JOURNAL OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4866, ISSN (o) 2303-4947
www.imvibl.org /JOURNALS / JOURNAL
Vol. 7(2017), 85-99
DOI: 10.7251/JIMVI1701085R

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

FUZZY PRIME IDEALS IN ORDERED $\Gamma-$SEMIRINGS

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Abstract. We introduce the notion of ideal, prime ideal, fuzzy ideal, fuzzy prime ideal of ordered $\Gamma-$semiring and study their properties and relations between them. We characterize the prime ideals of ordered $\Gamma-$semiring with respect to fuzzy ideals. And also characterize simple ordered$\Gamma-$semiring with respect to fuzzy prime ideals of ordered$\Gamma-$semiring.

1. Introduction

The notion of a semiring is an algebraic structure with two associative binary operations where one distributes over the other, was first introduced by H. S. Vandiver [**20**] in 1934 but semirings had appeared in earlier studies on the theory of ideals of rings. In structure, semirings lie between semigroups and rings. The results which hold in rings but not in semigroups hold in semirings since semiring is a generalization of ring. The study of rings shows that multiplicative structure of ring is an independent of additive structure whereas in semiring multiplicative structure of semiring is not an independent of additive structure of semiring. The additive and the multiplicative structure of a semiring play an important role in determining the structure of a semiring. The theory of rings and theory of semigroups have considerable impact on the development of theory of semirings. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory, in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches.

As a generalization of ring, the notion of a $\Gamma-$ring was introduced by N. Nobusawa [**17**] in 1964. In 1981, M. K. Sen [**18**] introduced the notion of a $\Gamma-$semigroup as a generalization of semigroup. The notion of a ternary algebraic system was

2010 *Mathematics Subject Classification.* 08A72, 20N25.
Key words and phrases. Fuzzy prime ideal, ordered $\Gamma-$ semiring .

introduced by Lehmer [**5**] in 1932, Lister [**7**] introduced the notion of a ternary ring . In 1995, M. Murali Krishna Rao [**9, 10, 11**] introduced the notion of a $\Gamma-$semiring as a generalization of $\Gamma-$ring, ring, ternary semiring and semiring. The set of all negative integers Z is not a semiring with respect to usual addition and multiplication but Z forms a $\Gamma-$semiring where $\Gamma = Z$. The important reason for the development of $\Gamma-$semiring is a generalization of results of rings, $\Gamma-$rings, semirings, semigroups and ternary semirings.

The fuzzy set theory was developed by L. A. Zadeh [**21**] in 1965. The fuzzification of algebraic structure was introduced by A. Rosenfeld [**17**] and he introduced the notion of fuzzy subgroups in 1971. K.L. N. Swamy and U. M. Swamy [**19**] studied fuzzy prime ideals in rings in 1988. In 1982, W. J. Liu [**6**] defined and studied fuzzy subrings as well as fuzzy ideals in rings. Applying the concept of fuzzy sets to the theory of $\Gamma-$ring, Y. B. Jun and C. Y. Lee [**2**] introduced the notion of fuzzy ideals in $\Gamma-$ring and studied the properties of fuzzy ideals of $\Gamma-$ring. D. Mandal [**8**] studied fuzzy ideals and fuzzy interior ideals in an ordered semiring. T. K. Dutta et al. [**1**] studied fuzzy ideals of $\Gamma-$semirings. M. Murali Krishna Rao [**13**] studied fuzzy soft $\Gamma-$semiring and fuzzy soft $k-$ideal over $\Gamma-$semiring. N. Kuroki [**4**] studied fuzzy interior ideals in semigroups. M. Murali krishna rao and B .Venkateswarlu [**15**] studied regular $\Gamma-$incline and field $\Gamma-$semiring. In 1988, Zhang [**22**] studied prime $L-$fuzzy ideals in rings where L is completely distributive lattice. The concept of $L-$fuzzy ideal and normal $L-$fuzzy ideal in semirings were studied by Jun, Neggers and Kim [**3**]. M.Murali Krishna Rao [**12**] studied $T-$fuzzy ideals of ordered $\Gamma-$semirings. In this paper, we introduce the notion of ideal, prime ideal, fuzzy ideal, fuzzy prime ideal in an ordered $\Gamma-$semiring and study their properties and relations between them. We characterize the prime ideals of ordered $\Gamma-$semiring with respect to fuzzy ideals.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions which are necessary for this paper.

