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NARUMI-KATAYAMA INDEX OF SOME DERIVED GRAPHS

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ABSTRACT. The Narumi-Katayama index of a graph G is equal to the product of degrees of all the vertices of G. In this paper, we examine the Narumi-Katayama index of some derived graphs such as a Mycielski graph, subdivision graphs, double graph, extended double cover graph, thorn graph, subdivision vertex join and edge join graphs.

1. INTRODUCTION

In Chemical graph theory a molecular graph is an unweighted, undirected graph without self loop or multiple edges such that its vertices corresponds to atoms and edges to the bonds between them. A topological index is a numeric quantity which is derived from a molecular graph and it does not depend on labeling or pictorial representation of a graph. Topological indices correlates the physico-chemical properties of molecular graph and are used for studying quantitative structure-activity (QSAR) and structure-property (QSPR) relationship for predicting different properties of chemical compounds and biological activities. In chemistry, biochemistry and nanotechnology different topological indices are used for modeling physico-chemical, pharmacologic, toxicologic, biological and other properties of chemical compounds.

There exist several types of such indices, especially those based on vertex degree which is one of the most widely used and have great application in chemical graph theory. Suppose G be a simple connected graph and $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of G. Let, n and m respectively denote the number of vertices and edges of G. Let, for any vertex $v \in V(G)$, $d_G(v)$ denotes

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its degree, that is the number of neighbor of v and $N(v)$ denotes the set of vertices which are the neighbors of the vertex v, so that $|N(v)| = d_G(v)$.

One of the oldest and well known topological indices is the first and second Zagreb indices, was first introduced by Gutman et al. in 1972 [**18**], where they have examined the dependence of the total π-electron energy on molecular structure. The first and second Zagreb indices of a graph are denoted by $M_1(G)$ and $M_2(G)$ and are respectively defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u)d_G(v).$$

These indices are extensively studied in chemical and mathematical literature. Interested readers are referred to [**16, 24, 32, 33, 34, 5**] for some recent results on this topic.

Todeschini et al. [**29, 30**] have introduced the multiplicative variants of additive graph invariants, which applied to the Zagreb indices would lead to the first and second Multiplicative Zagreb Indices. Thus the multiplicative Zagreb indices are defined as

$$\prod\nolimits_1 (G) = \prod_{uv \in E(G)} d_G(u)^2$$

and

$$\prod\nolimits_2 (G) = \prod_{uv \in E(G)} d_G(u)d_G(v).$$

The properties of these multiplicative Zagreb indices for trees were studied by Gutman [**20**]. These topological indices were subject to a large number of studies [**35, 25, 28**].

Related to multiplicative version of ordinary first Zagreb index, Eliasi, Iranmanesh and Gutman [**14**] introduced a new multiplicative graphical invariant and called multiplicative sum Zagreb index, which is defined as

$$\prod\nolimits_1{}^*(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)].$$

In 1984, Narumi and Katayama [**27**] introduced a multiplicative graph invariant for representing the carbon skeleton of a saturated hydrocarbon, and named it as a "simple topological index". Tomovic and Gutman [**31**] later renamed this index as "Narumi-Katayama index" or NK index and is denoted by $NK(G)$. The Narumi-Katayama index of a graph G is defined as the product of degrees of all its vertices, that is

$$NK(G) = \prod_{v \in V(G)} d_G(v).$$

Clearly, the Narumi-Katayama index is just the square root of the first multiplicative Zagreb index. In this paper, we compute Narumi-Katayama index of several classes of derived graphs such as Mycielski graph, subdivision graphs, double graph, extended double cover graph, thorn graph, subdivision vertex join and edge join graphs.

2. MAIN RESULTS

In this section, we proceed to introduce different derived graphs that are relevant for this study and hence present the behavior of the Narumi-Katayama index of these derived graphs. First, we recall a well-known inequalities.

LEMMA 2.1. *(A.M.-G.M. Inequality) Let $x_1, x_2, ..., x_n$ be non-negative numbers, then*

$$\frac{x_1 + x_2 + ... + x_n}{n} \geqslant \sqrt[n]{x_1 x_2 .... x_n}$$

with equality if and only if $x_1 = x_2 = ... = x_n$.

In the following first we determine Narumi-Katayama index of the Mycielski graph $\mu(G)$ of G.

