BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Vol. 8(2018), 1-9
DOI: 10.7251/BIMVI1801001P

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

4-PRIME CORDIAL GRAPHS OBTAINED FROM 4-PRIME CORDIAL GRAPHS

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Abstract. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a function. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leqslant 1$, $i, j \in \{1, 2, \ldots, k\}$ and $|e_f(0) - e_f(1)| \leqslant 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with admits a k-prime cordial labeling is called a k-prime cordial graph. In this paper we generate some new 4-prime cordial graphs derived from 4-prime cordial graphs.

1. Introduction

Graphs considered here are finite, simple and undirected only. Let G be a (p, q) graph where p refers the number of vertices of G and q refers the number of edges of G. The number of vertices of a graph G is called order of G, and the number of edges is called size of G. The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The join of two graphs $G_1 + G_2$ is obtained from G_1 and G_2 and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$. Sundaram, Ponraj, Somasundaram have introduced the notion of prime cordial labeling [**10**] and product cordial labeling [**11**]. Several authors were studied the behavior of prime cordial labeling of graphs [**1**]. Also Prajapati *et al.* have studied the edge product cordial labeling of some cycle related graphs [**7, 8**]. In this sequel Ponraj *et al.* [**3**] have introduced k-prime cordial labeling of graphs and studied some k-prime cordial and 3-prime cordial graphs. A 2-prime cordial labeling is a product cordial labeling. In [**4, 5, 6**] Ponraj *et al.* have

2010 *Mathematics Subject Classification.* 05C78.
Key words and phrases. Path,union of graphs, bipartite graph,complete graph.

studied the 4-prime cordial labeling behavior of complete graph, book, flower, mC_n, wheel, gear, double cone, helm, closed helm, butterfly graph, and friendship graph, union of two bipartite graphs, union of trees, durer graph, tietze graph, planar grid $P_m \times P_n$, subdivision of wheels and subdivision of helms. In this paper we have obtained some 4-prime cordial graphs from 4-prime cordial graphs. Terms not defined here follow from Harary [**2**].

2. Main results

THEOREM 2.1. *If G is a 4-prime cordial graph then $G \cup P_n$ is also a 4-prime cordial graph for $n \geqslant 5$.*

PROOF. Let G be a (p, q) 4-prime cordial graph with a 4-prime cordial labeling f. Let P_n be the path $v_1 v_2 \dots v_n$ and $V(G) = \{u_i : 1 \leqslant i \leqslant p\}$. The proof is divided into four cases.

Case 1. $p \equiv 0 \pmod 4$.

Let $p = 4t$. This forces, $v_f(1) = v_f(2) = v_f(3) = v_f(4) = t$. Now assign the label to the vertices of G using the labeling of f. Next assign the labels $2, 4$ alternatively to the vertices $u_1, u_2, \dots$, until we reach the vertex $u_{\lceil \frac{n}{2} \rceil}$. Next assign the labels $1, 3$ to the remaining vertices $u_{\lceil \frac{n}{2} \rceil + 1}, u_{\lceil \frac{n}{2} \rceil + 2}, \dots, u_n$ alternatively. We denote this vertex labeling by g.

Subcase 1. q is even.

Clearly the above vertex labeling g is a 4-prime cordial labeling of $G \cup P_n$.

Subcase 2. q is odd.

This case the following possible arises: $e_f(0) = e_f(1) + 1$ or $e_f(1) = e_f(0) + 1$. In the former cases, the above vertex labeling g is a 4-prime cordial labeling.

In the latter case, interchange the labels of u_{n-1} and u_n or u_{n-2} and u_{n-1} according as u_n received the label 3 or 1. Clearly the resulting vertex labeling is a 4-prime cordial labeling of $G \cup P_n$.

Case 2. $p \equiv 1 \pmod 4$.

Let $p = 4t + 1$. Any one of the types given below are arises.

TYPE A: $v_f(1) = t + 1$, $v_f(2) = v_f(3) = v_f(4) = t$.

TYPE B: $v_f(2) = t + 1$, $v_f(1) = v_f(3) = v_f(4) = t$.

TYPE C: $v_f(3) = t + 1$, $v_f(1) = v_f(2) = v_f(4) = t$.

