subseteq \eta^c$. Therefore $(Int^*\ Cl\ Int^*\ \eta)^c \subseteq (\eta)^c$ and $\eta \subseteq Cl^*\ Int\ Cl^*\ \eta$.

(iv) $\Rightarrow$ (v). We have $Int^*\ Cl\ Int^*\ \eta^c \subseteq \eta^c$ and we know that $Int^*\eta^c \subseteq Int^*\ Cl\ Int^*\ \eta^c$. So, $Int^*\ Cl\ Int^*\ \eta^c = Int^*\eta^c$.

(v) $\Rightarrow$ (vi). We have $Int^*\ Cl\ Int^*\ \eta^c \subseteq Int^*\eta^c \subseteq Cl\ Int^*\ \eta^c$. Let us take $\delta = Cl\ Int^*\eta^c$ for some δ closed set. Then $Int^*\ \delta \subseteq Int^*\ \eta^c \subseteq \delta$.

(vi) $\Rightarrow$ (iv). We have $Int^*\ Cl\ Int^*\ \eta^c \subseteq Int^*\eta^c \subseteq Cl\ Int^*\ \eta^c$. This implies that $Int^*\ Cl\ Int^*\ \eta^c \subseteq Int^*\eta^c \subseteq Int^*\ Cl\ Int^*\ \eta^c$ and $Int^*\ Cl\ Int^*\ \eta^c \subseteq \eta^c$. $\square$

Corollary 2.1. *Let* η *any subset of* X*, then the next properties are equivalent:*
(i): $\eta \in S\beta - O(X)$.
(ii): $Cl\eta = Cl\ Int^*\ Cl\ \eta$.
(iii): $\delta \subseteq Cl\ \eta \subseteq Cl\ \delta$, *where* δ *is a generalized open set.*
(iv): η^c *is infra-*β*-closed set.*
(v): $Int\ Cl^*\ Int\ \eta^c = Int\eta^c$.
(vi): $Int\ \delta \subseteq Int\ \eta^c \subseteq \delta$, *where* δ *is a generalized closed set.*

Definition 2.3. Let λ any set. Then

(1) $S\beta - Cl\ \lambda = \cap\ \{\mu : \mu \supseteq \lambda,\ \mu$ is a supra-β-closed set of $X\}$ is called a supra-β-closure.
(2) $S\beta - Int\ \lambda = \cup\ \{\mu : \mu \subseteq \lambda,\ \mu$ is a supra-β-open set of $X\}$ is called a supra-β-Interior.
(3) $I\beta - Cl\ \lambda = \cap\ \{\mu : \mu \supseteq \lambda,\ \mu$ is an infra-β-closed set of $X\}$. is called an infra-β-closure
(4) $I\beta - Cl\ \lambda = \cup\ \{\mu : \mu \subseteq \lambda,\ \mu$ is an infra-β-open set of $X\}$. is called an infra-β-interior

Proposition 2.1. *Let* λ *and* μ *be the sets in* X *and* $\lambda \subseteq \mu$*. Then the following statements hold:*

(1) $S\beta - Int(\lambda)\ \big(I\beta - Int(\lambda)\ \big)$ *is the largest supra-*β*-open* $\big($*infra-*β*-open*$\big)$ *set contained in* λ.
(2) $S\beta - Int\ \lambda \subseteq \lambda$, $\big(I\beta - Int\ \lambda \subseteq \lambda\big)$.
(3) $S\beta - Int\ \lambda \subseteq S\beta - Int\ \mu$, $\big(I\beta - Int\ \lambda \subseteq I\beta - Int\ \mu\big)$.

(4) $S\beta-Int(S\beta-Int\ \lambda)=S\beta-Int\ \lambda$, $\big(I\beta-Int(I\beta-Int\ \lambda)=I\beta-Int\ \lambda\big)$.
(5) $\lambda\in S\beta-O(X)\Leftrightarrow S\beta-Int\ \lambda=\lambda$, $\big(\lambda\in I\beta-O(X)\Leftrightarrow I\beta-Int\ \lambda=\lambda\big)$.