2.1. The Mycielski Graphs. The construction of the Mycielski graph was introduced in [**26**]. The Mycielski graph $\mu(G)$ of G contains G itself as an isomorphic subgraph, and also $(n + 1)$ additional vertices; a vertex u_i corresponding to each vertex v_i and another vertex w. Each vertex u_i is connected by an edge to the vertex w, so that these vertices form a subgraph in the form of a star $K_{1,n}$. In addition for each edge $v_i v_j$ of G, the Mycielski graph includes two edges $u_i v_j$ and $v_i u_j$. The vertex set of $\mu(G)$ is given by $V(\mu(G)) = V(G) \cup X \cup \{w\}$, where $V(G) = \{v_1, v_2, ..., v_n\}$ and $x = \{u_1, u_2, ..., u_n\}$. Thus $E(\mu(G)) = E(G) \cup \{v_i u_j : v_i v_j \in E(G)\} \cup \{u_i w : 1 \leqslant i \leqslant n\}$. Clearly, if G has n vertices and m edges then has $(2n+1)$ vertices and $(3m+n)$ edges. For different mathematical properties and applications of the Mycielski graph, we refer the reader to [**2, 3, 4, 17, 15, 21**].

THEOREM 2.1. *The Narumi-Katayama index of Mycielski graph $\mu(G)$ satisfies the following inequality*

$$NK(\mu(G)) \leqslant 2^n n NK(G) \left(\frac{2m}{n} + 1 \right)^n$$

with equality if and only if G is a regular graph.

PROOF. Let G be a nontrivial graph of order n and size m and let $\mu(G)$ be its Mycielski graph, then from the construction of Mycielski graph, for each $i = 1, 2, ..., n$, $d_{\mu(G)}(v_i) = 2d_G(v_i)$, $d_{\mu(G)}(u_i) = d_G(v_i) + 1$ and $d_{\mu(G)}(w) = n$ Then the

Narumi-Katayama index of Mycielski graph $\mu(G)$ is given by

$$\begin{aligned} NK(\mu(G)) &= d_{\mu(G)}(w)\prod_{i=1}^{n} d_{\mu(G)}(v_i)\prod_{i=1}^{n} d_{\mu(G)}(u_i) \\ &= n\prod_{i=1}^{n}\{2d_G(v_i)\}\prod_{i=1}^{n}\{d_G(v_i)+1\} \\ &= 2^n nNK(G)\prod_{i=1}^{n}\{d_G(v_i)+1\}. \end{aligned}$$

Now using Lemma 2.1, we have

$$\prod_{i=1}^{n}\{d_G(v_i)+1\} \leqslant \left[\frac{1}{n}\sum_{i=1}^{n}\{d_G(v_i)+1\}\right]^n = \left[\frac{1}{n}(2m+n)\right]^n$$

with equality if and only if G is a regular graph, so the desired result follows from above. □

Corollary 2.1. *If G be a r-regular graph with n vertices then*

$$NK(\mu(G)) = n(2r(r+1))^n.$$

2.2. Subdivision Graphs. Let G be a connected graph. We are now concerned with the following derived graphs of G by subdividing each edges of G so that the vertex set of these graphs are equal to $V(G) \cup E(G)$.

(a) $S(G)$ is obtained from G by replacing each edge of G by a path of length two.

(b) $R(G)$ is obtained from G by adding a new vertex corresponding to each edge of G, then joining each new vertex to the end vertices of the corresponding edge.

(c) $Q(G)$ is obtained from G by inserting a new vertex into each edge of G, then joining with edges those pairs of new vertices on adjacent edges of G.

(d) $T(G)$ has its vertices the edges and vertices of G. Adjacency in $T(G)$ is defined as adjacency or incidence for the corresponding elements of G, $T(G)$ is also called the total graph of G.

We refer the reader to [**6, 13, 36, 16**] for mathematical properties and applications of the these subdivision graphs. First we recall the following important relevant lemma.

Lemma 2.2. *(a) For every vertex $v \in V(G)$, we have $d_{S(G)}(v) = d_{Q(G)}(v) = d_G(v)$, $d_{R(G)}(v) = d_{T(G)}(v) = 2d_G(v)$.*

(b) For every vertex $v \in V(F(G))\backslash V(G)$, where $F = \{S, Q, R, T\}$, we have $d_{S(G)}(v) = d_{R(G)}(v) = 2$, $d_{Q(G)}(v) = d_{T(G)}(v) = d_{L(G)}(v) + 1$.