TYPE D: $v_f(4) = t + 1$, $v_f(1) = v_f(2) = v_f(3) = t$.

We now discuss the types one by one.

TYPE A:

Now consider the case $|v_g(1) - v_g(3)| = 2$, then choose the largest r such that $g(v_r) = 1$, and relabel the vertex v_r with 3.

Subcase 1. q is even or q is odd with $e_f(1) = e_f(0) + 1$.

Clearly the above resulting vertex labeling is a 4-prime cordial labeling of $G \cup P_n$.

Subcase 2. q is odd with $e_f(0) = e_f(1) + 1$.

In this case interchange the labels of v_1 and v_n, we get a 4-prime cordial labeling of $G \cup P_n$.

We now consider th case $|v_g(1) - v_g(4)| = 2$. In this case choose the largest r in such a way that $g(v_r) = 1$ and relabel the vertex v_r with 4. That is either v_n or v_{n-1} received the label 4. In the case of $e_f(0) = e_f(1) + 1$, clearly this resulting labeling is a 4-prime cordial labeling of $G \cup P_n$. For the case $e_f(1) = e_f(0) + 1$, choose the smallest integer s such that $g(v_s) = 1$ and interchange the labels of v_s and v_n or v_n and v_{n-1} according as v_n or v_{n-1} received the label 4. Obviously this labeling is a 4-prime cordial labeling of $G \cup P_n$. Clearly the case $|v_g(1) - v_g(2)| = 2$ is not arises.

TYPE B:

First consider the case $|v_g(2) - v_g(4)| = 2$. In this case, relabel the vertex v_1 with 4. If $e_f(0) = e_f(1) + 1$, clearly this resulting vertex labeling is a 4-prime cordial labeling also; otherwise, choose the smallest integers r such that $g(v_r) = 3$ and interchange the labels of v_r and v_n or v_{n-1} and v_n according as v_n received the label 1 or 3.

Consider the next case, $|v_g(2) - v_g(3)| = 2$. Interchange the labels of v_n and v_{n-1} or v_n and v_{n-1} where r is the smallest integer such that $g(v_r) = 1$, according as v_n received the label 3 or 1. After interchanging, relabel the vertex v_1 with 3.

When $e_f(0) = e_f(1) + 1$, clearly the resulting labeling is a 4-prime cordial labeling of $G \cup P_n$ also. For the case $e_f(1) = e_f(0) + 1$, interchange the labels of v_n and v_1 or v_{n-1} and v_1 according as v_n or v_{n-1} received the label 3. Obviously this resulting labeling is a 4-prime cordial labeling of $G \cup P_n$.

TYPE C and TYPE D:

In this type, clearly the labeling g satisfies the vertex condition. If $e_f(0) = e_f(1)+1$, then this labeling g is also a 4-prime cordial labeling of $G \cup P_n$; otherwise, interchange the labeling of v_r and v_{r+1} where r is the least positive integer such that $g(v_r) = 3$.

Case 3. $p \equiv 2 \pmod 4$.

Let $p = 4t + 2$. In this case any one of the following case arises.

TYPE A: $v_f(1) = v_f(2) = t + 1$, $v_f(3) = v_f(4) = t$.

TYPE B: $v_f(1) = v_f(3) = t + 1$, $v_f(2) = v_f(4) = t$.

TYPE C: $v_f(1) = v_f(4) = t + 1$, $v_f(2) = v_f(3) = t$.

TYPE D: $v_f(2) = v_f(3) = t + 1$, $v_f(1) = v_f(4) = t$.

TYPE E: $v_f(2) = v_f(4) = t + 1$, $v_f(1) = v_f(3) = t$.

TYPE F: $v_f(3) = v_f(4) = t + 1$, $v_f(1) = v_f(2) = t$.

We now discuss all these types one by one.

TYPE A:

Choose the least positive integer r such that $g(v_r) = 1$. Relabel the vertices v_1 and v_r with 4 and 3 respectively. Clearly if $e_f(1) = e_f(0) + 1$, then the resulting labeling is a 4-prime cordial labeling of $G \cup P_n$. For the case $e_f(0) = e_f(1) + 1$, interchange the labels of v_1 and v_n.

