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SIGN DOMINATING SWITCHED INVARIANTS OF A GRAPH

B. Chaluvaraju and V. Chaitra

Abstract. In this paper, we newly constructed the sign dominating outer (inner) switched graph $\mu_d^o(G)$ $(\mu_d^i(G))$ of a graph $G = (V, E)$ and establish their properties. Also we determine number of edges and its relation between $\mu_d^o(G)$ and $\mu_d^i(G)$ in some special classes of graphs are explored.

1. INTRODUCTION

All the graphs considered in this paper are finite, nontrivial, simple and undirected. Let $G = (V, E)$ be a simple graph with vertex set $V(G) = V$ of order $|V| = n$, edge set $E(G) = E$ of size $|E| = m$ and let v be a vertex of V. The *open neighborhood* of v is $N(v) = \{u \in V/uv \in E(G)\}$ and *closed neighborhood* of v is $N[v] = \{v\} \cup N(v)$. The graph G^c is called *complement* of a graph G, if G and G^c have the same vertex set and two vertices are adjacent in G if and only if they are not adjacent in G^c. A subset S of V is called *vertex independent set* if no two vertices in S are adjacent in G. A *clique* in a graph is an induced complete subgraph. The maximum order of a clique in the graph G is called the clique number of G, denoted by $\omega(G)$. A collection of independent edges of a graph G is called a matching of G. If there is a matching consists of all vertices of G it is called a *perfect matching.* For standard terminology and notation in graph theory, we refer [**6**].

A sign dominating function of a graph G is a function $f : V \to \{-1, 1\}$ such that $f(N[v]) \geqslant 1$ for all $v \in V$. The *sign domination number* of a graph G is $\gamma_s(G) = min\{w(f) : f$ is sign dominating function$\}$. The concept of sign domination was initiated by Dunbar et al. [**5**]. For complete review on theory of domination and its related parameters, we refer [**7**], [**8**] and [**13**].

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In a graph G, let $P = \{V_1, V_2, ..., V_k\}$ be a partition of V of order $k \geqslant 1$. The *k-complement* of a graph G denoted as G_k^P is defined as follows: for all V_i and V_j in $P, i \neq j$ remove the edges between V_i and V_j and add the edges which are not in G. The *k(i)-complement* of a graph G denoted as $G_{k(i)}^P$ is defined as: for each set V_r in the partition P, remove the edges of G inside V_r and add the missing edges between them. The concept of generalized complement of a graph was studied by Sampathkumar et al. [**9**] and [**10**]. Analogously, the concept of 2-complement and $2(i)$-complement of a graph is also known as switched graph, where switching of G assigns $+1$ or -1 to each vertex of a graph G. This type of switched graph was introduced by Van-Lint et al. [**12**].

Further, let V_1 and V_{-1} be set of vertices assigned 1 and -1 in G respectively. We denote $\rho^+(G) = \rho^+ = |V_1|$ and $\rho^-(G) = \rho^- = |V_{-1}|$. For more details on Generalized complements (switched invariants) and its related concept, we refer [**1**], [**3**], [**4**], [**5**] and [**11**].

2. Sign dominating outer switched graph

The sign dominating outer switched graph of G denoted as $\mu_d^o(G)$ is defined as: for V_1 and V_{-1} of G remove the edges between V_1 and V_{-1} and add the edges which are not there in G.

THEOREM 2.1. *Let G be a nontrivial graph. Then*

(i) *$\mu_d^o(G)$ is a totally disconnected graph if and only if G is totally disconnected graph.*

(ii) *V_1 is independent set if and only if $\mu_d^o(G)$ is totally disconnected graph.*

PROOF. (i) Let G be totally disconnected graph. Every vertex of G belongs to V_1. In construction of $\mu_d^o(G)$, no new edge is added or any edge is deleted. Hence $G \cong \mu_d^o(G)$. Conversely, if $\mu_d^o(G)$ is totally disconnected graph, then in G no two vertices of V_1(or V_{-1}) are connected. Also in G, vertices of V_1 and V_{-1} cannot be connected as a vertex v of V_1 which is adjacent to a vertex of V_{-1} should be adjacent to at least one vertex of V_1 such that $f(N[v]) \geqslant 1$. Hence G is totally disconnected graph.