In the following first we determine the Narumi-Katayama index of the subdivision graph $S(G)$, then the triangle parallel graph $R(G)$, then the line superposition graph $Q(G)$ and the total graph $T(G)$ respectively.

THEOREM 2.2. *The Narumi-Katayama index of Subdivision graphs S(G), R(G), Q(G) and T(G) are given by*
(a) $NK(S(G)) = 2^m NK(G)$.
(b) $NK(R(G)) = 2^{n+m} NK(G)$.
(c) $NK(Q(G)) = NK(G)\Pi_1{}^*(G)$.
(d) $NK(T(G)) = 2^n NK(G)\Pi_1{}^*(G)$.

PROOF. (a) Using Lemma 2.2 we have,

$$\begin{aligned} NK(S(G)) &= \prod_{v\in V(S(G))} d_{S(G)}(v) \\ &= \prod_{v\in V(G)} d_{S(G)}(v) \prod_{v\in V(S(G))\setminus V(G)} d_{S(G)}(v) \\ &= \prod_{v\in V(G)} d_G(v) \prod_{v\in V(S(G))\setminus V(G)} 2 \\ &= 2^m NK(G) \end{aligned}$$

which is the desired result.

(b) By using the facts in Lemma 2.2, we get

$$\begin{aligned} NK(R(G)) &= \prod_{v\in V(R(G))} d_{R(G)}(v) \\ &= \prod_{v\in V(G)} d_{R(G)}(v) \prod_{v\in V(R(G))\setminus V(G)} d_{R(G)}(v) \\ &= 2^m \prod_{v\in V(G)} 2d_G(v) \\ &= 2^{n+m} NK(G). \end{aligned}$$

Hence the desired result follows.

(c) Still using Lemma 2.2, we have

$$\begin{aligned} NK(Q(G)) &= \prod_{v\in V(Q(G))} d_{Q(G)}(v) \\ &= \prod_{v\in V(G)} d_{Q(G)}(v) \prod_{v\in V(Q(G))\setminus V(G)} d_{Q(G)}(v) \\ &= \prod_{v\in V(G)} d_G(v) \prod_{v\in V(Q(G))\setminus V(G)} \{d_{L(G)}(v)+2\} \\ &= NK(G) \prod_{(u,v)\in E(G)} \{d_G(u)+d_G(v)\} \\ &= NK(G)\Pi_1{}^*(G). \end{aligned}$$

which is the desired result.

(d) Again, using Lemma 2.2, we have

$$\begin{aligned} NK(T(G)) &= \prod_{v\in V(T(G))} d_{T(G)}(v) \\ &= \prod_{v\in V(G)} d_{T(G)}(v) \prod_{v\in V(T(G))\setminus V(G)} d_{T(G)}(v) \\ &= \prod_{v\in V(G)} 2d_G(v) \prod_{v\in V(T(G))\setminus V(G)} \{d_{L(G)}(v)+2\} \\ &= 2^n NK(G) \prod_{(u,v)\in E(G)} \{d_G(u)+d_G(v)\} \\ &= 2^n \Pi_1(G){\Pi_1}^*(G). \end{aligned}$$

Hence the desired result follows. □

Next in the following examples, we present the expressions of Narumi-Katayama index for subdivision graphs of three different classes of graphs, which are direct consequence of the previous theorem.

Example 2.1. (i) $NK(S(K_n)) = 2^{\frac{n(n-1)}{2}}(n-1)^n$.
(ii) $NK(R(K_n)) = 2^{\frac{n(n+1)}{2}}(n-1)^n$.
(iii) $NK(Q(K_n)) = 2^{\frac{n(n-1)}{2}}(n-1)^{\frac{n(n+1)}{2}}$.
(iv) $NK(T(K_n)) = 2^{\frac{n(n+1)}{2}}(n-1)^{\frac{n(n+1)}{2}}$.

Example 2.2. (i) $NK(S(C_n)) = 2^{2n}$.
(ii) $NK(R(C_n)) = 2^{3n}$.
(iii) $NK(Q(C_n)) = 2^{3n}$.
(iv) $NK(T(C_n)) = 2^{4n}$.