TYPE B:

Let r be the least integer such that $g(v_r) = 1$. Relabel v_r with 4. When $e_f(1) = e_f(0) + 1$, the resulting labeling is a 4-prime cordial labeling of $G \cup P_n$. In the case $e_f(0) = e_f(1) + 1$, interchange the labels of v_1 and v_n.

TYPE C and TYPE D:

In this type relabel the vertex v_r with 3, where r is the least integer such that $g(v_r) = 1$. Next proceed as in TYPE B we get a 4-prime cordial labeling of $G \cup P_n$.

TYPE E:

In this type relabel the vertex v_1 with 3. Next interchange the labels of v_1 and v_n or v_1 and v_{n-1} according as v_n received the label 1 or 3. Clearly this resulting labeling is a 4-prime cordial labeling of $G \cup P_n$ for the case, $e_f(0) = e_f(1) + 1$. For the case $e_f(1) = e_f(0) + 1$, interchange the labels of v_{n-2} and v_{n-3}, such vertices are exists as $n \geqslant 5$.

TYPE F:

Clearly the labeling g satisfies the vertex condition. For the case $e_f(0) = e_f(1)+1$, this labeling g is a 4-prime cordial labeling. In the case $e_f(1) = e_f(0)+1$, interchange the labels of v_{n-1} and v_n or v_{n-2} and v_{n-1} according as v_n received the label 3 or 1.

Case 4. $p \equiv 3 \pmod 4$.

This case any one of the following cases happen. Let $p = 4t + 3$.

TYPE A: $v_f(4) = t$, $v_f(1) = v_f(2) = v_f(3) = t + 1$.

TYPE B: $v_f(3) = t$, $v_f(1) = v_f(2) = v_f(4) = t + 1$.

TYPE C: $v_f(2) = t$, $v_f(1) = v_f(3) = v_f(4) = t + 1$.

TYPE D: $v_f(1) = t$, $v_f(2) = v_f(3) = v_f(4) = t + 1$.

Now we discuss one by one.

TYPE A:

Relabel the vertex v_1 with 4. In the case $e_f(0) = e_f(1)+1$, clearly the resulting labeling is a 4-prime cordial labeling of $G \cup P_n$. For the case $e_f(1) = e_f(0) + 1$, let v_r be the smallest positive integer such that $g(v_r) = 3$. Then interchange the labels of v_r and v_{r+1}.

TYPE B:

Relabel the vertex v_n or v_{n-1} with 3 according as v_n received the label 1 or 3. Next proceed as in TYPE A.

TYPE C and TYPE D:

Clearly the labeling g satisfies the vertex condition of 4-prime cordial labeling. For the edge condition, proceed as in TYPE A.

□

Remark 2.1. *When $n = 1$, using the 4-prime cordial labeling f of G assign the labels to the vertices of G in $G \cup P_1$. As $|E(G \cup P_1)| = |E(G)|$, assign any label to the vertex of P_1 with the 4-prime cordial vertex condition we get a 4-prime cordial labeling of $G \cup P_1$.*

THEOREM 2.2. *Let G be a (p,q) 4-prime cordial graph. Then $G \cup mK_{n,n}$ is 4-prime cordial for all even values of m.*

PROOF. Clearly $G \cup mK_{n,n}$ has $p+2mn$ vertices and $q+mn^2$ edges. Let $K^i_{n,n}$ be the i^{th} copy of $K_{n,n}$. Let $V(K^i_{n,n}) = V^i_1 \cup V^i_2$ where $V^i_1 = \{v^i_j : 1 \leqslant j \leqslant n\}$ and $V^i_2 = \{w^i_j : 1 \leqslant j \leqslant n\}$. Let f be a 4-prime cordial labelin of G. In $G \cup mK_{n,n}$, assign the label to the vertices $u_i\,(1 \leqslant i \leqslant n)$ using the 4-prime cordial labeling f. We now move to the bipartite graphs. First consider the $\frac{m}{2}$ copies of $K_{n,n}$. Assign the label 1 to all the vertices of the set $V^1_1, V^2_1, \ldots, V^{\frac{m}{2}}_1$ and 3 to all the vertices of the set $V^1_2, V^2_2, \ldots, V^{\frac{m}{2}}_2$. Next we now move to the another $\frac{m}{2}$ copies of $K_{n,n}$. Assign the label 2 to all vertices of the sets $V^{\frac{m}{2}+1}_1, V^{\frac{m}{2}+2}_1, \ldots, V^m_1$ and 4 to the all the vertices of the sets $V^{\frac{m}{2}+1}_2, V^{\frac{m}{2}+2}_2, \ldots, V^m_2$. We now prove this labeling g is a 4-prime cordial labeling of $G \cup mK_{n,n}$. $e_g(0) = e_f(0) + \frac{m}{2}n^2$ and $e_g(1) = e_f(1) + \frac{m}{2}n^2$. Also $v_g(1) = v_f(1) + \frac{m}{2}n$, $v_g(2) = v_f(2) + \frac{m}{2}n$, $v_g(3) = v_f(3) + \frac{m}{2}n$ and $v_g(4) = v_f(4) + \frac{m}{2}n$. Therefore