DEFINITION 2.1. A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called semiring provided

(i). Addition is a commutative operation.
(ii). Multiplication distributes over addition both from the left and from the right.
(iii). There exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

DEFINITION 2.2. Let M and Γ be two non-empty sets. Then M is called a $\Gamma-$semigroup if it satisfies

(i) $x\alpha y \in M$
(ii) $x\alpha(y\beta z) = (x\alpha y)\beta z$, for all $x, y, z \in M, \alpha, \beta \in \Gamma$.

Definition 2.3. Let $(M,+)$ and $(\Gamma,+)$ be commutative semigroups. A $\Gamma-$semigroup M is said to be $\Gamma-$semiring M if it satisfies the following axioms, for all $x,y,z \in M$ and $\alpha,\beta \in \Gamma$

(i) $x\alpha(y+z) = x\alpha y + x\alpha z$,
(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha+\beta)y = x\alpha y + x\beta y$.

Every semiring M is a $\Gamma-$semiring with $\Gamma = M$ and ternary operation as the usual semiring multiplication

Definition 2.4. A $\Gamma-$semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$, for all $x \in M$.

Example 2.1. Let M be the additive semi group of all $m \times n$ matrices over the set of non negative rational numbers and Γ be the additive semigroup of all $n \times m$ matrices over the set of non negative integers, then with respect to usual matrix multiplication M is a $\Gamma-$semiring.

Definition 2.5. Let M be a $\Gamma-$semiring and A be a non-empty subset of M. A is called a $\Gamma-$subsemiring of $\Gamma-$semiring M if A is a sub-semigroup of $(M,+)$ and $A\Gamma A \subseteq A$.

Definition 2.6. Let M be a $\Gamma-$semiring. A subset A of M is called a left (right) ideal of $\Gamma-$semiring M if A is closed under addition and $M\Gamma A \subseteq A$ ($A\Gamma M \subseteq A$). A is called an ideal of M if it is both a left ideal and a right ideal of M.

Definition 2.7. Let M be a non-empty set. A mapping $f : M \to [0,1]$ is called a fuzzy subset of $\Gamma-$semiring M. If f is not a constant function then f is called a non-empty fuzzy subset.

Definition 2.8. Let f be a fuzzy subset of a non-empty set M, for $t \in [0,1]$ the set $f_t = \{x \in M \mid f(x) \geqslant t\}$ is called a level subset of M with respect to f.

Definition 2.9. Let M be a $\Gamma-$semiring.A fuzzy subset μ of M is said to be fuzzy $\Gamma-$subsemiring of M if it satisfies the following conditions
(i) $\mu(x+y) \geqslant min\ \{\mu(x),\mu(y)\}$
(ii) $\mu(x\alpha y) \geqslant min\ \{\mu(x),\mu(y)\}$, for all $x,y \in M, \alpha \in \Gamma$.

Definition 2.10. A fuzzy subset μ of $\Gamma-$semiring M is called a fuzzy left (right) ideal of M if for all $x,y \in M, \alpha \in \Gamma$ it satisfies the following conditions
(i) $\mu(x+y) \geqslant min\{\mu(x),\mu(y)\}$
(ii) $\mu(x\alpha y) \geqslant \mu(y)$ $(\mu(x))$, for all $x,y \in M, \alpha \in \Gamma$.

Definition 2.11. A fuzzy subset μ of $\Gamma-$semiring M is called a fuzzy ideal of M if it satisfies the following conditions
(i) $\mu(x+y) \geqslant min\{\mu(x),\mu(y)\}$
(ii) $\mu(x\alpha y) \geqslant max\ \{\mu(x),\mu(y)\}$, for all $x,y \in M, \alpha \in \Gamma$.

Definition 2.12. For any two fuzzy subsets λ and μ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leqslant \mu(x)$ for all $x \in M$.

Definition 2.13. Let f and g be fuzzy subsets of $\Gamma-$semiring M. Then $f \circ g, f+g, f \cup g, f \cap g$, are defined by

$$f \circ g(z) = \begin{cases} \sup\limits_{z=x\alpha y} \{min\{f(x), g(y)\}\}, \\ 0, \quad \text{otherwise.} \end{cases} ; f+g(z) = \begin{cases} \sup\limits_{z=x+y} \{min\{f(x), g(y)\}\}, \\ 0, \quad \text{otherwise} \end{cases}$$

$$f \cup g(z) = max\{f(z), g(z)\} \ ; \ f \cap g(z) = min\{f(z), g(z)\}$$

$x, y \in M, \alpha \in \Gamma,$ for all $z \in M$.

Definition 2.14. A function $f : R \to M$ where R and M are $\Gamma-$semirings is said to be $\Gamma-$semiring homomorphism if $f(a+b) = f(a) + f(b)$ and $f(a\alpha b) = f(a)\alpha f(b)$ for all $a, b \in R, \alpha \in \Gamma$.

