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PERMUTING TRI-f-DERIVATIONS ON ALMOST DISTRIBUTIVE LATTICES

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ABSTRACT. In this paper, we introduce the concept of permuting tri-f-derivation in an Almost Distributive Lattice (ADL) and derive some important properties of permuting tri-f-derivation in ADLs.

1. Introduction

The notion of derivation in lattices was first given in G. Szasz [14] in 1974. Several authors worked on derivations in Lattices ([1], [2], [3], [4], [5], [6], [15], [16] and [17]). The concept of derivation in an ADL was introduced in our earlier paper [8]. Further, in an ADL we worked on f-derivations in [9], symmetric bi-derivations in [10], symmetric bi-f-derivations in [11] and permuting tri-derivations in [12]. The concept of permuting tri-f-derivations in lattices was introduced by H. Yazarli and M. A. Öztürk [17] in 2011.

In this paper, we introduce the concept of permuting tri-f-derivations in an ADL and investigate some important properties. If m is a maximal element in an ADL L, then we prove that $D(x, y, z) = fx$ when $fx \leqslant D(m, y, z)$ and if fm is also a maximal element of L, then we prove that $D(x, y, z) \geqslant D(m, y, z)$ when $fx \geqslant D(m, y, z)$. Also. we prove that $fx \wedge D(x \vee w, y, z) = D(x, y, z)$ when D is an isotone map and $fx \wedge D(x \vee w, y, z) \leqslant D(x, y, z)$ when f is either a join preserving or an increasing function on L. We establish a set of conditions which are sufficient for a permuting tri-f-derivation on an ADL with a maximal element to become an isotone when f is a homomorphism. Also, we prove

$$d(x \wedge y) = (fy \wedge dx) \vee D(x, x, y) \vee D(x, y, y) \vee (fx \wedge dy)$$

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where d is the trace of a permuting tri-f-derivation on an associative ADL L. Finally, we prove that the set $F_d(L) = \{x \in L/dx = fx\}$ is a weak ideal in an associative ADL L where f is a join preserving map on L.

2. Preliminaries

In this section, we recollect certain basic concepts and important results on Almost Distributive Lattices.

Definition 2.1. [7] *An algebra* $(L, \vee, \wedge)$ *of type* $(2, 2)$ *is called an Almost Distributive Lattice, if it satisfies the following axioms:*
L_1 : $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$ $(RD\wedge)$
L_2 : $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ $(LD\wedge)$
L_3 : $(a \vee b) \wedge b = b$
L_4 : $(a \vee b) \wedge a = a$
L_5 : $a \vee (a \wedge b) = a$ *for all* $a, b, c \in L$.

Definition 2.2. [7] *Let* X *be any non-empty set. Define, for any* $x, y \in L$, $x \vee y = x$ *and* $x \wedge y = y$. *Then* $(X, \vee, \wedge)$ *is an ADL and such an ADL, we call discrete ADL.*

Through out this paper L stands for an ADL $(L, \vee, \wedge)$ unless otherwise specified.

Lemma 2.1. [7] *For any* $a, b \in L$, *we have:*
(i) $a \wedge a = a$
(ii) $a \vee a = a$.
(iii) $(a \wedge b) \vee b = b$
(iv) $a \wedge (a \vee b) = a$
(v) $a \vee (b \wedge a) = a$.
(vi) $a \vee b = a$ *if and only if* $a \wedge b = b$
(vii) $a \vee b = b$ *if and only if* $a \wedge b = a$.

Definition 2.3. [7] *For any* $a, b \in L$, *we say that* a *is less than or equal to* b *and write* $a \leqslant b$, *if* $a \wedge b = a$ *or, equivalently,* $a \vee b = b$.

Theorem 2.1. [7] *For any* $a, b, c \in L$, *we have the following*
(i) *The relation* $\leqslant$ *is a partial ordering on* L.
(ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$. $(LD\vee)$
(iii) $(a \vee b) \vee a = a \vee b = a \vee (b \vee a)$.
(iv) $(a \vee b) \wedge c = (b \vee a) \wedge c$.
(v) *The operation* $\wedge$ *is associative in* L.
(vi) $a \wedge b \wedge c = b \wedge a \wedge c$.