Proposition 2.2. *Let λ and μ be the sets in X and $\lambda\subseteq\mu$. Then the following statements hold:*

(1) $S\beta-Cl(\lambda)$ $\big(I\beta-Cl(\lambda)\big)$ *is the smallest supra-β-closed $\big($infra-β-closed$\big)$ set containing λ.*
(2) $\lambda\subseteq S\beta-Cl(\lambda)$, $\big(\lambda\subseteq I\beta-Cl(\lambda)\ \big)$.
(3) $S\beta-Cl\ \lambda\subseteq S\beta-Cl\ \mu$, $\big(I\beta-Cl\ \lambda\subseteq I\beta-Cl\ \mu\ \big)$.
(4) $S\beta-Cl(S\beta-Cl\ \lambda)=S\beta-Cl\ \lambda$, $\big(I\beta-Cl(I\beta-Cl\ \lambda)=I\beta-Cl\ \lambda\big)$.
(5) $\lambda\in S\beta-C(X)\Leftrightarrow S\beta-Cl\ \lambda=\lambda$, $\big(\lambda\in I\beta-C(X)\Leftrightarrow I\beta-Cl\ \lambda=\lambda\ \big)$.

Theorem 2.2. *Let λ be a set of X. Then, the following properties are true:*

(a): $(S\beta-Int\ \lambda)^c=S\beta-Cl\ \lambda$, $\Big((I\beta-Int\ \lambda)^c=I\beta-Cl\ \lambda\ \Big)$.
(b): $(S\beta-Cl\ \lambda)^c=S\beta-Int\ \lambda$, $\Big((I\beta-Cl\ \lambda)^c=I\beta-Int\ \lambda\Big)$.
(c): $S\beta-Int\ \lambda\subseteq\lambda\cap Cl\ Int^*\ Cl\ \lambda$, $\Big(I\beta-Int\ \lambda\subseteq\lambda\cap Cl^*\ Int\ Cl^*\ \lambda\ \Big)$.
(d): $S\beta-Cl\ \lambda\supseteq\lambda\cup Int\ Cl^*\ Int\ \lambda$, $\Big(I\beta-Cl\ \lambda\supseteq\lambda\cup Int^*\ Cl\ Int^*\ \lambda\ \Big)$.

Proof. We will prove only (a) for $S\beta-Int\ \lambda$ and (d) for $I\beta-Cl\ \lambda$.
(a)

$$\begin{aligned}(I\beta-Int\ \lambda)^c &= \big(\cup\{v: v\subseteq\ \lambda,\ v\ is\ an\ \ supra-\beta-open\ set\ of\ X\}\big)^c\\ &= I\beta-Cl\ \lambda.\end{aligned}$$

(d) Since $\lambda\subseteq I\beta-Cl\ \lambda$ and $I\beta-Cl\ \lambda$ is an infra-β-closed set. Hence, $Int^*\ Cl\ Int^*(I\beta-Cl\ \lambda)\subseteq I\beta-Cl\ \lambda$. Then, $I\beta-Cl\ \lambda\supseteq\lambda\cup Int^*\ Cl\ Int^*\ \lambda$. □

Theorem 2.3. *If λ any sets in $X$and μ are closed (generalized closed) sets in space X such that $\mu\subseteq\lambda\subseteq Cl\ Int^*\ \mu$ $\big(\mu\subseteq\lambda\subseteq Cl^*\ Int\ \mu\big)$, then $\lambda\in S\beta-O(X)$ $\big(\lambda\in I\beta-O(X)\big)$.*

Proof. We can show that $Cl\ Int^*\ \mu\subseteq Cl\ Int^*\ Cl\lambda$ $\big(Cl^*\ Int\ \mu\subseteq Cl^*\ Int\ Cl^*\lambda\big)$ this implies that $\lambda\subseteq Cl\ Int^*\ Cl\ \lambda$ $\big(\ \lambda\subseteq Cl^*\ Int\ Cl^*\ \lambda\big)$, then $\lambda\in S\beta-O(X)$ $\big(\lambda\in I\beta-O(X)\big)$. □

Corollary 2.2. *If λ any sets in X and μ is a supra-preopen (infra-preopen) set in X such that $\mu\subseteq\lambda\subseteq Cl\ Int^*\ \mu$ $\big(\mu\subseteq\lambda\subseteq Cl^*\ Int\ \mu\big)$, then $\lambda\in S\beta-O(X)$ $\big(\lambda\in I\beta-O(X)\big)$.*

Corollary 2.3.

