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COMMON FIXED POINT THEOREM IN DISLOCATED GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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Abstract. In this paper we define dislocated generalized intuitiionistic fuzzy metric space and prove common fixed point theorems for weakly compatible maps in dislocated generalized intuitionistic fuzzy metric spaces.

1. Introduction

Hitzler and Seda [5] introduced the notion of dislocated metric space in which self distance of a point need not be equal to zero in 2000. They generalized the Banach contraction principle and studied common fixed points for maps satisfying certain contractive conditions. Also, Panthi [17] studied common fixed point theorem in this space. The notion of fuzzy sets was introduced by Zadeh [23] in 1965. The fuzzy metric space with the concept of fuzzy sets was introduced Kramosil and Michalek [10] , Kaleva and Seikkala [7] . Since then a number of fixed point theorems proved by different authors and many generalizations of this theorem have been established.

Recently, Park et al. ([18], [19]) introduced the Intuitionistic fuzzy metric spaces, and studied some results using weakly compatible maps in intuitionistic fuzzy metric spaces. Also, Park [19] proved common fixed point using type (α) compatible maps in IFMS.

In this paper, we define the dislocated generalized intuitionistic fuzzy metric space and prove a common fixed point for weakly compatible maps in dislocated generalized intuitionistic fuzzy metric space.

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Key words and phrases. Weakly compatible map, Intuitionistic fuzzy metric space, Dislocated generalized intuitionistic fuzzy metric spaces.

2. Preliminaries

Definition 2.1. *A 5 tuple $(X, M, N, *, \diamond)$ is called a generalized intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^3 \times [0, \infty) \rightarrow [0, 1]$ satisfying the following conditions for every $x, y, z, a \in X$ and $t, s > 0$:*

(1) $M(x, y, z, 0) = 0$,

(2) $M(x, x, x, t) = 1$,

(3) $M(x, y, z, t) = 1$ *if and only if* $x = y = z$,

(4) $M(x, y, z, t) = M(p\{x, y, z\}, t)$, *where p is a permutation function,*

(5) $M(x, y, a, t) * M(a, z, z, s) \leqslant M(x, y, z, t + s)$,

(6) $M(x, y, z, t) : (0, \infty) \rightarrow [0, 1]$ *is continuous,*

(7) $N(x, y, z, 0) = 1$,

(8) $N(x, x, x, t) = 0$,

(9) $N(x, y, z, t) = 0$ *if and only if* $x = y = z$,

(10) $N(x, y, z, t) = N(p\{x, y, z\}, t)$, *where p is a permutation function,*

(11) $N(x, y, a, t) \diamond N(a, z, z, s) \geqslant N(x, y, z, t + s)$,

(12) $N(x, y, z, t) : (0, \infty) \rightarrow [0, 1]$ *is continuous.*

If all conditions satisfy, then X is called an generalized intuitionistic fuzzy metric space.

If (1), (3) to (6), (7), (9) to (11) satisfy, then X is said to be a dislocated generalized intuitionistic fuzzy metric space in which self distance of a point need not be equal to zero in the sense of George and Veeramani [4].

Note that (M, N) is called an dislocated generalized intuitionistic fuzzy metric space on X. The functions $M(x, y, z, t)$ and $N(x, y, z, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 2.2. *Let $\{x_n\}$ be a sequence of dislocated generalized intuitionistic fuzzy metric space in X.*

(1) *$\{x_n\}$ is converges to a point $x \in X$ if*
$\lim_{n \to \infty} M(x, x, x_n, t) = 1$ *and* $\lim_{n \to \infty} N(x, x, x_n, t) = 0$;

(2) *A sequence $\{x_n\}$ is called a Cauchy sequence in X if for given $\epsilon > 0$, there exists $n_0 \in N$ such that for all $m, n \geqslant n_0$. We have*
$M(x_n, x_n, x_m, t) > 1 - \epsilon$ *and* $N(x_n, x_n, x_m, t) > \epsilon$;