Theorem 2.2. [7] *For any* $a, b \in L$, *the following are equivalent.*
(i) $(a \wedge b) \vee a = a$
(ii) $a \wedge (b \vee a) = a$
(iii) $(b \wedge a) \vee b = b$
(iv) $b \wedge (a \vee b) = b$

(v) $a \wedge b = b \wedge a$
(vi) $a \vee b = b \vee a$
(vii) *The supremum of* a *and* b *exists in* L *and equals to* $a \vee b$
$(viii)$ *There exists* $x \in L$ *such that* $a \leqslant x$ *and* $b \leqslant x$
(ix) *The infimum of* a *and* b *exists in* L *and equals to* $a \wedge b$.

Definition 2.4. [7] *L is said to be associative, if the operation $\vee$ in L is associative.*

Theorem 2.3. [7] *The following are equivalent:*
(i) *L is a distributive lattice.*
(ii) *The poset $(L, \leqslant)$ is directed above.*
(iii) $a \wedge (b \vee a) = a$, *for all* $a, b \in L$.
(iv) *The operation $\vee$ is commutative in L.*
(v) *The operation $\wedge$ is commutative in L.*
(vi) *The relation $\theta := \{(a, b) \in L \times L \mid a \wedge b = b\}$ is anti-symmetric.*
(vii) *The relation θ defined in (vi) is a partial order on L.*

Lemma 2.2. [7] *For any $a, b, c, d \in L$, we have the following:*
(i) $a \wedge b \leqslant b$ *and* $a \leqslant a \vee b$
(ii) $a \wedge b = b \wedge a$ *whenever* $a \leqslant b$.
(iii) $[a \vee (b \vee c)] \wedge d = [(a \vee b) \vee c] \wedge d$.
(iv) $a \leqslant b$ *implies* $a \wedge c \leqslant b \wedge c$, $c \wedge a \leqslant c \wedge b$ *and* $c \vee a \leqslant c \vee b$.

Definition 2.5. [7] *An element $0 \in L$ is called zero element of L, if $0 \wedge a = 0$ for all $a \in L$.*

Lemma 2.3. [7] *If L has 0, then for any $a, b \in L$, we have the following:*
(i) $a \vee 0 = a$, (ii) $0 \vee a = a$ *and* (iii) $a \wedge 0 = 0$.
(iv) $a \wedge b = 0$ *if and only if* $b \wedge a = 0$.

Definition 2.6. [13] *Let L be a non-empty set and $x_0 \in L$. If for $x, y \in L$ we define*
$x \wedge y = y$ *if* $x \neq x_0$
$x \wedge y = x$ *if* $x = x_0$ *and*
$x \vee y = x$ *if* $x \neq x_0$
$x \vee y = y$ *if* $x = x_0$,
then $(L, \vee, \wedge, x_0)$ is an ADL with x_0 as zero element. This is called discrete ADL with zero.

An element $x \in L$ is called maximal if, for any $y \in L$, $x \leqslant y$ implies $x = y$.

We immediately have the following.

Lemma 2.4. [7] *For any $m \in L$, the following are equivalent:*
(1) m is maximal
(2) $m \vee x = m$ for all $x \in L$
(3) $m \wedge x = x$ for all $x \in L$.

Definition 2.7. [7] *A nonempty subset I of L is said to be an ideal if and only if it satisfies the following:*

(1) $a, b \in I \Rightarrow a \vee b \in I$
(2) $a \in I,\ x \in L \Rightarrow a \wedge x \in I$.

Definition 2.8. [7] *A nonempty subset I of L is said to be an initial segment of L if, $a \in L$ and $x \in L$ such that $x \leqslant a$ imply that $x \in L$.*

Definition 2.9. [10] *A nonempty subset I of L is said to be a weak ideal if and only if it satisfies the following:*
(1) $a, b \in I \Rightarrow a \vee b \in I$
(2) I is an initial segment of L.

Observe that every ideal of L ia weak ideal, but not converse.

Definition 2.10. [7] *A function $f : L \to L$ is said to be an ADL homomorphism if it satisfies the following:*
(1) $f(x \wedge y) = fx \wedge fy$,
(2) $f(x \vee y) = fx \vee fy$ *for all* $x, y \in L$.

Definition 2.11. *A function $d : L \to L$ is called an isotone, if $dx \leqslant dy$ for any $x, y \in L$ with $x \leqslant y$.*

3. Permuting tri-f-derivations in ADLs.

We begin this paper with the following definition of a permuting map in an *ADL*.