Definition 2.15. Let A be a non-empty subset of M. The characteristic function of A is a fuzzy subset of M, defined by

$$\chi_{_A}(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A. \end{cases}$$

Definition 2.16. Let M be a $\Gamma-$semiring. An element $a \in M$ is said to be idempotent if $a = a\alpha a$, for all $\alpha \in \Gamma$.

Definition 2.17. Let M be a $\Gamma-$semiring. Every element of M, is an idempotent of M then M is said to be idempotent $\Gamma-$semiring M.

Definition 2.18. A $\Gamma-$semiring M is called a simple $\Gamma-$semiring if it has no proper ideals.

3. Fuzzy Ideals and Fuzzy Prime Ideals in an Ordered $\Gamma-$semiring

In this section, we introduce the notion of ordered $\Gamma-$semiring, ideal, prime ideal, fuzzy ideal, fuzzy prime ideal in an ordered $\Gamma-$semiring and we study some of their properties.

Definition 3.1. A $\Gamma-$semiring M is called an ordered $\Gamma-$semiring if it admits a compatible relation $\leqslant$. i.e. $\leqslant$ is a partial ordering on M satisfies the following conditions. If $a \leqslant b$ and $c \leqslant d$ then

(i) $a + c \leqslant b + d$ (ii) $a\alpha c \leqslant b\alpha d$ (iii) $c\alpha a \leqslant d\alpha b$, for all $a, b, c, d \in M, \alpha \in \Gamma$.

Definition 3.2. An ordered $\Gamma-$semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0,$ for all $x \in M, \alpha \in \Gamma$.

Definition 3.3. Let M be an ordered $\Gamma-$semiring. An element $1 \in M$ is said to be unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$.

Definition 3.4. An ordered $\Gamma-$semiring M is said to be commutative $\Gamma-$semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$.

Definition 3.5. Let M be an ordered $\Gamma-$semiring. An element $a \in M$ is said to be an idempotent of M if $a = a\alpha a$ for all $\alpha \ \in \ \Gamma$.

EXAMPLE 3.1. Let $M = [0,1], \Gamma = N,\ +$ and ternary operation be defined as $x + y = \max\{x, y\}, x\gamma y = \min\{x, \gamma, y\}$ for all $x, y \in M, \gamma \in \Gamma$. Then M is an ordered $\Gamma-$semiring with respect to usual ordering.

DEFINITION 3.6. An ordered $\Gamma-$semiring M is said to be totally ordered $\Gamma-$semiring M if any two elements of M are comparable.

DEFINITION 3.7. In an ordered $\Gamma-$semiring M

(i) $(M, +)$ is positively ordered if $a + b \geqslant a, b$ for all $a, b \in M$.
(ii) $(M, +)$ is negatively ordered if $a + b \leqslant a, b$ for all $, a, b \in M$.
(iii) $\Gamma-$semigroup M is positively ordered if $a\alpha b \geqslant a, b$ for all $\alpha \in \Gamma, a, b \in M$.
(iv) $\Gamma-$semigroup M is negatively ordered if $a\alpha b \leqslant a, b$ for all $\alpha \in \Gamma, a, b \in M$.

THEOREM 3.1. *Let M be an ordered $\Gamma-$semiring M, the zero element of an ordered $\Gamma-$semiring $M$is the least element of M.*

PROOF. Let M be an ordered $\Gamma-$semiring M with the zero element. We have $0 + x = x,$ for all $x \in M \Rightarrow\ 0 \leqslant x$. Hence 0 is the least element of M. □

THEOREM 3.2. *If M is an ordered $\Gamma-$semiring M with unity 1 then 1 is the greatest element of M.*

PROOF. Let M be an ordered $\Gamma-$semiring M with unity 1 and $x \in M$. by definition of unity, there exists $\alpha \in \Gamma$ such that $x = x\alpha 1 \leqslant 1$. Hence 1 is the greatest element of M. □

The following theorem follows from theorems 3.5 and 3.6.

THEOREM 3.3. *Let M be an ordered $\Gamma-$semiring M with unity 1 and zero element 0. If $a \in M$ then $0 \leqslant a \leqslant 1$.*

DEFINITION 3.8. A non-empty subset A of an ordered $\Gamma-$semiring M is called a $\Gamma-$subsemiring M if $(A, +)$ is a subsemigroup of $(M, +)$ and $a\alpha b \in A$ for all $a, b \in A$ and $\alpha \in \Gamma$.