Example 2.3. (i) $NK(S(P_n)) = 2^{2n-3}$.
(ii) $NK(R(P_n)) = 2^{3n-3}$.
(iii) $NK(Q(P_n)) = 3^2.2^{3n-8}$.
(iv) $NK(T(C_n)) = 3^2.2^{4n-8}$.

2.3. Double Graph and Extended Double cover. In this section, we find the expressions of the Narumi-Katayama index of double graph and extended double cover graphs. Let $G = (V, E)$ be a simple connected graph with $V = \{v_1, v_2, ..., v_n\}$. The double graph G^* of a given graph G is constructed by making two copies of G (including the initial edge set of each) and adding edges u_1v_2 and u_2v_1 for every edge uv of G. The extended double cover of G, denoted by G^{**} is the bipartite graph with bipartion (X, Y) where $X = \{x_1, x_2, ..., x_n\}$ and $Y = \{y_1, y_2, ..., y_n\}$ in which x_i and y_i are adjacent if and only if $i = j$. For example the extended double cover of the complete graph is the complete bipartite graph. The extended double cover was introduced by Alon in [**1**]. We refer the reader to [**10, 12, 22**] for mathematical properties and applications of the these double graph and extended double cover graphs. In the following we find the Narumi-Katayama index of double graph and extended double cover graph respectively.

THEOREM 2.3. *Let G be a simple connected graph with order n, then the Narumi-Katayama index of G^* is given by*

$$NK(G^*) = 4^n NK(G)^2.$$

PROOF. From the construction of double graph of G it is clear that $d_{G^*}(x_i) = d_{G^*}(y_i) = 2d_G(v_i)$ for i=1,2,,n. Thus the Narumi-Katayama index of the double graph G^* is given by

$$NK(G^*) = \prod_{i=1}^{n} d_{G^*}(x_i) \prod_{i=1}^{n} d_{G^*}(y_i) = \left\{ \prod_{i=1}^{n} 2d_G(v_i) \right\}^2 = 2^{2n} NK(G)^2$$

which is the desired result. □

EXAMPLE 2.4. Let G_{2n} be the double graph of P_n. Then the NK-index of G_{2n} is given by

$$NK(G_{2n}) = 4^{2(n-1)}.$$

THEOREM 2.4. *The Narumi-Katayama index of G^{**} satisfies the following inequality*

$$NK(G^{**}) \leqslant \left(\frac{2m}{n} + 1 \right)^{2n}$$

with equality if and only if G is a regular graph.

PROOF. If G is a graph with n vertices and m edges then from definition of extended double cover graph G^{**} consists of $2n$ vertices and $(n + 2m)$ edges and

$$d_{G^{**}}(x_i) = d_{G^{**}}(y_i) = d_G(v_i) + 1$$

for $i = 1, 2, , n$. Then using the definition of Narumi-Katayama index, for extended double cover it is given by

$$NK(G^{**}) = \prod_{i=1}^{n} d_{G^{**}}(x_i) \prod_{i=1}^{n} d_{G^{**}}(y_i) = \left\{ \prod_{i=1}^{n} (d_G(v_i) + 1) \right\}^2$$

Now using Lemma 2.1 we have

$$\left\{ \prod_{i=1}^{n} (d_G(v_i) + 1) \right\}^2 \leqslant \left(\frac{2m}{n} + 1 \right)^{2n}$$

with equality if and only if all the vertices of G are of same degree. This completes the proof. □

EXAMPLE 2.5. Let H_{2n} be the double graph of P_n. Then the NK-index of H_{2n} is given by

$$NK(H_{2n}) = 16.3^{2(n-2)}.$$

2.4. Thorn Graph. The concept of thorn graph was first introduced by Gutman [**19**] and is obtained by joining a number of edges or thorn to each vertex of of the given graph G. A thorn graph is denoted by G^T, so that $V(G^T) = V(G) \cup V_1 \cup V_2 \cup ... \cup V_n$ be the vertex set of G^T, where V_i, $i = 1, 2, ..., n$ be the set of vertices of degree one attached to the vertex v_i in G^T. Let p_i be the number of thorns attached to the vertex v_i in G^T. Thus if v_{ij} denote the vertices of the set V_i ($i = 1, 2, ..., n$ and $j = 1, 2, ..., p_i$), then $d_{G^T}(v_i) = d_G(v_i) + p_i$ and $|V(G^T)| = n + z$ where $z = \sum\limits_{i=1}^{n} p_i$. For different study regarding thorn graphs we refer the reader to [**7, 8, 9, 6, 11**]. In the following we find an upper bound of the Narumi-Katayama index of the thorn graph G^T and consider some particular cases.