(ii) If $\mu_d^o(G)$ is totally disconnected graph, then it is obvious that V_1 is independent set as every vertex of $\mu_d^o(G)$ is assigned 1. Conversely, suppose V_1 is independent set. To prove $\mu_d^o(G)$ is totally disconnected, we shall prove G is totally disconnected. Suppose G is not totally disconnected and a vertex $v \in V_1$ be adjacent to a vertex of V_{-1}. But this vertex v should be adjacent to another vertex in V_1 as $f(N[v]) \geqslant 1$ which is a contradiction to V_1 being independent. Hence G is totally disconnected. $\square$

THEOREM 2.2. *Let G be a nontrivial graph. Then, there is no perfect matching between vertices of V_1 and V_{-1}.*

PROOF. In a graph G, a vertex assigned -1 is adjacent to at least two vertices assigned 1, there cannot be a perfect matching between V_1 and V_{-1}. Thus the required result follows. $\square$

THEOREM 2.3. *For any nontrivial graph G,*

$$(\mu_d^o(G))^c \cong \mu_d^o(G^c).$$

PROOF. Let u and v be two non adjacent vertices of G. Then they are adjacent in G^c. We prove the result in following cases:

Case 1. If u and v belongs to same set V_1 or V_{-1}, then they are non adjacent in $\mu_d^o(G)$, implies they are adjacent in $(\mu_d^o(G))^c$. Also they are adjacent in $\mu_d^o(G^c)$.

Case 2. If u and v belongs to different sets, then they are adjacent in $\mu_d^o(G)$, implies they non adjacent in $(\mu_d^o(G))^c$. Also they are non adjacent in $\mu_d^o(G^c)$.

From above two cases, the required result follows. □

THEOREM 2.4. *Let $G = K_{p,q}$ be a complete bipartite graph with bipartition P_1 and P_2 such that $|P_1| = p$ and $|P_2| = q$ with $p \leqslant q$. If $\left\lfloor \frac{q}{2} \right\rfloor = r$, then*

$$m(\mu_d^o(G)) = (q - r)(p + r).$$

PROOF. Let $G = K_{p,q}$. Since $p \leqslant q$, degree of every vertex of P_1 is greater than or equal to degree of every vertex of P_2. Let every vertex of P_1 be assigned 1. Since a vertex assigned -1 should be adjacent to at least two vertices assigned 1, number of vertices assigned -1 in P_2 should be $\left\lfloor \frac{q}{2} \right\rfloor = r$. p vertices of P_1 and $(q - r)$ vertices of P_2 which are assigned 1 forms an induced bipartite graph. Now in $\mu_d^o(G)$, $(q - r)$ vertices assigned 1 and r vertices assigned -1 are adjacent. These $(q - r)$ vertices are adjacent to p vertices in $\mu_d^o(G)$. Hence $\mu_d^o(G)$ is complete bipartite graph K_{r_1,r_2}, where $\mid r_1 \mid= q - r$ and $\mid r_2 \mid= r + p$. □

To prove our next result we make use of the following result due to Bohdan Zelinka [**2**].

THEOREM 2.5. *Let $G = K_{p,q}$ be a complete bipartite graph with bipartition P_1 and P_2 such that $|P_1| = p$ and $|P_2| = q$, with $p \leqslant q$. Then*

(i) *for $p = 1, \gamma_s(G) = q + 1$.*

(ii) *for $2 \leqslant p \leqslant 3$, $\gamma_s(G) = \begin{cases} p & \text{if } q \text{ is even,} \\ p+1 & \text{if } q \text{ is odd.} \end{cases}$*

(iii) *for $p \geqslant 4$, $\gamma_s(G) = \begin{cases} 4 & \text{if both } p \text{ and } q \text{ are even,} \\ 6 & \text{if both } p \text{ and } q \text{ are odd,} \\ 5 & \text{if one out of } p \text{ or } q \text{ is even.} \end{cases}$*