DEFINITION 3.9. [**7**] Let M be an ordered $\Gamma-$semiring. A non-empty subset A of M is called a left (right) ideal of M if A is closed under addition, $M\Gamma A \subseteq A$ $(A\Gamma M \subseteq A)$ and if for any $a \in M,\ b \in I,\ a \leqslant b \Rightarrow a \in I$. A is called an ideal of M if it is both a left ideal and a right ideal of M.

DEFINITION 3.10. Let M be an ordered $\Gamma-$semiring. A $\Gamma-$subsemiring P of M is called a prime ideal of M if

(i) $a \leqslant b, a \in M, b \in P \Rightarrow a \in P$
(ii) $a\gamma b \in P, a, b \in M, \gamma \in \Gamma \Rightarrow a \in P$ or $b \in P$

DEFINITION 3.11. Let M be an ordered $\Gamma-$semiring. A fuzzy subset μ of M is called a fuzzy $\Gamma-$subsemiring of M if

(i) $\mu(x + y) \geqslant min\{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) \geqslant min\{\mu(x), \mu(y)\}$
(iii) $x \leqslant y \Rightarrow \mu(x) \geqslant \mu(y),$ for all $x, y \in M, \alpha \in \Gamma$.

DEFINITION 3.12. Let μ be a non-empty fuzzy subset of an ordered $\Gamma-$semiring M. Then μ is called a fuzzy prime ideal of M if

(i) $\mu(x+y) \geqslant min\{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) = max\{\mu(x), \mu(y)\}$
(iii) $x \leqslant y \Rightarrow \mu(x) \geqslant \mu(y),$ for all $x, y \in M, \alpha \in \Gamma$.

DEFINITION 3.13. Let R and M be ordered $\Gamma-$semirings. Then f is a mapping from R to M is called a homomorphism of ordered $\Gamma-$semirings R and S if

(i) $f(a+b) = f(a) + (b)$
(ii) $f(a\alpha b) = f(a)\alpha f(b)$
(iii) $a \leqslant b \Rightarrow f(a) \leqslant f(b),$ for all $a, b \in R, \alpha \in \Gamma$.

THEOREM 3.4. *In an ordered $\Gamma-$semiring M, the following are equivalent*

(i) *μ is a fuzzy left ideal of an ordered $\Gamma-$semiring M*
(ii) *$\gamma \circ \mu \subseteq \mu$ where γ is the characteristic function of M and $x \leqslant y \Rightarrow \mu(x) \geqslant \mu(y)$, for all $x, y \in M$.*

PROOF. Suppose (i) holds. Let $x \in M$. Then

$$\begin{aligned}\gamma \circ \mu(x) =& \sup_{x=y\alpha z,\ \alpha\in\Gamma,\ y,z\in M} min\{\gamma(y), \mu(z)\}\\ \leqslant& \sup_{x=y\alpha z,\ \alpha\in\Gamma,\ y,z\in M} min\{1, \mu(y\alpha z)\}\\ =& \sup_{x=y\alpha z,\ \alpha\in\Gamma,\ y,z\in M} min\{1, \mu(x)\}\\ =&\mu(x).\end{aligned}$$

Otherwise $\gamma \circ \mu(x) = 0 \leqslant \mu(x)$. Therefore $\gamma \circ \mu \subseteq \mu$. By definition of fuzzy left ideal of an ordered $\Gamma-$semiring M, we have $x \leqslant y \Rightarrow \mu(x) \geqslant \mu(y)$.

Now suppose (ii) holds. Let $x, y \in M, \alpha \in \Gamma$. Since $\gamma \circ \mu \subseteq \mu$,

$$\begin{aligned}\mu(x+y) \geqslant& \gamma \circ \mu(x+y)\\ =& \sup_{x+y=c+d} \min\{\gamma(c), \mu(d)\}\\ \geqslant& \min\{\mu(x), \mu(y)\}\\ \mu(x\alpha y) \geqslant& \gamma \circ \mu(x\alpha y)\\ =& \sup_{x\alpha y=c\alpha d} \min\{\gamma(c), \mu(d)\}\\ \geqslant& \min\{\gamma(x), \mu(y)\}\\ =& \min\{1, \mu(y)\} = \mu(y).\end{aligned}$$

Hence μ is a fuzzy left ideal of M. □

The proof of the following theorem follows by routine verification.