(3) *X is complete if every Cauchy sequence is converges in X.*

Definition 2.3. *Let (A, B) be a pair of self-maps of dislocated generalized intuitionistic fuzzy metric space in X. Then (A, B) is said to be weakly compatible, if for $x \in X$, $Ax = Bx$ implies that $ABx = BAx$.*

Lemma 2.1. *Let $(X, M, N, *, \diamond)$ be a dislocated generalized intuitionistic fuzzy metric space and*

$$M(x,y,z,kt) \geqslant M(x,y,z,t) \text{ and } N(x,y,z,kt) \leqslant N(x,y,z,t)$$

for all $x,y,z \in X$, $t > 0$, and for a number $k \in (0,1)$. Then $x = y = z$.

For more details on dislocated generalized intuitionistic fuzzy metric space, one can read [3], [5], [8-9], [11-17], [20].

LEMMA 2.2. *Let $(X, M, N, *, \diamond)$ be a generalized intuitionistic fuzzy metric spaces. Then for any $t > 0$ and for every $x, y \in X$ we have*

$$M(x,x,y,t) = M(x,y,y,t) \text{ and } N(x,x,y,t) = N(x,y,y,t).$$

3. Main Results

THEOREM 3.1. *Let X be a complete dislocated generalized intuitionistic fuzzy metric space with t-norm $*$, t-conorm $\diamond$, defined by*

$$\alpha * \beta = min\{\alpha,\ \beta\},\ \ \alpha \diamond \beta = max\{\alpha,\ \beta\}.$$

Also, let A, B, S and T be four self continuous maps on X satisfying following conditions:

(1) $T(X) \subset A(X)$, $S(X) \subset B(X)$;

(2) (S, A) and (T, B) are weakly compatible;

(3) there exists $k \in (0,1)$ such that

$$M(Sx,Ty,Ty,kt) \geqslant$$
$$min\{M(Ax,Ty,Ty,2t), M(By,Sx,Sx,t), M(Ax,By,By,t)\},$$
$$N(Sx,Ty,Ty,kt) \leqslant max\{N(Ax,Ty,Ty,2t), N(By,Sx,Sx,t), N(Ax,By,By,t\}$$

for all $x, y \in X$, $t > 0$. Then A, B, S and T have a unique common fixed point in dislocated generalized intuitionistic fuzzy metric space in X.

PROOF. Let x_0 be an arbitrary point of dislocated generalized intuitionistic fuzzy metric space in X. We can inductively construct sequence $\{x_n\} \cdot \{y_n\} \subset X$ such that $y_{2n} = Bx_{2n+1} = Sx_{2n}, y_{2n+1} = Ax_{2n+2} = Tx_{2n+1}$ $(n = 0, 1, 2, ...)$.

First, we prove that $\{x_n\}$ is Cauchy sequence. If $y_{2n} = y_{2n+1}$ for some $n \in \mathbf{N}$, then $Bx_{2n+1} = Tx_{2n+1}$. Therefore x_{2n+1} is coincidence point of B and T. Also, if $y_{2n+1} = y_{2n+2}$ for some $n \in \mathbf{N}$, then $Ax_{2n+2} = x_{2n+2}$. Hence x_{2n+2} is a coincidence point of A and S. Assume that $y_{2n} \neq y_{2n+1}$. Then, from (3) we have

$$\begin{aligned}
M(y_{2n}, y_{2n+1}, y_{2n+1}, kt) &= M(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}, kt) \\
&\geqslant min\{M(Ax_{2n}, Tx_{2n+1}, Tx_{2n+1}, 2t), M(Bx_{2n+1}, Sx_{2n}, Sx_{2n}, t), \\
&\qquad M(Ax_{2n}, Bx_{2n+1}, Bx_{2n+1}, t)\} \\
&\geqslant min\{M(y_{2n-1}, y_{2n+1}, y_{2n+1}, t), M(y_{2n}, y_{2n}, y_{2n}, t), M(y_{2n-1}, y_{2n}, y_{2n}, t)\} \\
&\geqslant min\{M(y_{2n-1}, y_{2n}, y_{2n}, t) * M(y_{2n}, y_{2n+1}, y_{2n+1}, t), 1, M(y_{2n-1}, y_{2n}, y_{2n}, t)\}
\end{aligned}$$