Definition 3.1. [12]

(i) *A map $D : L \times L \times L \to L$ is called permuting map if*

$$D(x, y, z) = D(x, z, y) = D(y, z, x) = D(y, x, z) = D(z, x, y) = D(z, y, x)$$

for all $x, y, z \in L$.

(ii) *D is called an isotone map if, for any $x, y, z, w \in L$ with $x \leqslant w$, $D(x, y, z) \leqslant D(w, y, z)$.*

(iii) *The mapping $d : L \to L$ defined by $dx = D(x, x, x)$ for all $x \in L$, is called the trace of D.*

Definition 3.2. [12] *A permuting map $D : L \times L \times L \to L$ is called a permuting tri-derivation on L, if*

$$D(x \wedge w, y, z) = [w \wedge D(x, y, z)] \vee [x \wedge D(w, y, z)]$$

for all $x, y, z, w \in L$.

Now, the following definition gives the notion of permuting tri-f-derivation in an *ADL*.

Definition 3.3. *A permuting map $D : L \times L \times L \to L$ is called a permuting tri-f-derivation on L, if there exists a function $f : L \to L$ such that*

$$D(x \wedge w, y, z) = [fw \wedge D(x, y, z)] \vee [fx \wedge D(w, y, z)] \text{ for all } x, y, z, w \in L.$$

Observe that a permuting tri-f-derivation D on L also satisfies

$$D(x, y \wedge w, z) = [fw \wedge D(x, y, z)] \vee [fy \wedge D(x, w, z)] \text{ and}$$
$$D(x, y, z \wedge w) = [fw \wedge D(x, y, z)] \vee [fz \wedge D(x, y, w)]$$

for all $x, y, z, w \in L$.

EXAMPLE 3.1. Every permuting tri-derivation on L is a permuting tri-f - derivation, where $f : L \to L$ is the identity map.

EXAMPLE 3.2. Let L be an ADL with 0 and $0 \neq a \in L$. If we define a mapping $D : L \times L \times L \to L$ by $D(x, y, z) = a$ for all $x, y, z \in L$ and $f : L \to L$ by $fx = a$ for all $x \in L$, then D is a permuting tri-f-derivation on L but not a permuting tri-derivation on L..

EXAMPLE 3.3. Let L be an ADL with atleast two elements. If we define a mapping $D : L \times L \times L \to L$ by $D(x, y, z) = (x \vee y) \vee z$, then D is not a permuting tri-f-derivation on L, since it is not a permuting map on L.

EXAMPLE 3.4. Let L be an ADL with at least three elements and $a \in L$. If we define the mapping $D : L \times L \times L \to L$ by $D(x, y, z) = (x \vee y \vee z) \wedge a$ for all $x, y, z \in L$ and $f : L \to L$ by $fx = a$ for all $x \in L$, then D is a permuting tri-f-derivation on L but, not a permuting tri-derivation on L.

LEMMA 3.1. *Let D be a permuting tri-f-derivation on L. Then the following identities hold:*

(1) $D(x, y, z) = fx \wedge D(x, y, z) = fy \wedge D(x, y, z) = fz \wedge D(x, y, z)$ *for all* $x, y, z \in L$
(2) *If L has 0 and $f0 = 0$, then $D(0, y, z) = 0$ for all $y, z \in L$*
(3) $(fx \vee fy) \wedge D(x \wedge w, y, z) = D(x \wedge w, y, z)$ *for all* $x, y, z, w \in L$
(4) $fx \wedge dx = dx$ *for all* $x \in L$.

PROOF. Let $x, y, z, w \in L$.

(1) $D(x, y, z) = D(x \wedge x, y, z) = [fx \wedge D(x, y, z)] \vee [fx \wedge D(x, y, z)] = fx \wedge D(x, y, z)$. Similarly, $fy \wedge D(x, y, z) = D(x, y, z) = fz \wedge D(x, y, z)$.

(2) Suppose L has 0 and $f0 = 0$. Now by (1) above, $D(0, y, z) = f0 \wedge D(0, y, z) = 0 \wedge D(0, y, z) = 0$.

(3) $(fx \vee fw) \wedge D(x \wedge w, y, z) = (fx \vee fw) \wedge [[fw \wedge D(x, y, z)] \vee [fx \wedge D(w, y, z)]] = [fw \wedge D(x, y, z)] \vee [fx \wedge D(w, y, z)] = D(x \wedge w, y, z)$.