THEOREM 2.6. *Let $G = K_{p,q}$ be a complete graph with $p \leqslant q$. If $\left\lfloor \frac{q}{2} \right\rfloor = r$, $r_1 = q - r$ and $r_2 = p + r$, then*

(i) *for $r_2 = 1, \gamma_s(\mu_d^o(G)) = r_1 + 1$.*

(ii) *for* $2 \leqslant r_2 \leqslant 3$, $\gamma_s(\mu_d^o(G)) = \begin{cases} r_2 & \text{if } r_1 \text{ is even,} \\ r_2+1 & \text{if } r_1 \text{ is odd.} \end{cases}$

(iii) *for* $r_2 \geqslant 4$, $\gamma_s(\mu_d^o(G)) = \begin{cases} 4 & \text{if both } r_1 \text{ and }_2 \text{ are even,} \\ 6 & \text{if both } r_1 \text{ and } r_2 \text{ are odd,} \\ 5 & \text{if one out of } r_1 \text{ or } r_2 \text{ is even.} \end{cases}$

PROOF. From Theorem 2.4, if G is complete bipartite graph, then $\mu_d^o(G)$ is also complete bipartite graph isomorphic to K_{r_1,r_2}, where $r_1 = q - r$ and $r_2 = p + r$. From Theorem 2.5, the desired results follows. □

THEOREM 2.7. *Let G be a nontrivial graph. If $G \cong K_n$ with $n \geqslant 3$ vertices, then*

(i) $\mu_d^o(G) \ncong K_n$.

(ii) $\rho^-(G) = \begin{cases} \dfrac{n-2}{2} & \text{if } n \text{ is even,} \\ \dfrac{n-1}{2} & \text{if } n \text{ is odd.} \end{cases}$

(iii) $\mu_d^o(G) = G_{\rho^+} + G_{\rho^-}$, *where G_{ρ^+} and G_{ρ^-} are clique graphs of G with*

$$m(\mu_d^o(G)) = \begin{cases} \rho^+(G) = \dfrac{n+2}{2} \text{ and } \rho^-(G) = \dfrac{n-2}{2} & \text{if } n \text{ is even,} \\ \rho^{+(G)} = \dfrac{n+1}{2} \text{ and } \rho^-(G) = \dfrac{n-1}{2} & \text{if } n \text{ is odd.} \end{cases}$$

(iv) $m(\mu_d^o(G)) = \begin{cases} \dfrac{n^2-2n+4}{4} & \text{if } n \text{ is even,} \\ \dfrac{n^2-2n+1}{4} & \text{if } n \text{ is odd.} \end{cases}$

(v) $\gamma_s(\mu_d^o(G)) = \begin{cases} 2 & \text{if both } \rho^+(G) \text{ and } \rho^-(G) \text{ are odd,} \\ 4 & \text{if both } \rho^+(G) \text{ and } \rho^-(G) \text{ are even,} \\ 3 & \text{if one is odd and other is even.} \end{cases}$

PROOF. (i) If possible let $\mu_d^o(G) \cong K_n$ with $n \geqslant 3$ vertices, then we consider following two cases:

Case 1. Let v be a vertex of $V_1(G)$ and w be a vertex of $V_{-1}(G)$. In $\mu_d^o(G)$ adjacency of vertices within V_1 and within V_{-1} are retained as it is in G. Remove edge connecting vertices v and w and connect v to vertices other than w in V_{-1}. Hence in $\mu_d^o(G)$, v is not connected to w which is a contradiction to $\mu_d^o(G)$ being complete graph.

Case 2. Suppose $V = V_1$ or V_{-1}. As $V \neq V_{-1}$ implies $V = V_1$. Hence $G \cong \mu_d^o(G)$. But G is not a complete graph with $V = V_1$ for $n \geqslant 3$. Hence $\mu_d^o(G) \ncong K_n$ for $n \geqslant 3$.