THEOREM 2.5. *The Narumi-Katayama index of G^T satisfies the following inequality*

$$NK(G^T) \leqslant \left(\frac{2m+z}{n}\right)^n$$

with equality if and only if $d_G(v_1) + p_1 = d_G(v_2) + p_2 = ... = d_G(v_n) + p_n$.

PROOF. Using the definition of Narumi-Katayama index for thorn graph G^T and using lemma 1, we have

$$\begin{aligned} NK(G^T) &= \prod_{i=1}^{n} d_{G^T}(v_i) = \prod_{i=1}^{n} (d_G(v_i) + p_i) \\ &\leqslant \left[\frac{1}{n}\sum_{i=1}^{n} (d_G(v_i) + p_i)\right]^n = \left(\frac{2m+z}{n}\right)^n \end{aligned}$$

Clearly, in the above inequality equality holds if and only if $d_G(v_1)+p_1 = d_G(v_2)+p_2 = ... = d_G(v_n) + p_n$. □

Now from the previous theorem the following corollaries are follows.

COROLLARY 2.2. *Let G^T be the thorn graph where $p_i = t$, for all i, then*

$$NK(G^T) \leqslant \left(\frac{m}{n} + t\right)^n$$

with equality if G is a regular graph.

COROLLARY 2.3. *Let G^T be the thorn graph where p_i ($\geqslant 1$) is equal to the degree of the corresponding vertex v_i, for all i, then*

$$NK(G^T) \leqslant \left(\frac{3m}{n}\right)^n$$

with equality if G is a regular graph.

COROLLARY 2.4. *Let G^T be the thorn graph where $d_G(v_i) + p_i = \lambda$, for all i, then*

$$NK(G^T) = \left(\frac{n\lambda - m}{n}\right)^n.$$

2.5. The Subdivision vertex-join graph. The subdivision-vertex join [**23**] of two vertex disjoint graphs G_1 and G_2 with n_1 and n_2 vertices and m_1 and m_2 edges is the graph denoted by $G_1\dot{\vee}G_2$ and is obtained from $S(G_1)$ and G_2 by joining each vertices of G_1 with every vertex of G_2.

THEOREM 2.6. *The Narumi-Katayama index of $G_1\dot{\vee}G_2$ satisfies the following inequality*

$$NK(G_1\dot{\vee}G_2) \leqslant 2^{m_1}\left[\frac{1}{n_1}(2m_1+n_1n_2)\right]^{n_1}\left[\frac{1}{n_2}(2m_2+n_2{n_1}^2)\right]^{n_2}$$

with equality if and only if both G_1 and G_2 are both regular graphs.

PROOF. From the definition of subdivision-vertex join of two graphs G_1 and G_2 it is clear that,

$$\deg_{G_1\dot{\vee}G_2}(v) = \begin{cases} d_{G_2}(v)+n_1, if\ v\in V(G_2) \\ d_{G_1}(v)+n_2, if\ v\in V(G_2) \\ 2, if\ v\in V(S(G_1)\backslash V(G_2) \end{cases}$$

Thus the Narumi-Katayama index of subdivision-vertex join of G_1 and G_2 is given by

$$\begin{aligned} NK(G_1\dot{\vee}G_2) &= \prod_{v\in V(G_1)} d_{G_1\dot{\vee}G_2}(v)\prod_{v\in V(G_2)} d_{G_1\dot{\vee}G_2}(v)\prod_{v\in V(S(G)_1)\backslash V(G_1)} d_{G_1\dot{\vee}G_2}(v) \\ &= \prod_{v\in V(G_1)}(d_{G_1}(v)+n_2)\prod_{v\in V(G_2)}(d_{G_2}(v)+n_1)\prod_{v\in V(S(G)_1)\backslash V(G_1)} 2 \end{aligned}$$