(4) By (1) above, we get that $fx \wedge D(x, x, x) = D(x, x, x)$. Thus $fx \wedge dx = dx$. $\square$

THEOREM 3.1. *Let D be a permuting tri-f-derivation on L and m be a maximal element in L. Then the following hold:*

(1) *If $x, y, z \in L$ such that $fx \leqslant D(m, y, z)$, then $D(x, y, z) = fx$.*
(2) *If $x, y, z \in L$ such that $fx \geqslant D(m, y, z)$ and fm is a maximal element in L, then $D(x, y, z) \geqslant D(m, y, z)$.*

PROOF. (1) Let $x, y, z \in L$ with $fx \leqslant D(m, y, z)$. Then $D(x, y, z) = D(m \wedge x, y, z) = [fx \wedge D(m, y, z)] \vee [fm \wedge D(x, y, z)] = fx \vee [fm \wedge D(x, y, z)] = (fx \vee fm) \wedge [fx \vee D(x, y, z)] = (fx \vee fm) \wedge fx = fx$, by Lemma 3.1.

(2) Let $x, y, z \in L$ with $fx \geqslant D(m, y, z)$. Then $D(x, y, z) = D(m \wedge x, y, z) =$

$[fx \wedge D(m,y,z)] \vee [fm \wedge D(x,y,z)] = D(m,y,z) \vee D(x,y,z)$. Thus $D(x,y,z) \geqslant D(m,y,z)$. □

THEOREM 3.2. *Let D be a permuting tri-f-derivation on L where f is an increasing function on L. If $x,y,z \in L$ such that $w \leqslant x$ and D(x,y,z)=fx, then $D(w,y,z) = fw$.*

PROOF. Let $x,y,z \in L$ with $w \leqslant x$ and $D(x,y,z) = fx$. Since f is an increasing function on L, $fw \leqslant fx$. Now $D(w,y,z) = D(x \wedge w,y,z) = [fw \wedge D(x,y,z)] \vee [fx \wedge D(w,y,z)] = [fw \wedge fx] \vee [fx \wedge fw \wedge D(w,y,z)] = fw \vee [fw \wedge D(w,y,z)] = fw$. □

THEOREM 3.3. *Let D be a permuting tri-f-derivation on L. Then for any $x,y,z,w \in L$, the following hold:*

(1) *If D is an isotone map on L, then $fx \wedge D(x \vee w,y,z) = D(x,y,z)$.*
(2) *If f is either a join preserving or an increasing function on L, then $fx \wedge D(x \vee w,y,z) \leqslant D(x,y,z)$.*

PROOF. Let $x,y,z,w \in L$.
(1) Suppose D is an isotone map on L. Then $D(x,y,z) \leqslant D(x \vee w,y,z)$. Now $D(x,y,z) = D((x \vee w) \wedge x,y,z) = [fx \wedge D(x \vee w,y,z)] \vee [f(x \vee w) \wedge D(x,y,z)] = [fx \wedge D(x \vee w,y,z)] \vee [f(x \vee w) \wedge D(x,y,z) \wedge D(x \vee w,y,z)] = [fx \vee [f(x \vee w) \wedge D(x,y,z)]] \wedge D(x \vee w,y,z) = [[fx \vee f(x \vee w)] \wedge fx] \wedge D(x \vee w,y,z) = fx \wedge D(x \vee w,y,z)$.
(2) **Case**(i): Suppose f is a join preserving map on L. Then
$D(x,y,z) = D((x \vee w) \wedge x,y,z) = [fx \wedge D(x \vee w,y,z)] \vee [f(x \vee w) \wedge D(x,y,z)] = [fx \wedge D(x \vee w,y,z)] \vee [(fx \vee fw) \wedge fx \wedge D(x,y,z)] = [fx \wedge D(x \vee w,y,z)] \vee D(x,y,z)$. Thus $fx \wedge D(x \vee w,y,z) \leqslant D(x,y,z)$.
Case(ii): Suppose f is an increasing function on L. Then $fx \leqslant f(x \vee w)$. Now $D(x,y,z) = D((x \vee w) \wedge x,y,z) = [fx \wedge D(x \vee w,y,z)] \vee [f(x \vee w) \wedge D(x,y,z)] = [fx \wedge D(x \vee w,y,z)] \vee [f(x \vee w) \wedge fx \wedge D(x,y,z)] = [fx \wedge D(x \vee w,y,z)] \vee [fx \wedge D(x,y,z)] = [fx \wedge D(x \vee w,y,z)] \vee D(x,y,z)$. Hence $fx \wedge D(x \vee w,y,z) \leqslant D(x,y,z)$. □