(ii) Let v be a vertex of a graph G. Then consider the following two cases:

Case 3. If n is even, then v is adjacent to odd number of vertices. Out of which $\frac{n-2}{2}$ vertices are assigned 1 and $\frac{n-2}{2}$ vertices are assigned -1. The remaining $(n-1)^{th}$ vertex cannot be assigned -1 as it makes the weight of every vertex either 0 or -2 depending on v being assigned 1 or -1. So the $(n-1)^{th}$ vertex is assigned 1 and v is also assigned 1. Hence $\rho^-(G) = \frac{n-2}{2}$.

Case 4. If n is odd, then v is adjacent to even number of vertices. Out of which $\frac{n-1}{2}$ vertices are assigned 1, remaining $\frac{n-1}{2}$ vertices are assigned -1 and v is assigned 1 so as $f(N[v]) \geqslant 1$ for all $v \in V$. Hence $\rho^-(G) = \frac{n-1}{2}$.

(iii) Any vertex v in a graph G is adjacent to $n-1$ vertices, in $\mu_d^o(G)$ we remove edges between vertices of V_1 and V_{-1} and no new edge is added. Hence $\mu_d^o(G) = G_{\rho^+} + G_{\rho^-}$, where G_{ρ^+} is graph induced by vertices of V_1 and G_{ρ^-} is graph induced by vertices of V_{-1} in G. Also in $\mu_d^o(G)$, each G_{ρ^+} and G_{ρ^-} is complete. From (ii), if n is even, then $\rho^+(G) = \frac{n+2}{2}$ and $\rho^-(G) = \frac{n-2}{2}$. And if n is odd, then $\rho^+(G) = \frac{n+1}{2}$ and $\rho^-(G) = \frac{n-1}{2}$.

(iv) *when n is even:*

$$\rho^+(G) = \frac{n+2}{2} \quad \textit{and} \quad \rho^-(G) = \frac{n-2}{2}.$$
$$m(\mu_d^o(G)) = \frac{1}{2}\left[\frac{n+2}{2}\left(\frac{n+2}{2}-1\right) + \frac{n-2}{2}\left(\frac{n-2}{2}-1\right)\right].$$
$$m(\mu_d^o(G)) = \frac{1}{4}\left(n^2 - 2n + 4\right).$$

when n is odd:

$$\rho^+(G) = \frac{n+1}{2} \quad \textit{and} \quad \rho^-(G) = \frac{n-1}{2}.$$
$$m(\mu_d^o(G)) = \frac{1}{2}\left[\frac{n+1}{2}\left(\frac{n+1}{2}-1\right) + \frac{n-1}{2}\left(\frac{n-1}{2}-1\right)\right].$$
$$m(\mu_d^o(G)) = \frac{1}{4}\left(n^2 - 2n + 1\right).$$

(v) Since

$$\gamma_s(K_n) = \begin{cases} 1 & \text{if } n \text{ is odd,} \\ 2 & \text{if } n \text{ is even.} \end{cases}$$

Case 5. If both $\rho^+(G)$ and $\rho^-(G)$ are odd, then

$$\gamma_s(\mu_d^o(G)) = \gamma_s(G_{\rho^+}) + \gamma_s(G_{\rho^-}) = 2.$$

Case 6. If both $\rho^+(G)$ and $\rho^-(G)$ are even, then

$$\gamma_s(\mu_d^o(G)) = \gamma_s(G_{\rho^+}) + \gamma_s(G_{\rho^-}) = 4.$$

Case 7. If one of $\rho^+(G)$ or $\rho^-(G)$ is odd and the other is even, then

$$\gamma_s(\mu_d^o(G)) = \gamma_s(G_{\rho^+}) + \gamma_s(G_{\rho^-}) = 3.$$

From all the above cases, the required results follows. □

THEOREM 2.8. *For a cycle C_n with $n \geqslant 3$ vertices,*

$$m(\mu_d^o(G)) = -r^2 + r(n-4) + n,$$

where $\left\lfloor \frac{n}{3} \right\rfloor = r$.

PROOF. Let $G \cong C_n$ with $n \geqslant 3$ vertices. In a cycle C_n, number of vertices assigned -1 are $\left\lfloor \frac{n}{3} \right\rfloor = r$ and number of vertices assigned 1 are $n - r$.