which implies $M(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \geqslant M(y_{2n-1}, y_{2n}, y_{2n}, t)$ and

$$\begin{aligned} N(y_{2n}, y_{2n+1}, y_{2n+1}, kt) &= N(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}, kt) \\ &\leqslant max\{N(Ax_{2n}, Tx_{2n+1}, Tx_{2n+1}, 2t), N(Bx_{2n+1}, Sx_{2n}, Sx_{2n}, t), \\ &\qquad N(Ax_{2n}, Bx_{2n+1}, Bx_{2n+1}, t)\} \\ &\leqslant max\{N(y_{2n-1}, y_{2n+1}, y_{2n+1}, t), N(y_{2n}, y_{2n}, y_{2n}, t), N(y_{2n-1}, y_{2n}, y_{2n}, t)\} \\ &\leqslant max\{N(y_{2n-1}, y_{2n}, y_{2n}, t) \diamond N(y_{2n}, y_{2n+1}, y_{2n+1}, t), 0, N(y_{2n-1}, y_{2n}, y_{2n}, t)\} \end{aligned}$$

and

$$N(y_{2n}, y_{2n+1}, y_{2n+1}, kt) \leqslant N(y_{2n-1}, y_{2n}, y_{2n}, t),$$

and in general case

$$M(y_n, y_{n+1}, y_{n+1}, kt) \geqslant M(y_{n-1}, y_n, y_n, t),$$

and

$$N(y_n, y_{n+1}, y_{n+1}, kt) \leqslant N(y_{n-1}, y_n, y_n, t).$$

Therefore, as $n \to \infty$

$$M(y_n, y_{n+1}, y_{n+1}, t) \geqslant M(y_{n-1}, y_n, y_n, \frac{t}{k}) \geqslant ... \geqslant M(y_0, y_1, y_1, \frac{t}{k^n}) \to 1,$$

$$N(y_n, y_{n+1}, y_{n+1}, t) \leqslant N(y_{n-1}, y_n, y_n, \frac{t}{k}) \leqslant ... \leqslant N(y_0, y_1, y_1, \frac{t}{k^n}) \to 0,$$

Hence, for $t > 0$ and $\epsilon \in (0, 1)$, we can choose $n_0 \in N$ such that for all $n \geqslant n_0$, we have

$$M(y_n, y_{n+1}, y_{n+1}, t) > 1 - \epsilon$$

and

$$N(y_n, y_{n+1}, y_{n+1}, t) < \epsilon.$$

Suppose that for $m \in \mathbf{N}$ and $n \geqslant n_0$,

$$M(y_n, y_{n+m}, y_{n+m}, t) > 1 - \epsilon, N(y_n, y_{n+m}, y_{n+m}, t) < \epsilon.$$

Then

$$M(y_n, y_{n+m+1}, y_{n+m+1}, t) \geqslant$$
$$min\{M(y_n, y_{n+m}, y_{n+m}, \frac{t}{2}), M(y_{n+m}, y_{n+m+1}, \frac{t}{2})\} > 1 - \epsilon$$

and

$$[N(y_n, y_{n+m+1}, y_{n+m+1}, t) \leqslant$$
$$max\{N(y_n, y_{n+m}, y_{n+m}, \frac{t}{2}), N(y_{n+m}, y_{n+m+1}, \frac{t}{2})\} < \epsilon.$$

Hence $\{y_n\} \subset X$ is a Cauchy sequence in a complete dislocated generalized intuitionistic fuzzy metric space. So, there exists a point $z \in X$ such that $y_n \to z$.