THEOREM 3.4. *Let D be a permuting tri-f-derivation on L and m be a maximal element in L. If f is a homomorphism on L, then the following are equivalent.*

(1) *D is an isotone map on L*
(2) *$D(x,y,z) = fx \wedge D(m,y,z)$ for all $x,y,z \in L$*
(3) *D is a join preserving map on L*
(4) *D is a meet preserving map on L.*

PROOF. Let f be a homomorphism on L and $x,y,z \in L$.
(1) $\Rightarrow$ (2) : $D(x,y,z) = D(m \wedge x,y,z) = [fx \wedge D(m,y,z)] \vee [fm \wedge D(x,y,z)]$. Thus $fx \wedge D(m,y,z) \leqslant D(x,y,z)$. On the other hand,
$fx \wedge D(x \wedge m,y,z) = fx \wedge [[fm \wedge D(x,y,z)] \vee [fx \wedge D(m,y,z)]] = [fx \wedge fm \wedge D(x,y,z)] \vee [fx \wedge D(m,y,z)] = [fm \wedge fx \wedge D(x,y,z)] \vee [fx \wedge D(m,y,z)] = [f(m \wedge x) \wedge D(x,y,z)] \vee [fx \wedge D(m,y,z)] = [fx \wedge D(x,y,z)] \vee [fx \wedge D(m,y,z)] = D(x,y,z) \vee [fx \wedge D(m,y,z)] = D(x,y,z)$. Since D is an isotone map on L, $D(x \wedge m,y,z) \leqslant D(m,y,z)$. Thus $D(x,y,z) = fx \wedge D(x \wedge m,y,z) \leqslant fx \wedge D(m,y,z)$. Hence

$D(x,y,z) = fx \wedge D(m,y,z)$.

(2) $\Rightarrow$ (3) : $D(x \vee w, y, z) = f(x \vee w) \wedge D(m,y,z) = (fx \vee fw) \wedge D(m,y,z) = (fx \wedge D(m,y,z)) \vee (fy \wedge D(m,y,z)) = D(x,y,z) \vee D(w,y,z)$. Thus D is a join preserving map on L.

(2) $\Rightarrow$ (4) : $D(x \wedge w, y, z) = f(x \wedge w) \wedge D(m,y,z) = fx \wedge fw \wedge D(m,y,z) = D(x,y,z) \wedge D(w,y,z)$. Thus D is a meet preserving map on L.

(3) $\Rightarrow$ (1) and (4) $\Rightarrow$ (1) are trivial. □

Theorem 3.5. *Let d be the trace of the permuting tri-f-derivation D on an associative ADL L. Then $d(x \wedge y) = (fy \wedge dx) \vee D(x,x,y) \vee D(x,y,y) \vee (fx \wedge dy)$ for all $x, y, z \in L$.*

Proof. Let $x, y, z \in L$. Then
$fy \wedge D(x, x \wedge y, x \wedge y) = fy \wedge [[fy \wedge D(x,x,x \wedge y)] \vee [fx \wedge D(x,y,x \wedge y)]] = [fy \wedge D(x,x,x \wedge y)] \vee D(x,y,x \wedge y) = [fy \wedge [[fy \wedge D(x,x,x)] \vee [fx \wedge D(x,x,y)]]] \vee [[fy \wedge D(x,y,x)] \vee [fx \wedge D(x,y,y)]] = (fy \wedge dx) \vee D(x,x,y) \vee D(x,y,x) \vee D(x,y,y) = (fy \wedge dx) \vee D(x,x,y) \vee D(x,y,y)$.

Again, $fx \wedge D(y, x \wedge y, x \wedge y) = fx \wedge [[fy \wedge D(y,x,x \wedge y)] \vee [fx \wedge D(y,y,x \wedge y)]] = D(y,x,x \wedge y) \vee [fx \wedge D(y,y,x \wedge y)] = [fy \wedge D(y,x,x)] \vee [fx \wedge D(y,x,y)] \vee [fx \wedge [[fy \wedge D(y,y,x)] \vee [fx \wedge D(y,y,y)]]] = D(y,x,x) \vee D(y,x,y) \vee D(y,y,x) \vee (fx \wedge dy) = D(y,x,x) \vee D(x,y,y) \vee (fx \wedge dy)$.