THEOREM 3.5. *Let M be an ordered $\Gamma-$semiring. Then I is a left ideal of M if and only if the characteristic function χ_I is a fuzzy left ideal of M.*

THEOREM 3.6. *Let μ and γ be fuzzy prime ideals of ordered $\Gamma-$semiring M. Then $\mu \cap \gamma$ is also a fuzzy prime ideal of M.*

Proof. Let μ and γ be fuzzy prime ideals of an ordered $\Gamma-$semiring M and $x, y \in M, \alpha \in \Gamma$.

$$\begin{aligned}
\mu \cap \gamma(x+y) &= min\{\mu(x+y), \gamma(x+y),\} \\
&\geqslant min\Big\{min\{\mu(x), \mu(y)\}, min\{\gamma(x), \gamma(y)\}\Big\} \\
&= min\Big\{min\{\mu(x), \gamma(x)\}, min\{\mu(y), \gamma(y)\}\Big\} \\
&= min\Big\{\mu \cap \gamma(x), \mu \cap \gamma(y)\Big\} \\
\mu \cap \gamma(x\alpha y) &= min\{\mu(x\alpha y), \gamma(x\alpha y),\} \\
&= min\Big\{max\{\mu(x), \mu(y)\}, max\{\gamma(x), \gamma(y)\}\Big\} \\
&= max\Big\{min\{\mu(x), \gamma(x)\}, min\{\mu(y), \gamma(y)\}\Big\} \\
&= max\Big\{\mu \cap \gamma(x), \mu \cap \gamma(y)\Big\}
\end{aligned}$$

If $x \leqslant y$ then $\mu(x) \geqslant \mu(y)$ and $\gamma(x) \geqslant \gamma(y)$

$$\begin{aligned}
\Rightarrow \mu \cap \gamma(x) &= min\Big\{\mu(x), \gamma(x)\Big\} \\
&\geqslant \min\Big\{\mu(y), \gamma(y)\Big\} \\
&= \mu \cap \gamma(y).
\end{aligned}$$

Hence $\mu \cap \gamma$ is a fuzzy prime ideal of ordered M. □

Proofs of the following theorems are similar to Theorems in [**14**], so we omit the proofs.

Theorem 3.7. *A fuzzy subset μ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M if and only if for any $t \in [0, 1]$ such that $\mu_t \neq \phi$, is a prime ideal of ordered M.*

Theorem 3.8. *Let M be an ordered $\Gamma-$semiring. Then I is a prime ideal of M if and only if the characteristic fuzzy subset χ_I is a fuzzy prime ideal of ordered M.*

Let M be an ordered $\Gamma-$semiring and I be a non-empty subset of M. Then the set $\{x \in M \mid x \leqslant y$ for some $y \in I\}$, is denoted by $(I]$. Obviously $I \subseteq (I]$. If $A \subseteq B$ then $(A] \subseteq (B]$. If I is an ideal of ordered $\Gamma-$semiring M then $(I] = I$. If $I = \{a\}$ then $(I] = I_a$. Let μ be a fuzzy subset of ordered $\Gamma-$semiring M and $I \subseteq M$. Then the set $\{x \in M \mid \mu(x) \geqslant \mu(y)$ for some $y \in I\}$ is denoted by $(I]_\mu$ If μ is a fuzzy ideal of ordered $\Gamma-$semiring M then $(I] = (I]_\mu$. The set $\{x \in M \mid \mu(x) \geqslant \mu(a)\}$ is denoted by $I_{\mu(a)}$.

Theorem 3.9. *Let μ be a fuzzy right (left) ideal of an ordered $\Gamma-$semiring M. Then $I_{\mu(a)}$ is a right (left) ideal of M, for all $a \in M$.*

Proof. Let μ be a fuzzy right ideal of an ordered $\Gamma-$semiring M. Then $I_{\mu(a)} \neq \phi$, since $a \in I_{\mu(a)}$.

$$\begin{aligned} \text{Let } b, c \in I_{\mu(a)} &\Rightarrow \mu(b) \geqslant \mu(a) \text{ and } \mu(c) \geqslant \mu(a) \\ &\Rightarrow \mu(b+c) \geqslant min\mu\{\mu(b), \mu(c)\} \geqslant \mu(a) \\ &\Rightarrow b + c \in I_{\mu(a)} \\ \text{Now } \mu(b\alpha x) &\geqslant \mu(b) \geqslant \mu(a), \text{ for all } x \in M, \alpha \in \Gamma \end{aligned}$$

Then $b\alpha x \in I_{\mu(a)}$, Let $b \in I_{\mu(a)}$ and $c \leqslant b$.

$$\begin{aligned} &\Rightarrow \mu(b) \geqslant \mu(a) \text{ and } \mu(c) \geqslant \mu(b) \\ &\Rightarrow \mu(c) \geqslant \mu(b) \geqslant \mu(a) \end{aligned}$$

Therefore $c \in I_{\mu(a)}$.