$$\begin{aligned} |e_g(0) - e_g(1)| &= |e_f(0) - e_f(1)| \\ &\leqslant 1 \end{aligned}$$

as f is a 4-prime cordial labeling of G. Also

$$\begin{aligned} |v_g(r) - v_g(s)| &= |v_f(r) - v_f(s)| \\ &\leqslant 1 \end{aligned}$$

for all $r, s \in \{1,2,3,4\}$ since f is a 4-prime cordial labeling. □

THEOREM 2.3. *If G is a (p,q) 4-prime cordial graph then $G \cup mK_{1,n}$, is also 4-prime cordial for all even values of m and all values of n .*

PROOF. Note that the order and size of $G \cup mK_{1,n}$ are $p+m(n+1)$ and $q+mn$ respectively. Let f be a 4-prime cordial labeling of G. Let $K^i_{1,n}$ be the i^{th} copy of the star and $V(K^i_{1,n}) = \{v_i, v^i_j : 1 \leqslant j \leqslant n\}$ and $V(G) = \{u_i : 1 \leqslant i \leqslant p\}$. With the use of 4-prime cordial labeling f, assign the label to the vertices $u_i\,(1 \leqslant i \leqslant p)$. Next move to the stars.

Case 1. $m \equiv 0 \pmod 4$.

Assign the label 1 to the vertice $v_1, v_3, \ldots, v_{\frac{m}{2}-1}$ and 3 to the vertices v_2, v_4, $\ldots$, $v_{\frac{m}{2}}$. Next assign the label 3 to the vertices v^1_1, v^1_2, $\ldots$, v^1_n, v^3_1, v^3_2, $\ldots$, v^3_n, $\ldots$, $v^{\frac{m}{2}-1}_1$, $v^{\frac{m}{2}-1}_2$, $\ldots$, $v^{\frac{m}{2}-1}_n$ and 1 to the vertices v^2_1, v^2_2, $\ldots$, v^2_n, v^4_1, v^4_2, $\ldots$, v^4_n, $\ldots$, $v^{\frac{m}{2}}_1$, $v^{\frac{m}{2}}_2$, $\ldots$, $v^{\frac{m}{2}}_n$. Next consider another $\frac{m}{2}$ copies of the stars. Assign the label 2 to the vertices $v_{\frac{m}{2}+1}$, $v_{\frac{m}{2}+3}$, $\ldots$, v_{m-1} and 4 to the vertices v_2, v_4, $\ldots$, v_m. Now assign the label 4 to the vertices $v^{\frac{m}{2}+1}_1$, $v^{\frac{m}{2}+1}_2$, $\ldots$, $v^{\frac{m}{2}+1}_n$, $v^{\frac{m}{2}+3}_1$, $v^{\frac{m}{2}+3}_2$, $\ldots$, $v^{\frac{m}{2}+3}_n$, $\ldots$,v^{m-1}_1, v^{m-1}_2, $\ldots$, v^{m-1}_n and 2 to the vertices $v^{\frac{m}{2}+2}_1$, $v^{\frac{m}{2}+2}_2$, $\ldots$, $v^{\frac{m}{2}+2}_n$, $\ldots$,v^m_1, v^m_2, $\ldots$, v^m_n. Let g denote the resulting labeling. $e_g(0) = e_f(0) + \frac{m}{2}n$ and