Thus $d(x \wedge y) = D(x \wedge y, x \wedge y, x \wedge y) = [fy \wedge D(x, x \wedge y, x \wedge y)] \vee [fx \wedge D(y, x \wedge y, x \wedge y)] = (fy \wedge dx) \vee D(x,x,y) \vee D(x,y,y) \vee (fx \wedge dy)$. □

Theorem 3.6. *Let d be the trace of the join preserving permuting tri-f-derivation D on an associative ADL L. If f is a join preserving map on L, then $F_d(L) = \{x \in L | dx = fx\}$ is a weak ideal in L.*

Proof. Suppose f is a join preserving map on L. Let $x \in L, y \in F_d(L)$ and $x \leqslant y$. Since f is a join preserving, f is an increasing function on L and hence $fx \leqslant fy$. Now, by Theorem 3.5,
$dx = d(y \wedge x) = (fx \wedge dy) \vee D(y,y,x) \vee D(y,x,x) \vee (fy \wedge dx) = fx \vee D(y,y,x) \vee D(y,x,x) \vee (fy \wedge dx) = fx \vee (fy \wedge dx) = fy \wedge fx = fx$. Thus $x \in F_d(L)$.

Let $x, y \in F_d(L)$. Then $d(x \vee y) = D(x \vee y, x \vee y, x \vee y) = D(x, x \vee y, x \vee y) \vee D(y, x \vee y, x \vee y) = D(x,x,x \vee y) \vee D(x,y,x \vee y) \vee D(y,x,x \vee y) \vee D(y,y,x \vee y) = dx \vee D(x,x,y) \vee D(x,y,x) \vee D(x,y,y) \vee D(y,x,x) \vee D(y,x,y) \vee D(y,y,x) \vee dy = dx \vee D(x,x,y) \vee D(x,y,y) \vee D(x,x,y) \vee D(x,y,y) \vee dy = fx \vee D(x,x,y) \vee D(x,y,y) \vee D(x,x,y) \vee D(x,y,y) \vee fy = fx \vee fy = f(x \vee y)$. Thus $x \vee y \in F_d(L)$. Hence $F_d(L)$ is a weak ideal in L. □

Lemma 3.2. *Let L be an associative ADL with 0 and D a join preserving permuting tri-f-derivation on L and d the trace of D. If $dx = 0$ for all $x \in L$, then $D = 0$.*

Proof. Suppose $dx = 0$ for all $x \in L$. Let $x, y, z \in L$. Then we have $d(x \vee y) = D(x \vee y, x \vee y, x \vee y) = dx \vee D(x,x,y) \vee D(x,y,y) \vee dy$. Thus $D(x,x,y) \vee D(x,y,y) = 0$.

Therefore $D(x,x,y) = 0$ for all $x,y \in L$. In particular, $D(x \vee z, x \vee z, y) = 0$ and hence $D(x,y,z) = 0$. Therefore $D = 0$. □

Let us recall the definition of a prime ADL in the following.

DEFINITION 3.4. [12] *An ADL L with 0 is said to be a prime ADL if, for* $a, b \in L$, $a \wedge b = 0$ *implies either* $a = 0$ *or* $b = 0$.

THEOREM 3.7. *Let L be an associative prime ADL and* d_1, d_2 *be the traces of join preserving permuting tri-*f_1*, tri-*f_2*-derivations* D_1, D_2 *on L, respectively. If* $d_1x \wedge d_2x = 0$ *for all* $x \in L$*, then either* $D_1 = 0$ *or* $D_2 = 0$.

PROOF. Suppose $d_1x \wedge d_2x = 0$ for all $x \in L$. Assume that $d_1 \neq 0$ and $d_2 \neq 0$. Then $d_1y \neq 0$ and $d_2z \neq 0$ for some $y, z \in L$. Now, $d_1(y \vee z) = D_1(y \vee z, y \vee z, y \vee z) = d_1y \vee D_1(y,y,z) \vee D_1(y,z,z) \vee d_1z \neq 0$ and $d_2(y \vee z) = D_2(y \vee z, y \vee z, y \vee z) = d_2y \vee D_2(y,y,z) \vee D_2(y,z,z) \vee d_2z \neq 0$. But, by our assumption $d_1(y \vee z) \wedge d_2(y \vee z) = 0$. This is a contradiction, (since L is a prime ADL). Thus $d_1 = 0$ or $d_2 = 0$ and hence by Lemma 3.2, either $D_1 = 0$ or $D_2 = 0$. □

Finally we conclude this paper with the following theorem.