Construction of $\mu_d^o(G)$ is as follows: In G, since degree of vertex is 2, no two vertices assigned -1 are adjacent as $f(N[v]) \geqslant 1$ for every vertex $v \in V$.

Step 1. A vertex assigned -1 is adjacent to exactly two vertices assigned 1. In $\mu_d^o(G)$, remove two edges for one vertex assigned -1. Since there are r such vertices, number of edges removed are $2r$ and number of edges remaining in $\mu_d^o(G)$ are $n - 2r$.

Step 2. In $\mu_d^o(G)$ a vertex assigned -1 is adjacent to $n - r - 2$ vertices so we connect these by $n-r-2$ edges. Repeat this process for all r vertices assigned -1. Therefore number of edges connecting vertices assigned -1 and vertices assigned 1 in $\mu_d^o(G)$ are $r(n - r - 2)$ edges.

From above two steps, $m(\mu_d^o(G)) = n - 2r + r(n - r - 2) = -r^2 + r(n-4) + n$ follows. □

THEOREM 2.9. *For a path P_n with $n \geqslant 2$ vertices,*

$$m(\mu_d^o(G)) = -r^2 + n(r+1) - 4r - 1,$$

where $\left\lceil \frac{n-4}{3} \right\rceil = r$.

PROOF. Let $G \cong P_n$ with $n \geqslant 2$ vertices, then end vertices and support vertices of a graph G cannot be assigned -1 as $f(N[v]) \geqslant 1$. Hence such vertices belongs to V_1. After this assignment number of vertices left for assignment are $n-4$. In a path, since degree of any vertex other than end vertex is 2, a vertex assigned -1 should have exactly two neighbours assigned 1. So for a minimum of 3 vertices, only one vertex as -1 provided $f(N[v]) \geqslant 1$ for every $v \in V(G)$. Hence $\rho^-(G) = \left\lceil \frac{n-4}{3} \right\rceil = r(say)$ and $\rho^+(G) = n - r$.

Construction of $\mu_d^o(G)$ is as follows:

Step 1. *Remove the edges between vertices of V_1 and V_{-1}*:

A vertex assigned -1 is adjacent to exactly two vertices assigned 1, so remove two edges connecting them. This process is repeated for r vertices of V_{-1}. Then total number of edges deleted are $2r$ and edges left in $\mu_d^o(G)$ are $n - 1 - 2r$.

Step 2. *Add edges between vertices of V_{-1} and V_1 which are non adjacent in G*:

A vertex of V_{-1} is adjacent to exactly two vertices of V_1 and no vertex assigned -1. Hence in $\mu_d^o(G)$ a vertex of V_{-1} is adjacent to $n-r-2$ vertices of V_1. This process is repeated for r vertices of V_{-1}. Hence total edges added are $r(n-r-2)$.

From above two steps, $m(\mu_d^o(G)) = -r^2 + n(r+1) - 4r - 1$ follows. □

Theorem 2.10. *Let G be a nontrivial graph with $m(\mu_d^o(G)) = m(G)$. Then one of the following condition holds:*

(i) $G \cong \mu_d^o(G)$.
(ii) $\rho^-(G) = 0$.
(iii) *For every $v \in V_{-1}$, number of vertices adjacent to v is same as number of vertices of V_1 non adjacent to v.*

Proof. For a graph G,
(i) If $G \cong \mu_d^o(G)$, then $m(G) = m(\mu_d^o(G))$.
(ii) If $\rho^-(G) = 0$, then every vertex of G is assigned 1. Hence in $\mu_d^o(G)$ no new edges are added or deleted.
(iii) If any two vertices of V_{-1}(or V_1) are adjacent in G, then they are adjacent in $\mu_d^o(G)$. Now if a vertex v of V_{-1} is adjacent to k vertices of V_1 in G, then in $\mu_d^o(G)$, k edges are removed and if v is non adjacent to k vertices of V_1, then k edges are added in $\mu_d^o(G)$. This holds for every vertex of V_{-1}. Hence total number of edges added and deleted are same in $\mu_d^o(G)$. Therefore $m(\mu_d^o(G)) = m(G)$. □

Theorem 2.11. *For any nontrivial graph G,*

$$\gamma_s(\mu_d^o(G)) \leqslant n.$$

Further equality is obtained if every vertex of G is an end vertex or a support vertex.