Therefore, the subsequences $x_{2n} \to z$, $Bx_{2n+1} \to z$, $Tx_{2n+1} \to z$, and $Sx_{2n+2} \to z$. Since $T(X) \subset A(X)$, there exists a point $u \in X$ such that $Au = z$. So,

$$\begin{aligned} M(Su, z, z, kt) &= M(Su, Tx_{2n+1}, Tx_{2n+1}, kt) \\ &\geqslant min\{M(Au, Tx_{2n+1}, Tx_{2n+1}, 2t), M(Bx_{2n+1}, Su, Su, t), M(Au, Bx_{2n+1}, Bx_{2n+1}, t)\} \\ &= min\{M(z, Tx_{2n+1}, Tx_{2n+1}, 2t), M(Bx_{2n+1}, Su, Su, t), M(z, Bx_{2n+1}, Bx_{2n+1}, t)\} \end{aligned}$$

and

$$\begin{aligned} N(Su, z, z, kt) &= N(Su, Tx_{2n+1}, Tx_{2n+1}, kt) \\ &\leqslant max\{N(Au, Tx_{2n+1}, Tx_{2n+1}, 2t), N(Bx_{2n+1}, Su, Su, t), N(Au, Bx_{2n+1}, Bx_{2n+1}, t)\} \\ &= max\{N(z, Tx_{2n+1}, Tx_{2n+1}, 2t), N(Bx_{2n+1}, Su, Su, t), N(z, Bx_{2n+1}, Bx_{2n+1}, t)\} \end{aligned}$$

Taking limit as $n \to \infty$, we get

$$M(Su, z, z, kt) \geqslant M(Su, z, z, t), \text{ and } N(Su, z, z, kt) \leqslant N(Su, z, z, t).$$

Thus $Su = z$, that is $Su = Au = z$.

Again, since $S(X) \subset B(X)$, there exists a point $v \in X$ such that $z = Bv$. If $z \neq Tv$, then

$$\begin{aligned} M(z, Tv, Tv, kt) &= M(Su, Tv, Tv, kt) \\ &\geqslant min\{M(Au, Tv, Tv, 2t), M(Bv, Su, Su, t), M(Au, Bv, Bv, t)\} \\ &= min\{M(z, Tv, Tv, 2t), M(z, z, z, t), M(z, z, z, t)\} = M(z, Tv, Tv, t) \end{aligned}$$

and

$$\begin{aligned} N(z, Tv, Tv, kt) &= N(Su, Tv, Tv, kt) \\ &\leqslant max\{N(Au, Tv, Tv, 2t), N(Bv, Su, Su, t), N(Au, Bv, Bv, t)\} \\ &= max\{N(z, Tv, Tv, 2t), N(z, z, z, t), N(z, z, z, t)\} = N(z, Tv, Tv, t) \end{aligned}$$

which is a contradiction. So. we get $Tv = Bv = z$. Hence $Su = Au = Tv = Bv = z$. Since (S, A) is weakly compatible, then $SAu = ASu$ implies $Sz = Az$.

Second, we prove that z is the fixed point of S. If $z \neq z$, then

$$\begin{aligned} M(Sz, z, z, kt) &= M(Sz, Tv, Tv, kt) \\ &\geqslant min\{M(Az, Tv, Tv, 2t), M(Bv, Sz, Sz, t), M(Az, Bv, Bv, t)\} \\ &= min\{M(Sz, z, z, 2t), M(z, Sz, Sz, t), M(Sz, z, z, t)\} \\ &= M(Sz, z, z, t) \end{aligned}$$

and

$$\begin{aligned} N(Sz, z, z, kt) &= N(Sz, Tv, Tv, kt) \\ &\leqslant max\{N(Az, Tv, Tv, 2t), N(Bv, Sz, Sz, t), N(Az, Bv, Bv, t)\} \\ &= max\{N(Sz, z, z, 2t), N(z, Sz, Sz, t), N(Sz, z, z, t)\} \\ &= N(Sz, z, z, t) \end{aligned}$$