Now using the inequality between arithmetic and geometric mean we have

$$\prod_{v\in V(G_1)}(d_{G_1}(v)+n_2) \leqslant \left[\frac{1}{n_1}\sum_{v\in V(G_1)}(d_{G_1}(v)+n_2)\right]^{n_1} = \left[\frac{1}{n_1}(2m_1+n_1n_2)\right]^{n_1}$$

with equality if and only if G_1 is regular. Similarly we have

$$\prod_{v\in V(G_1)}(d_{G_2}(v)+n_1) \leqslant \left[\frac{1}{n_2}(2m_2+n_2n_1)\right]^{n_2}$$

with equality if and only if G_2 is a regular graph . Hence from above the desired result follows. □

COROLLARY 2.5. *If G_i be a r_i-regular graph for $i = 1, 2$ then the Narumi-Katayama index of $G_1\dot{\vee}G_2$ is given by*

$$NK(G_1\dot{\vee}G_2) = 2^{\frac{n_1r_1}{2}}(r_1+n_2)^{n_1}(r_2+n_1)^{n_2}.$$

EXAMPLE 2.6. $NK(K_p\dot{\vee}K_q) = 2^{\frac{p(p-1)}{2}}(p+q-1)^p(p+q-1)^q.$

2.6. The Subdivision edge-join graph. The subdivision-edge join [**23**] of two vertex disjoint graphs G_1 and G_2 with n_1 and n_2 vertices and m_1 and m_2 edges is the graph denoted by $G_1 \veebar G_2$ and is obtained from $S(G_1)$ and G_2 by joining each vertices of $S(G_1)\backslash V(G_1)$ with every vertex of G_2.

THEOREM 2.7. *The Narumi-Katayama index of* $G_1 \veebar G_2$ *satisfies the following inequality*

$$NK(G_1 \veebar G_2) \leqslant NK(G_1)(2+n_2)^{m_1}\left(\frac{2m_2}{n_2}+m_1\right)^{n_2}$$

with equality if and only if G_2 *is regular.*

PROOF. From the definition of subdivision-vertex join of two graphs G_1 and G_2 it is clear that [**23**],

$$d_{G_1 \veebar G_2}(v) = \begin{cases} d_{G_1}(v), if\ v \in V(G_1) \\ d_{G_2}(v)+m_1, if\ v \in V(G_2) \\ 2+n_2, if\ v \in V(S(G_1)\backslash V(G_2) \end{cases}$$

Thus the Narumi-Katayama index of subdivision-vertex join of G_1 and G_2 is given by

$$\begin{aligned} NK(G_1 \veebar G_2) &= \prod_{v\in V(G_1)} d_{G_1 \veebar G_2}(v) \prod_{v\in V(G_2)} d_{G_1 \veebar G_2}(v) \prod_{v\in V(S(G)_1)\backslash V(G_1)} d_{G_1 \veebar G_2}(v) \\ &= \prod_{v\in V(G_1)} d_{G_1}(v) \prod_{v\in V(G_2)} (d_{G_2}(v)+m_1) \prod_{v\in V(S(G)_1)\backslash V(G_1)} (2+n_2) \\ &= NK(G_1)(2+n_2)^{m_1} \prod_{v\in V(G_2)} (d_{G_2}(v)+m_1). \end{aligned}$$

Now using the Lemma 1, we have

$$\prod_{v\in V(G_2)} (d_{G_2}(v)+m_1) \leqslant \left[\frac{1}{n_2}\sum_{v\in V(G_2)} (d_{G_2}(v)+m_1)\right]^{n_2} = \left[\frac{1}{n_2}(2m_2+n_2m_1)\right]^{n_2}$$

with equality if and only if G_2 is a regular graph. Hence the desired result follows from above. □

COROLLARY 2.6. *If* G_2 *be a r-regular graph and* G_1 *be any arbitrary graph, then the Narumi-Katayama index of* $G_1 \veebar G_2$ *is given by*

$$NK(G_1 \veebar G_2) = NK(G_1)(r+m_1)^{n_2}(2+n_2)^{m_1}.$$

3. Conclusion

In this paper, we compute Narumi-Katayama index of several classes of derived graphs such as Mycielski graph, subdivision graphs, double graph, extended double cover graph, thorn graph, subdivision vertex join and edge join graphs. For further study, Narumi-Katayama index of some other derived graphs and for different composite graphs can be computed.

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