$e_g(1) = e_f(1) + \frac{m}{2}n$. Therefore,

$$\begin{aligned} |e_g(0) - e_g(1)| &= |e_f(0) - e_f(1)| \\ &\leqslant 1 \end{aligned}$$

as f is a 4-prime cordial labeling. $v_g(1) = v_f(1) + \frac{m}{4}$, $v_g(2) = v_f(2) + \frac{m}{4}$, $v_g(3) = v_f(3) + \frac{m}{4}$ and $v_g(4) = v_f(4) + \frac{m}{4}$. This implies

$$\begin{aligned} |v_g(r) - v_g(s)| &= |v_f(r) - v_f(s)| \\ &\leqslant 1 \end{aligned}$$

for all $r, s \in \{1, 2, 3, 4\}$ since f is a 4-prime cordial labeling.

Case 2. $m \equiv 1 \pmod 4$.

As $m-1 \equiv 0 \pmod 4$, assign the label to the first $m-2$ copies of the stars with the similar technique adopted in case 1. Next assign the labels $1, 2$ respectively to the vertices v_{m-1} and v_m. Now we assign the label 3 to the vertices v_1^{m-1}, v_2^{m-1}, $\ldots$, $v_{\lceil \frac{n}{2} \rceil}^{m-1}$ and 1 to the remaining vertices $v_{\lceil \frac{n}{2} \rceil+1}^{m-1}$, $v_{\lceil \frac{n}{2} \rceil+2}^{m-1}$, $\ldots$, v_n^{m-1} of the $(m-1)^{th}$ star. Next assign 2 to the vertices v_1^m, v_2^m, $\ldots$, $v_{\lceil \frac{n}{2} \rceil}^m$ and 4 to the vertices $v_{\lceil \frac{n}{2} \rceil+1}^m$, $v_{\lceil \frac{n}{2} \rceil+2}^m$, $\ldots$, v_n^m. Let g be this resulting vertex labeling.

Subcase 1. n is odd.

Obviously the labeling g is a 4-prime cordial labeling of $G \cup mK_{1,n}$.

Subcase 2. n is even and $p \equiv 0 \pmod 4$.

In this case also the labeling g is a 4-prime cordial labeling.

Subcase 3. n is even and $p \equiv 1 \pmod 4$.

Let $p = 4t + 1$. In the type $v_f(1) = t + 1$, $v_f(2) = v_f(3) = v_f(4) = t$, relabel the vertex v_n^{m-1} with 2. In the type $v_f(2) = t + 1$, $v_f(1) = v_f(3) = v_f(4) = t$, relabel the vertex v_m with 4. The remaining two types, the vertex condition of g automatically satisfied.

Subcase 4. n is even and $p \equiv 2 \pmod 4$.

Let $p = 4t + 2$. In the type $v_f(1) = v_f(2) = t + 1$, $v_f(3) = v_f(4) = t$, relabel the vertex v_{m-1} with 3 and v_n^m with 4. In the type $v_f(1) = v_f(3) = t + 1$, $v_f(2) = v_f(4) = t$, relabel the vertex v_n^{m-1} with 4. In the type $v_f(1) = v_f(4) = t+1$, $v_f(2) = v_f(3) = t$, relabel the vertex v_n^{m-1} with 3. For the type $v_f(2) = v_f(3) = t + 1$, $v_f(1) = v_f(4) = t$, relabel the vertex v_n^m with 4. For the type $v_f(2) = v_f(4) = t+1$, $v_f(1) = v_f(3) = t$, relabel the vertex v_n^m with 4 and interchange the labels of v_1^2 with v_n^m. In the remaining type, the vertex condition of g is automatically satisfied.

Subcase 5. n is even and $p \equiv 3 \pmod 4$.

In the type $v_f(1) = v_f(2) = v_f(3) = t + 1$, $v_f(4) = t$, relabel the vertex v_n^{m-1} with 4. For the type $v_f(1) = v_f(2) = v_f(4) = t + 1$, $v_f(3) = t$, relabel the vertex v_n^{m-1} with 3. For the type $v_f(1) = v_f(3) = v_f(4) = t + 1$, $v_f(2) = t$, relabel the vertex v_n^{m-1} with 2. In the type $v_f(2) = v_f(3) = v_f(4) = t + 1$, $v_f(1) = t$, the vertex condition of g is obviously satisfied.