- *If λ any sets in X and μ are open (generalized open) sets in space X such that $Int\ Cl^*\ \mu\subseteq\lambda\subseteq\mu$ $\big(Int^*\ Cl\ \mu\subseteq\lambda\subseteq\mu\big)$, then $\lambda\in S\beta-C(X)$ $\big(\lambda\in I\beta-C(X)\big)$.*
- *If λ any sets in X and μ is a supra-preclosed (infra-preclosed) set in X such that $Int\ Cl^*\ \mu\subseteq\lambda\subseteq\mu$ $\big(Int^*\ Cl\ \mu\subseteq\lambda\subseteq\mu\big)$, then $\lambda\in S\beta-C(X)$ $\big(\lambda\in I\beta-C(X)\big)$.*

REMARK 2.1. Let τ be the class of open set of X, then

- $\tau = Int\big(SB - O(X)\big)$.
- $\tau = Int\big(IB - O(X)\big)$.

THEOREM 2.4.

(a): *The arbitrary union of infra-β-open (supra-β-open) set is an infra-β-open (supra-β-open) set.*

(b): *The arbitrary intersection of infra-β-closed (supra-β-closed) set is an infra-β-closed (supra-β-closed) set.*

PROOF. We will prove for infra-β-open sets.

(a): Let $\{\lambda_i\}$ be family of infra-β-open set. Then, for each i, $\lambda_i \subseteq Cl^*\ Int\ Cl^*\lambda_i$ and $\cup\lambda_i \subseteq \cup(Cl^*\ Int\ Cl^*\ \lambda_i) \leqslant Cl^*\ Int\ Cl^*\ (\cup\lambda_i)$. Hence $\cup\lambda_i$ is an infra-β-open set.

(b): Obvious.

□

REMARK 2.2. The intersection of infra-β-open (supra-β-open) sets need not be infra-β-open (supra-β-open) set.

The union of supra-β-closed sets need not be supra-β-closed set.

These are illustrated by the following example:

EXAMPLE 2.1. Consider the space X where, $X = \{a, b, c, d\}$. Let B_1 and B_2 be sets of X defined as:

$$B_1 = \{a, b\} \quad B_2 = \{c, d\}$$

Let $\tau = \{\phi, B_1, B_2, X\}$. We can see

- B_1 and B_2 are supra-β-open (infra-β-open) set.
- But $B_1 \cap B_2$ is not supra-β-open (infra-β-open) set.

EXAMPLE 2.2. Consider the space X where, $X = \{a, b, c, d\}$. Let B_1, B_2, B_3 and B_4 be sets of X defined as:

$$B_1 = \{a, b\}, \quad B_2 = \{a, b, c\},\ B_3 = \{a, c\}, \quad B_4 = \{a\}$$

Let $\tau = \{\phi, B_1, B_2, B_3, B_4, X\}$. We can see

- B_1 and B_3 are supra-β-closed set.
- But $B_1 \cup B_2$ is not supra-β-closed set.

THEOREM 2.5. *Let λ be a set of a topological space X. Then the following statements hold:*

(a): *If λ is an infra-β- open (infra-β- closed) set, then λ is a β-open (β-closed) set.*

(b): *If λ is a β- open (β- closed) set, then λ is a supra-β-open (supra-β-closed) set.*

(c): *If λ is a supra-preopen (supra-preclosed) set, then λ is a supra-β-open (supra-β-closed) set.*

(d): *If λ is an infra-preopen (infra-preclosed) set, then λ is an infra-β-open (infra-β-closed) set.*

(e): *If λ is a supra-semiopen (supra-semiclosed) set, then λ is a supra-β-open (supra-β-closed) set.*
(f): *If λ is an infra-semiopen (infra-semiclosed) set, then λ is an infra-β-open (infra-β-closed) set.*
(g): *If λ is an open (closed) set, then λ is a supra-β- open (supra-β- closed) set.*
(h): *If λ is an open (closed) set, then λ is an infra-β- open (infra-β- closed) set.*

PROOF. It is clear from Definition 1.1, Definition 1.2, Definition 2.1, Definition 2.2 and basic relations. □

The following "Implication Diagram 1" illustrates the relation of different classes of open sets.