Proof. If G is a graph with n vertices, then $\mu_d^o(G)$ is also a graph with n vertices for which $\gamma_s(\mu_d^o(G)) \leqslant n$ is obvious. If every vertex of G is either an end vertex or support vertex, then these vertices belong to V_1. The equality follows. □

3. Sign dominating inner switched graph

The Sign dominating inner switched graph of G denoted as $\mu_d^i(G)$ is defined as: remove the edges of G inside V_1, V_{-1} and add the missing edges joining vertices inside V_1 and V_{-1}.

Theorem 3.1. *Let G be a nontrivial graph. Then*

(i) $\mu_d^i(G) \cong K_n$ *if* $G \cong K_n^c$.
(ii) $\mu_d^i(G) \ncong K_n^c$.

Proof. (i) Let G be totally disconnected graph, then every vertex of G belongs to V_1. In $\mu_d^i(G)$, every vertex of G is connected to remaining $n-1$ vertices of G. Hence $\mu_d^i(G)$ is complete graph.

(ii) On the contrary, if $\mu_d^i(G) \cong K_n^c$, then following cases arise

Case 1. In G, there are no edges between vertices of V_1 and between vertices of V_{-1}.

Case 2. In G, $\langle V_1 \rangle$ is complete.

Case 3. In G, $\langle V_{-1} \rangle$ is complete.

Since a vertex of V_{-1} should be adjacent to atleast two vertices of V_1. Hence, Case 1 and Case 3 are not possible. If $\langle V_1 \rangle$ is complete, then $\gamma_s(G)$ is not minimum, which is a contradiction of our assumption. Thus the results follows. □

THEOREM 3.2. *For any nontrivial graph G,*

(i) $\mu_d^i(G))^c \cong \mu_d^i(G^c)$.

(ii) $(\mu_d^o(G))^c \cong \mu_d^i(G)$.

(iii) $m(\mu_d^o(G)) + m(\mu_d^i(G)) = \binom{n}{2}$.

PROOF. (i) Let u and v be two non adjacent vertices of a graph G. Then they are adjacent in G^c. We prove the result in following cases:

Case 1. If vertices u and v belongs to same set V_1 or V_{-1}, then they are adjacent in $\mu_d^i(G)$, implying that they are non adjacent in $(\mu_d^i(G))^c$. Also they are non adjacent in $\mu_d^i(G^c)$.

Case 2. If u and v belongs to different sets, then they are non adjacent in $\mu_d^i(G)$, implying they are adjacent in $(\mu_d^i(G))^c$. Also they are adjacent in $\mu_d^i(G^c)$.

From above two cases (i) follows.

(ii) Let u and v be two non adjacent vertices in $\mu_d^o(G)$.

$\Longleftrightarrow$ u and v are adjacent in $(\mu_d^o(G))^c$.

$\Longleftrightarrow$ If both u and v belongs to V_1 or V_{-1}, then they are non adjacent in G, implies they are adjacent in $\mu_d^i(G)$.

$\Longleftrightarrow$ If u and v belongs to different sets, then they are adjacent in G implies they are adjacent in $\mu_d^i(G)$. Thus (ii) follows.

(iii) From (ii), as graph $\mu_d^i(G)$ is complement of $\mu_d^o(G)$, sum of their edges should be equal to nC_2. □

THEOREM 3.3. *Let $G \cong K_{p,q}$ be a complete bipartite graph with bipartition P_1 and P_2 such that $|P_1| = p$ and $|P_2| = q$ with $p \leqslant q$. If $r = \left\lfloor \frac{q}{2} \right\rfloor$, then*

$$m(\mu_d^i(G)) = \frac{1}{2}\left[p(p-1) + q(q-1) + 2r(p-q+r)\right].$$

PROOF. From Theorems 3.2 and 2.4,

$$m(\mu_d^i(G)) = \frac{(p+q)(p+q-1)}{2} - m(\mu_d^o(G)).$$
$$m(\mu_d^i(G)) = \frac{(p+q)(p+q-1)}{2} - (p+r)(q-r).$$
$$m(\mu_d^i(G)) = \frac{p(p-1) + q(q-1) + 2r(p-q+r)}{2}.$$