Hence $G \cup mK_{1,n}$ is 4-prime cordial for all even values of m. □

THEOREM 2.4. *Let G be a (p,q) 4-prime cordial graph with a 4-prime cordial labeling f. Let $f(u)=2$ and $f(v)=4$. The graph $G(u,v)$ obtained from G by identifying u with central vertex of the star $K_{1,n}$ and v with central vertex of the another star $K_{1,n}$ is 4-prime cordial for all values of n.*

PROOF. Using the 4-prime cordial labeling f, assign the labels to the vertices of G. Consider the first copy of the star $K_{1,n}$. Assign the label to the pendent vertices of the star with $1,3$ alternatively. Next we move to the second copy. Assign the label $2,4$ alternatively to the pendent vertices of the second star. Let g denote the resulting vertex labeling. $e_g(0)=e_f(0)+n$ and $e_g(1)=e_f(1)+n$. This implies

$$\begin{aligned}|e_g(0)-e_g(1)| &= |e_f(0)-e_f(1)|\\ &\leqslant 1\end{aligned}$$

$v_g(1)=v_f(1)+\frac{n}{2}$, $v_g(2)=v_f(2)+\frac{n}{2}$, $v_g(3)=v_f(3)+\frac{n}{2}$ and $v_g(4)=v_f(4)+\frac{n}{2}$. This implies

$$\begin{aligned}|v_g(r)-v_g(s)| &= |v_f(r)-v_f(s)|\\ &\leqslant 1\end{aligned}$$

for all $r,s\in\{1,2,3,4\}$. Hence $G(u,v)$ is 4-prime cordial. □

THEOREM 2.5. *Let G be a (p,q) 4-prime cordial graph and f be a 4-prime cordial labeling of G, $f(u)=2$. Let $G(u)$ be the graph obtained from G by identifying u with the central vertex of the star $K_{1,n}$. (i) If $p\equiv 0 \pmod 4$ then $G(u)$ is 4-prime cordial for all n (ii) If $p\equiv 1 \pmod 4$ then $G(u)$ is 4-prime cordial for all $n\equiv 0,1$ (mod 4).*

PROOF. Let $u_1,u_2,\ldots,u_n$ be the pendent vertices of the star $K_{1,n}$. Use of 4-prime cordial labeling f, assign the label to the vertices of G. Next assign the labels $2,4$ alternatively $u_1,u_2,u_3,u_4,\ldots$ until reach the vertex $u_{\lceil\frac{n}{2}\rceil}$. Next assign the labels $1,3$ alternatively to the vertices $u_{\lceil\frac{n}{2}\rceil+1},u_{\lceil\frac{n}{2}\rceil+2},\ldots,u_n$. Let g be the resulting vertex labeling.

Case 1. $p\equiv 0 \pmod 4$.

Let $p=4t$. This implies $v_f(1)=v_f(2)=v_f(3)=v_f(4)=t$.

Subcase 1. $n\equiv 0 \pmod 4$.

Here $v_g(1)=v_f(1)+\frac{n}{4}=t+\frac{n}{4}$, similarly, $v_g(2)=t+\frac{n}{4}$, $v_g(3)=t+\frac{n}{4}$ and $v_g(4)=t+\frac{n}{4}$. $e_g(0)=e_f(0)+\frac{n}{2}$ and $e_g(1)=e_f(1)+\frac{n}{2}$. This forces $|e_g(0)-e_g(1)|\leqslant 1$.

Subcase 2. $n\equiv 1 \pmod 4$.

Here $v_g(1)=v_f(1)+\frac{n-1}{4}=t+\frac{n-1}{4}$, $v_g(2)=t+\frac{n+3}{4}$, $v_g(3)=t+\frac{n-1}{4}$ and $v_g(4)=t+\frac{n-1}{4}$. This implies $|v_g(r)-v_g(s)|\leqslant 1$ for $r,s\in\{1,2,3,4\}$. When $e_f(1)=e_f(0)+1$ or $e_f(0)=e_f(1)$, clearly $|e_g(0)-e_g(1)|\leqslant 1$. On the other hand, namely $e_f(0)=e_f(1)+1$,relabel u_1 with 3 in g, the resulting labeling obviously a 4-prime cordial labeling.