Hence $I_{\mu(a)}$ is a right ideal of an ordered $\Gamma-$semiring M. Similarly we can prove the result for left ideal of M. □

Theorem 3.10. *If μ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M then $I_{\mu(a)}$ is a prime ideal of M.*

Proof. By Theorem 3.9, $I_{\mu(a)}$ is an ideal of an ordered $\Gamma-$semiring M.

$$\begin{aligned} \text{Let } c\alpha b \in I_{\mu(a)},\ c, b \in M, \alpha \in \Gamma. &\text{Suppose } b \notin I_{\mu(a)}. \\ &\Rightarrow \mu(c\alpha b) \geqslant \mu(a), \mu(b) < \mu(a) \\ &\Rightarrow max\{\mu(c), \mu(b)\} \geqslant \mu(a), \mu(b) < \mu(a) \\ &\Rightarrow \mu(c) \geqslant \mu(a), \text{ since } \mu(b) < \mu(a) \end{aligned}$$

Therefore $c \in I_{\mu(a)}$.

Hence $I_{\mu(a)}$ is a prime ideal of M. □

Definition 3.14. An ordered $\Gamma-$semiring M is called a fuzzy simple if for any fuzzy ideal μ of $M, x \in M$ then $\mu(x) \geqslant \mu(a)$, or $\mu(x) \leqslant \mu(a)$ for all $a \in M$.

Theorem 3.11. *In an ordered $\Gamma-$semiring M, M is a simple if and only if M is a fuzzy simple.*

Proof. Let μ be a fuzzy ideal of a simple ordered $\Gamma-$semiring M and $x, a \in M$. By Theorem 3.18, $I_{\mu(a)}$ is an ideal of $\Gamma-$semiring M. Since M is a simple, $I_{\mu(a)} = M \Rightarrow \mu(x) \geqslant \mu(a)$, for all $a \in M$. Therefore M is a fuzzy simple, Conversely suppose that I is a proper ideal of M. Then χ_I is a fuzzy ideal of M. Since M is a fuzzy simple, $x \in M$.

$$\begin{aligned} \text{We have } &\chi_I(x) \geqslant \chi_I(a) \text{ or } \chi_I(x) \leqslant \chi_I(a), \text{ for all } a \in M \\ &\Rightarrow \chi_I(x) \geqslant \chi_I(a) \text{ or } \chi_I(x) \leqslant \chi_I(a), \text{ for all } a \in I \\ &\Rightarrow \chi_I(x) = 1 \\ &\Rightarrow x \in I. \text{Therefore } I = M, \text{ which is a contradiction.} \end{aligned}$$

Hence M is a simple ordered $\Gamma-$semiring M. □

THEOREM 3.12. *Let I be a $\Gamma-$subsemiring of an idempotent ordered $\Gamma-$semiring M in which $\Gamma-$semigroup M is non positively ordered. Then $(I]$ is an ordered ideal of ordered $\Gamma-$semiring M generated by I and $(I]$ is a prime ideal of ordered $\Gamma-$semiring M.*

PROOF. Let I be a $\Gamma-$subsemiring of an idempotent ordered $\Gamma-$semiring M in which $\Gamma-$semigroup M is non positively ordered. Obviously $I \subseteq (I]$. Let $x, y \in (I]$ and $\alpha \in \Gamma$. Then there exist $a, b \in I$ such that $x \leqslant a, y \leqslant b \Rightarrow x + y \leqslant a + b \Rightarrow x + y \in I$. Suppose $x \in (I], r \in M$ and $\alpha \in \Gamma$. Then there exists $y \in I$ such that $x \leqslant y \Rightarrow x\alpha\gamma \leqslant y\alpha r \leqslant y \Rightarrow x\alpha r \in (I]$. Let $x \in (I]$ and $y \leqslant x$. Since $x \in (I]$, there exists $z \in I$ such that $x \leqslant z \Rightarrow y \leqslant z$. Therefore $y \in (I]$.Thus I is an ideal of M

Let J be an ideal of an ordered $\Gamma-$semiring M containing I. Since J is an ideal, $(J] = J \Rightarrow (I] \subseteq (J] = J$. Hence $(I]$ is an ideal of M generated by I. Let $x\alpha y \in (I], x, y \in M, \alpha \in \Gamma$ there exists $z \in I$ such that $x\alpha y \leqslant z$.

$$\begin{aligned} \text{Suppose } \ x, y \notin (I] &\Rightarrow x \geqslant i, y \geqslant i, \quad \text{for all } i \in I \\ &\Rightarrow x > z, y > z, \ \text{since } z \in I \\ &\Rightarrow x\alpha y > z\alpha z, \ \text{for all } \alpha \in \Gamma \\ &\Rightarrow x\alpha y > z, \ \text{which is a contradiction.} \end{aligned}$$