□

THEOREM 3.4. *For any graph $G \cong K_n$ with $n \geqslant 3$ vertices,*

$$m(\mu_d^i(G)) = \begin{cases} \dfrac{n^2-4}{4} & \text{if } n \text{ is even,} \\ \dfrac{n^2-1}{4} & \text{if } n \text{ is odd.} \end{cases}$$

PROOF. From Theorem 3.2, $m(\mu_d^i(G)) = \dfrac{n(n-1)}{2} - m(\mu_d^o(G))$.
From Theorem 2.7, *for n being even*

$$m(\mu_d^i(G)) = \frac{n(n-1)}{2} - \frac{n^2-2n+4}{4} = \frac{n^2-4}{4},$$

and for n being odd

$$m(\mu_d^i(G)) = \frac{n(n-1)}{2} - \frac{n^2-2n+1}{4} = \frac{n^2-1}{4}.$$

□

THEOREM 3.5. *For a cycle C_n with $n \geqslant 3$ vertices,*

$$m(\mu_d^i(G)) = \frac{1}{2}\left[n^2 - 3n + 2r^2 - 2r(n-4)\right],$$

where $r = \left\lfloor \dfrac{n}{3} \right\rfloor$.

PROOF. From Theorem 3.2 and 2.8,

$$m(\mu_d^i(G)) = \frac{n(n-1)}{2} - m(\mu_d^o(G)).$$
$$m(\mu_d^i(G)) = \frac{n(n-1)}{2} - [r^2 + r(n-4) + n].$$
$$m(\mu_d^i(G)) = \frac{1}{2}\left[n^2 - 3n + 2r^2 - 2r(n-4)\right].$$

□

THEOREM 3.6. *For a path P_n with $n \geqslant 2$ vertices,*

$$m(\mu_d^i(G)) = \frac{1}{2}\left[n^2 + 2r^2 - 3n - 2nr + 2(4r+1)\right],$$

where $r = \left\lceil \dfrac{n-4}{3} \right\rceil$.

PROOF. From Theorem 3.2 and 2.9,

$$m(\mu_d^i(G)) = \frac{n(n-1)}{2} - m(\mu_d^o).$$
$$m(\mu_d^i(G)) = \frac{n(n-1)}{2} - [r^2 + n(r+1) - 4r - 1].$$
$$m(\mu_d^i(G)) = \frac{1}{2}\left[n^2 + 2r^2 - 3n - 2nr + 2(4r+1)\right].$$

□

THEOREM 3.7. *Let G be a nontrivial graph. Then $m(\mu_d^o(G)) = m(\mu_d^i(G))$ if and only if $(2n-1)^2 - 1$ is a multiple of 16.*

PROOF. If $m(\mu_d^o(G)) = m(\mu_d^i(G)) = k$, then from Theorem 3.2,

$$2k = \frac{n(n-1)}{2}$$
$$n^2 - n - 4k = 0$$
$$n = \frac{1 \pm \sqrt{1+16k}}{2}$$

On simplifying, n is a positive integer for $(2n-1)^2 - 1$ a multiple of 16. Conversely, if $(2n-1)^2 - 1$ is a multiple of $16k$, then $n = \frac{1+\sqrt{16k+1}}{2}$. For $n = 4, 5, \ldots$, we can generate the graph with $m(\mu_d^o(G)) = m(\mu_d^i(G))$. □

Open problem: Characterize the graphs for which

$$m(G) = m(\mu_d^i(G)).$$

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Department of Mathematics, Bangalore University, Jnana Bharathi Campus, Bangalore - 560 056, India
E-mail address: bchaluvaraju@gmail.com

Department of Mathematics, B. M. S. College of Engineering, Basavangudi, Bangalore - 560 019, India
E-mail address: chaitrashok@gmail.com