THEOREM 3.8. *Let L be an associative prime ADL and* d_1, d_2 *be the traces of join preserving permuting tri-*f_1*, tri-*f_2*-derivations* D_1, D_2 *on L, respectively such that* $d_1of_2 = d_1$ *and* $f_1od_2 = d_2$. *Suppose one of the following condition hold*

(1) $D_1(d_2x, f_2x, f_2x) = 0$ *for all* $x \in L$
(2) $D_1(d_2x, d_2x, f_2x) = 0$ *for all* $x \in L$
(3) $d_1od_2 = 0$, *then either* $D_1 = 0$ *or* $D_2 = 0$.

PROOF. (1) Suppose $D_1(d_2x, f_2x, f_2x) = 0$ for all $x \in L$. Let $x \in L$. Since $f_2x \wedge d_2x = d_2x$, we get that
$[f_1(d_2x) \wedge D_1(f_2x, f_2x, , f_2x,)] \vee [f_1(f_2x) \wedge D_1(d_2x, f_2x, f_2x)] = D_1(f_2x \wedge d_2x, f_2x, f_2x) = 0$. Thus $(f_1od_2)x \wedge (d_1of_2)x = 0$. Therefore $d_2x \wedge d_1x = 0$.

(3) Suppose $D_1(d_2x, d_2x, f_2x) = 0$ for all $x \in L$. Let $x \in L$. Again since $f_2x \wedge d_2x = d_2x$, we get that $[f_1(d_2x) \wedge D_1(f_2x, d_2x, f_2x)] \vee [f_1(f_2x) \wedge D_1(d_2x, d_2x, f_2x)] = D_1(f_2x \wedge d_2x, d_2x, f_2x) = 0$. Thus $(f_1od_2)x \wedge D_1(f_2x, d_2x, f_2x) = 0$. Therefore $d_2x \wedge D_1(f_2x, d_2x, f_2x) = 0$. Thus $[d_2x \wedge f_1(d_2x) \wedge (d_1of_2)x] \vee [d_2x \wedge f_1(f_2x) \wedge D_1(f_2x, d_2x, f_2x)] = d_2x \wedge D_1(f_2x, f_2x \wedge d_2x, f_2x) = 0$. Therefore $d_2x \wedge (f_1od_2)x \wedge (d_1of_2)x = 0$ and hence $d_2x \wedge d_1x = 0$.

(2) Suppose $d_1od_2 = 0$. Then $d_1(d_2x) = 0$ for all $x \in L$. So that, $D_1(d_2x, d_2x, d_2x) = 0$ for all $x \in L$. Let $x \in L$. Again since $f_2x \wedge d_2x = d_2x$, we get that $[f_1(d_2x) \wedge D_1(d_2x, d_2x, f_2x)] \vee [f_1(f_2x) \wedge D_1(d_2x, d_2x, d_2x)] = D_1(d_2x, d_2x, f_2x \wedge d_2x) = 0$. Therefore $d_2x \wedge D_1(d_2x, d_2x, f_2x) = 0$. Thus $[d_2x \wedge f_1(d_2x) \wedge D_1(d_2x, f_2x, f_2x)] \vee [d_2x \wedge f_1(f_2x) \wedge D_1(d_2x, d_2x, f_2x)] = d_2x \wedge D_1(d_2x, f_2x \wedge d_2x, f_2x) = 0$. Hence $d_2x \wedge D_1(d_2x, f_2x, f_2x) = 0$. So that $[d_2x \wedge f_1(d_2x) \wedge (d_1of_2)x] \vee [d_2x \wedge f_1(f_2x) \wedge D_1(f_2x, d_2x, f_2x)] = d_2x \wedge D_1(f_2x, f_2x \wedge d_2x, f_2x) = 0$ and hence $d_2x \wedge d_1x = 0$. Therefore, $d_2x \wedge d_1x = 0$ for all $x \in L$ in all three cases. By Theorem 3.7, we get that either $D_1 = 0$ or $D_2 = 0$. □

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