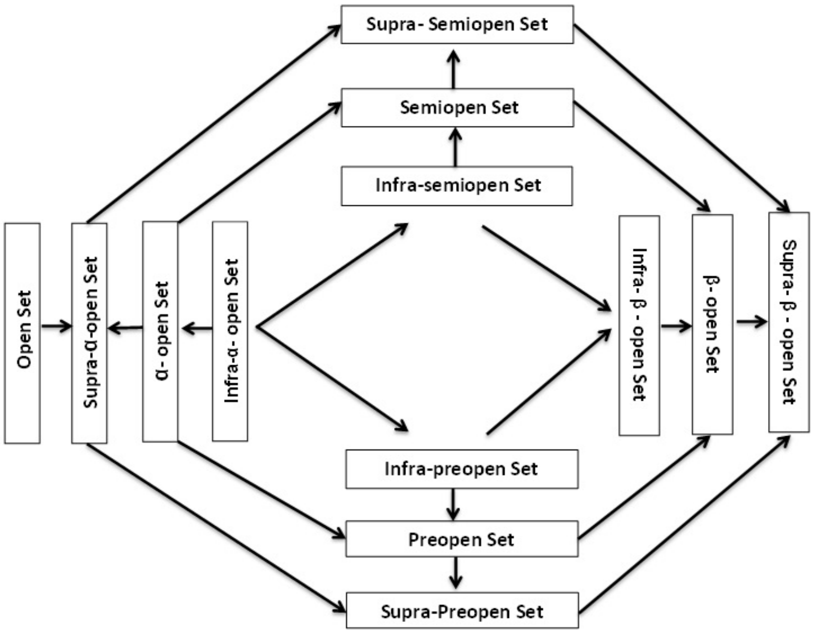

Diagram 1

REMARK 2.3. The converse of these relations need not be true, in general as shown by the following examples.

EXAMPLE 2.3. Consider the space X where, $X = \{a, b, c\}$. Let B_1 and B_2 be sets of X defined as:

$$B_1 = \{a\} \quad B_2 = \{a, c\}$$

Let $\tau = \{\phi, B_1, X\}$. We can see

- B_2 is a α-open set which is not an infra-β-open set.

- B_2 is a β-open set which is not an infra-β-open set.

EXAMPLE 2.4. Consider the space X where, $X = \{a, b, c, d\}$. Let B_1, B_2, B_3 and B_4 be sets of X defined as:

$$B_1 = \{b\} \quad B_2 = \{a, b\} \quad B_3 = \{a, b, c\} \quad B_4 = \{a, c, d\}$$

Let $\tau = \{\phi, B_1, B_2, B_3, X\}$. We can see

- B_4 is a supra-β-open set which is not infra-β-open set.
- B_4 is a supra-β-open set which is not β-open set.
- B_4 is a supra-β-open set which is not supra-preopen set

EXAMPLE 2.5. Consider the space X where, $X = \{a, b, c\}$. Let B_1, B_2 and B_3 be sets of X defined as:

$$B_1 = \{a\} \quad B_2 = \{b, c\} \quad B_3 = \{a, b\}$$

Let $\tau = \{\phi, B_1, B_2, X\}$. We can see

- B_3 is a supra-β-open set which is not infra-preopen set.
- B_3 is a supra-β-open set which is not infra-semiopen set.

REMARK 2.4. We can reach the converse relations in Diagram (1) as shown in next theorem.

THEOREM 2.6. *The following statements are true:*

i: *Each supra-β-open set which is infra-semiclosed is supra-semiopen set.*
ii: *Each infra-β-open set which is supra-semiclosed is infra-semiopen set.*
iii: *Each supra-β-open set which is infra-α-closed is closed set.*
iv: *Each supra-β-closed set which is infra-semiopen is supra-semiclosed set.*
v: *Each infra-β-closed set which is supra-semiopen is infra-semiclosed set.*
vi: *Each supra-β-closed set which is infra-α-open is open set.*

PROOF.

i: If A is supra-β-open and infra-semiclosed set, this implies that $A \subseteq Cl\ Int^*\ Cl\ A$ and $Int^*\ Cl\ A \subseteq A$. Therefore, $Cl\ Int^*\ Cl\ A \subseteq Cl\ Int^*\ A$, then $A \subseteq Cl\ Int^*\ A$. This show that A is supra-semiopen set.
ii: If A is infra-β-open and supra-semiclosed set, this implies that $A \subseteq Cl^*\ Int\ Cl^*\ A$ and $Int\ Cl^*\ A \subseteq A$. Therefore, $Cl^*\ Int\ Cl^*\ A \subseteq Cl^*\ Int\ A$, then $A \subseteq Cl^*\ Int\ A$. This show that A is infra-semiopen set.
iii: If A is supra-β-open and infra-α-closed set, this implies that $A \subseteq Cl\ Int^*\ Cl\ A$ and $Cl\ Int^*\ Cl\ A \subseteq A$. Then, A is closed set.
iv: If A is supra-β-closed and infra-semiopen set, this implies that $Int\ Cl^*\ Int\ A \subseteq A$ and $A \subseteq Cl^*\ Int\ A$. Therefore, $Int\ Cl^*\ A \subseteq Int\ Cl^*\ Int\ A$, then $Int\ Cl^*\ A \subseteq A$. This show that A is supra-semiclosed set.
v: If A is infra-β-closed and supra-semiopen set, this implies that $Int^*\ Cl\ Int^*\ A \subseteq A$ and $A \subseteq Cl\ Int^*\ A$. Therefore, $Int^*\ Cl\ A \subseteq Int^*\ Cl\ Int^*\ A$, then $Int^*\ Cl\ A \subseteq A$. This show that A is infra-semiclosed set.