Hence $(I]$ is a prime ideal of M. □

COROLLARY 3.1. *Let I be a $\Gamma-$subsemiring of an idempotent ordered $\Gamma-$semiring M in which $\Gamma-$semigroup M is non positively ordered and μ be a fuzzy ideal of $\Gamma-$semiring M. Then $(I]_\mu$ is an ideal of $\Gamma-$semiring M generated I and $(I]_\mu$ is a prime ideal.*

THEOREM 3.13. *Let $f : R \to S$ be a homomorphism of ordered $\Gamma-$semirings R and S. If μ is a fuzzy prime ideal of S then $f^{-1}(\mu)$ is a fuzzy prime ideal of R.*

PROOF. Let $f : R \to S$ be a homomorphism of ordered $\Gamma-$semirings R and S and μ be a fuzzy prime ideal of S.

$$\begin{aligned} f^{-1}(\mu)(0) &= \mu(0_s) \geqslant \mu(x) \neq 0, \ \text{for some } x \in S. \\ f^{-1}(\mu)[r+s] &= \mu\big[f(r+s)\big] \\ &= \mu\big[f(r) + f(s)\big] \\ &\geqslant min\{\mu(f(r)), \mu(f(s))\} \\ &= min\{f^{-1}(\mu)(r), f^{-1}(\mu)(s)\}, \ \text{for all } \ r, s \in R. \\ f^{-1}(\mu)[r\alpha s] &= \mu\big[f(r\alpha s)\big] \\ &= \mu\big\{f(r)\alpha f(s)\big\} \\ &= \max\{\mu(f(r)), \mu(f(s))\} \\ &= \max\{f^{-1}(\mu)(r), f^{-1}(\mu)(s)\}, \ \text{for all } \ r, s \in R, \alpha \in \Gamma. \end{aligned}$$

If $r \leqslant s$ then $f(r) \leqslant f(s)$. Now $f^{-1}(\mu)(r) = \mu(f(r)) \geqslant \mu(f(s)) = f^{-1}(\mu)(s)$. Then $f^{-1}(\mu)$ is a prime ideal of R. Hence the theorem. □

The proof of the following theorem is a straightforward verification

THEOREM 3.14. *Let M_1 and M_2 be ordered $\Gamma-$semirings. If we define as follows*

(i) $(x_1, x_2) + (y_1, y_2) = (x_1 + x_2, y_1 + y_2)$
(ii) $(x_1, x_2)\alpha(y_1, y_2) = (x_1\alpha y_1, x_2\alpha y_2)$
(iii) *If* $(x_1, y_1) \leqslant (x_2, y_2)$ *then* $x_1 \leqslant x_2$ *and* $y_1 \leqslant y_2$, *for all* $x_1, y_1 \in M_1,\ x_2, y_2 \in M_2, \alpha \in \Gamma$

Then $M_1 \times M_2$ is ordered $\Gamma-$semiring

DEFINITION 3.15. A fuzzy relation on any set M is a fuzzy subset of $M \times M$. If μ is a fuzzy relation on a set M and γ is a fuzzy subset of M then μ is a fuzzy relation on M and

$$\mu(x, y) \leqslant min\{\gamma(x), \gamma(y)\}, \text{ for all } x, y \in M.$$

DEFINITION 3.16. Let γ be a fuzzy subset on a set M. Then μ_γ is said to be strongest fuzzy relation on M if

$$\mu_\gamma(x, y) = min\{\gamma(x), \gamma(y)\}, \text{ for all } x, y \in M.$$

THEOREM 3.15. *Let μ_γ be the strongest fuzzy relation on ordered $\Gamma-$semiring M. Then γ is a fuzzy prime ideal of M if and only if μ_γ is a fuzzy prime ideal of $M \times M$.*

PROOF. Let γ be a fuzzy prime ideal of ordered $\Gamma-$semiring M, $(x_1, x_2), (y_1, y_2) \in M \times M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}
\mu_\gamma\big\{(x_1, x_2) + (y_1, y_2)\big\} &= \mu_\gamma\big\{(x_1 + y_1), (x_2 + y_2)\big\} \\
&= \min\big\{\gamma(x_1 + y_1), \gamma(x_2 + y_2)\big\} \\
&\geqslant min\Big\{min\{\gamma(x_1), \gamma(y_1)\}, min\{\gamma(x_2), \gamma(y_2)\}\Big\} \\
&= \min\Big\{min\{\gamma(x_1), \gamma(x_2)\}, min\{\gamma(y_1), \gamma(y_2)\}\Big\} \\
&= \min\Big\{\mu_\gamma(x_1, x_2), \mu_\gamma(y_1, y_2)\Big\}
\end{aligned}$$