Subcase 3. $n\equiv 2 \pmod 4$.

Here $v_g(1) = v_f(1) + \frac{n+2}{4} = t + \frac{n+2}{4}$, similarly, $v_g(2) = t + \frac{n+2}{4}$, $v_g(3) = t + \frac{n-2}{4}$ and $v_g(4) = t + \frac{n-2}{4}$. It follows that g satisfies the vertex condition of 4-prime cordial labeling. Also $e_g(0) = e_f(0) + \frac{n}{2}$ and $e_g(1) = e_f(1) + \frac{n}{2}$. This implies $|e_g(0) - e_g(1)| \leqslant 1$ as f is a 4-prime cordial labeling.

Subcase 4. $n \equiv 3 \pmod 4$.

In this case $v_g(1) = v_f(1) + \frac{n+1}{4} = t + \frac{n+1}{4}$, $v_g(2) = t + \frac{n+1}{4}$, $v_g(3) = t + \frac{n-3}{4}$ and $v_g(4) = t + \frac{n+1}{4}$. Clearly $|v_g(r) - v_g(s)| \leqslant 1$ for all $r, s \in \{1, 2, 3, 4\}$. Clearly if $e_g(1) = e_f(0) + 1$ or $e_g(1) = e_f(0)$, the labeling g is 4-prime cordial labeling; otherwise relabel u_1, with 3, the resulting labeling is a 4-prime cordial labeling.

Case 2. $p \equiv 1 \pmod 4$.

Let $p = 4t + 1$. In this case $v_f(1) = t + 1$ or $v_f(2) = t + 1$ or $v_f(3) = t + 1$ or $v_f(4) = t + 1$.

Subcase 1. $n \equiv 0 \pmod 4$.

The labeling g is obviously a 4-prime cordial labeling.

Subcase 2. $n \equiv 1 \pmod 4$ and $v_f(x) = t + 1$, $x \in \{1, 3, 4\}$.

Clearly if either $e_f(1) = e_f(0) + 1$ or $e_f(1) = e_f(0)$, then g is a 4-prime cordial labeling. In this case $e_f(0) = e_f(1) + 1$, we get a 4-prime cordial labeling as follows: If $x = 1$ or 4, then relabel u_1 with 3; if $x = 3$, then relabel u_1 with 1.

Subcase 3. $n \equiv 1 \pmod 4$ and $v_f(2) = t + 1$.

In this case relabel u_1 with 4. When $e_f(1) = e_f(0) + 1$ or $e_f(1) = e_f(0)$, clearly g itself a 4-prime cordial labeling; otherwise relabel u_1 with 1, we get a 4-prime cordial labeling. □

THEOREM 2.6. *Let G be a $(4t, q)$ 4-prime cordial graph. Then $G + K_1$ and $G + 2K_1$ are also 4-prime cordial.*

PROOF. Let f be a 4-prime cordial labeling of G. Consider the graph $G + K_1$. In $G + K_1$, assign the labels to the vertices of G as in G. Next assign the label 2 to the vertex of K_1. We call this labeling g. Then $v_g(1) = v_f(1) = t$, $v_g(2) = v_f(1) + 1 = t + 1$, $v_g(3) = v_f(3) = t$, and $v_g(4) = v_f(4) = t$. Also $e_g(0) = e_f(0) + 2t$, $e_g(1) = e_f(1) + 2t$. As f is a 4-prime cordial labeling, g is also 4-prime cordial labeling. Next consider the graph $G + 2K_1$. Use of 4-prime cordial labeling g of $G + K_1$, assign the label to the vertices of $G + K_1$ and assign 4 to the remaining vertex of K_1. It is easy to verify that this labeling is a 4-prime cordial labeling of $G + 2K_1$. □

ACKNOWLEDGEMENT. *The authors thank to the referee for his careful reading and commands which make the paper for better presentation.*

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Receibed by editors 20.10.2016; Revised version 29.04.2017; Accepted 01.06.2017;
Available online 12.06.2017.

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