vi: If A is supra-β-closed and infra-α-open set, this implies that $Int\ Cl^*\ Int\ A \subseteq A$ and $A \subseteq Int\ Cl^*\ Int\ A$. Then, A is open set.

$\square$

REMARK 2.5. In an indiscreet Topology X, each

- supra-β-open set is supra preopen set.
- infra-β-open set is infra preopen set.

3. Conclusion

In this paper, we introduced the new topological notions, supra-β-open (closed) and infra-β-open (closed) sets. The connections between these notions and other topological notions are studied. The results in this paper are just the beginning of a new structure and not only will form the theoretical basis for further applications of topology on fuzzy sets or soft sets but also will lead to the development of information system and various fields in computational topology for geometric design, computer-aided geometric design, engineering design research and mathematical science. Also, topology plays a significant role in space-time geometry and high-energy physics see, [**3, 4, 5, 6, 7**].

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud. β-open sets and β-continuous mapping. *Bull. Fac. Sci. Assiut Univ.*, **12**(1)(1983), 77-90.

[2] W. Dunham. A New Closure Operator for non-T_1 topology. *Kyuungpook Math. J.*, **22**(1982), 55-60.

[3] MS. El Naschie. On the uncertainty of Cantorian geometry and the two-slit experiment, *Chaos, Solitons and Fractals*, **9**(3)(1998), 517-529.

[4] MS. El Naschie. Quantum gravity from descriptive set theory. *Chaos, Solitons and Fractals*, **19**(5)(2004), 1339-1344.

[5] MS. El Naschie. Quantum gravity, Clifford algebras, fuzzy set theory and the fundamental constants of nature. *Chaos, Solitons and Fractals*, **20**(3)(2004), 437-450.

[6] MS. El Naschie. On a fuzzy Kahler-like manifold which is consistent with the two slit experiment. *Int. J. Nonlinear Sci. Numer Simul*, **6**(2)(2005), 95-98.

[7] MS. El Naschie. Topics in the mathematical physics of E-infinity theory. *Chaos, Solitons and Fractals*, **30**(3)(2006), 656-663.

[8] Hakeem A. Othman and Md. Hanif Page. ON An Infra-α-Open Sets. *Global J. Math. Anal., 4(3)(2016), 12-16.*

[9] N. Levine. Semi open sets and semi-continuouity in topological space. *Amer. Math. Monthly*, **70**(1)(1981), 36-41.

[10] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb. On Precontinuous and weak precontinuous Mappings. *Proc. Math. Phys. Soc. Egypt*, **53**(1982), 47-53.

[11] O. Njastad. Some Classes of Nearly Open sets. *Pacific J. Math.*, **15**(3)(1965), 961-970.

[12] S. Pious Missier and P. Anbarasi Rodrigo. Some Notions of Nearly Open Sets in Toological Spaces. *Inter. J. Math. Archive*, **4**(12)(2013), 12-18.

[13] A. Robert and S. Pious Missier. A New Class of Nearly Open Sets. *Inter. J. Math. Archive*, **3**(7)(2012), 2575-2582.

[14] T. Selvi and A. Punitha Dharani. Some new class of nearly closed and open sets. *Asian J. Current Engineering and Maths.*, **1**(5)(2012), 305-307.

Received by editors 25.04.2016; Available online 05.09.2016.

Department of Mathematics, Al-Qunfudah Center for Scientific Research, AL-Qunfudhah University college, Umm Al-Qura University, KSA.

Department of Mathematics, Rada'a College of Education and Science,
Albida University, Albida, Yemen
E-mail address: hakim_albdoie@yahoo.com, haoali@uqu.edu.sa