which is a contraction. So we have $Sz = z$. Hence, $Az = Sz = z$. Since (T, B) is weakly compatible, then $TBv = BTv$ implies $Tz = Bz$. Also, we prove that z is the fixed point of T. If $z \neq Tz$, then

$$\begin{aligned} M(z,Tz,Tz,kt) &= M(Sz,Tz,Tz,kt) \\ &\geqslant min\{M(Az,Tz,Tz,2t),M(Bz,Sz,Sz,t),M(Az,Sz,Sz,t)\} \\ &= min\{M(z,Tz,Tz,2t),M(Tz,z,z,t),M(z,z,z,t)\} \\ &= M(z,Tz,Tz,t) \end{aligned}$$

and

$$\begin{aligned} N(z,Tz,Tz,kt) &= N(Sz,Tz,Tz,kt) \\ &\leqslant max\{N(Az,Tz,Tz,2t),N(Bv,Sz,Sz,t),N(Az,Sz,Sz,t)\} \\ &= max\{N(z,Tz,Tz,2t),N(Tz,z,z,t),N(z,z,z,t\} \\ &= N(z,Tz,Tz,t) \end{aligned}$$

which is a a contradiction. So we have $z = Tz$. Hence $z = Tz = Bz = Az = Sz$. We know that z is the common fixed point of the self maps A, B, S and T.

Finally, we show that z is a unique common fixed point. Let $z, w(z \neq w)$ be two common fixed point of the self maps A, B, S and T. Then

$$\begin{aligned} M(z,w,w,kt) = M(Sz,Tw,Tw,kt) \\ \geqslant min\{M(Az,Tw,Tw,2t),M(Bw,Sz,Sz,t),M(Az,Bw,Bw,t)\} \\ = min\{M(z,w,w,2t),M(w,z,z,t),M(z,w,w,t)\} = M(z,w,w,t) \end{aligned}$$

and

$$\begin{aligned} N(z,w,w,kt) = N(Sz,Tw,Tw,kt) \\ \leqslant max\{N(Az,Tw,Tw,2t),N(Bw,Sz,Sz,t),N(Az,Bw,Bw,t)\} \\ = max\{N(z,w,w,2t),N(w,z,z,t),N(z,w,w,t)\} = N(z,w,w,t) \end{aligned}$$

which is a contradiction. So, we have $z = w$. Hence z is a unique common fixed point of the self maps A, B, S and T. □

Example. Let

$$X = [0,1],\ M(x,y,z,t) = \frac{t}{t + D(x,y,z)},\ N(x,y,z,t) = \frac{D(x,y,z)}{t + D(x,y,z)}.$$

Let A, B, S and T be four self maps on X defined by $Ax = x$, $Bx = x$, $Tx = \dfrac{x}{5}$ and $Sx = 0$ for all x in X. Clearly the maps A, B, S and T satisfies all conditions of Theorem 3.1 and $x = 0$ is the unique common fixed point of A, B, S and T.

Corollary 3.1. *Let X be a complete dislocated generalized intuitionistic fuzzy metric space and S, T be two self continuous maps on X satisfying for all $x, y \in X$, $t > 0$*

$$M(Sx,Ty,Ty,t) \geqslant min\{M(x,Ty,Ty,2t),M(y,Sx,Sx,t),M(x,y,y,t)\},$$

$$N(Sx,Ty,Ty,t) \leqslant max\{N(x,Ty,Ty,2t),N(y,Sx,Sx,t),N(x,y,y,t)\}.$$

Then S and T have a unique common fixed point in dislocated generalized intuitionistic fuzzy metric space in X.

Proof. From Theorem 3.1, we obtain the result of Corollary 3.2 as $A = B = I$ (identity map). □

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