$$\begin{aligned}
\mu_\gamma\big\{(x_1, x_2)\alpha(y_1, y_2)\big\} &= \mu_\gamma\big\{(x_1\alpha y_1), (x_2\alpha y_2)\big\} \\
&= \min\big\{\gamma(x_1\alpha y_1), \gamma(x_2\alpha y_2)\big\} \\
&= \min\Big\{max\{\gamma(x_1), \gamma(y_1)\}, max\{\gamma(x_2), \gamma(y_2)\}\Big\} \\
&= \max\Big\{min\{\gamma(x_1), \gamma(x_2)\}, min\{\gamma(y_1), \gamma(y_2)\}\Big\} \\
&= \max\Big\{\mu_\gamma(x_1, x_2), \mu_\gamma(y_1, y_2)\Big\}.
\end{aligned}$$

Suppose $(x_1, x_2) \leqslant (y_1, y_2)$ Then $x_1 \leqslant y_1,\ x_2 \leqslant y_2 \Rightarrow \gamma(x_1) \geqslant \gamma(y_1), \gamma(x_2) \geqslant \gamma(y_2)$ and

$$\begin{aligned}\mu_\gamma(x_1, x_2) &= min\{\gamma(x_1), \gamma(x_2)\}\\ &\geqslant min\{\gamma(y_1), \gamma(y_2)\}\\ &= \mu_\gamma(y_1, y_2).\end{aligned}$$

Hence μ_γ is a fuzzy prime ideal of $M \times M$.

Conversely suppose that μ_γ is a fuzzy prime ideal of ordered $\Gamma-$semiring M. $(x_1, x_2), (y_1, y_2) \in M \times M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}min\big\{\gamma\{(x_1 + y_1), \gamma(x_2 + y_2)\}\big\} &= \mu_\gamma\big\{(x_1 + y_1), (x_2 + y_2)\big\}\\ &= \mu_\gamma\big\{(x_1, x_2) + (y_1, y_2)\big\}\\ &\geqslant \min\Big\{\mu_\gamma(x_1, x_2), \mu_\gamma(y_1, y_2)\Big\}\\ &= \min\Big\{\ \min\{\gamma(x_1), \gamma(x_2)\}, min\{\gamma(y_1), \gamma(y_2)\}\Big\}.\end{aligned}$$

Now put $x_1 = x, x_2 = 0, y_1 = y, y_2 = 0$ then we get

$$\begin{aligned}\min\{\gamma(x + y), \gamma(0)\} &\geqslant \min\Big\{\ \min\{\gamma(x), \gamma(0)\}, \min\{\gamma(y), \gamma(0)\}\Big\}\\ \Rightarrow\ \gamma(x + y) &= \min\{\gamma(x), \gamma(y)\},\ \text{since } \gamma(x) \leqslant \gamma(0),\ \text{for all}\ \ x \in M.\end{aligned}$$

$$\begin{aligned}\min\big\{\gamma\{(x_1\alpha y_1), \gamma(x_2\alpha y_2)\}\big\} &= \mu_\gamma\big\{(x_1\alpha y_1), (x_2\alpha y_2)\big\}\\ &= \mu_\gamma\big\{(x_1, x_2)\alpha(y_1, y_2)\big\}\\ &= max\Big\{\mu_\gamma(x_1, x_2), \mu_\gamma(y_1, y_2)\Big\}\\ &= max\Big\{\ \min\{\gamma(x_1), \gamma(x_2)\}, \min\{\gamma(y_1), \gamma(y_2)\}\Big\}.\end{aligned}$$

Now put $x_1 = x, x_2 = 0, y_1 = y, y_2 = 0$, then we get

$$\begin{aligned}\min\{\gamma(x\alpha y), \gamma(0)\} &= \max\Big\{\ \min\{\gamma(x), \gamma(0)\}, min\{\gamma(y), \gamma(0)\}\Big\}\\ &= \max\Big\{\gamma(x), \gamma(y)\Big\}.\end{aligned}$$

Therefore $\gamma(x\alpha y) = \max\{\gamma(x), \gamma(y)\}$.

Suppose $x \leqslant y, x, y \in M$. Then

$$\begin{aligned}&(x, 0) \leqslant (y, 0)\\ \Rightarrow&\mu_\gamma(x, 0) \geqslant \mu_\gamma(y, 0)\\ \Rightarrow&\min\{\gamma(x), \gamma(0)\} \geqslant min\{\gamma(y), \gamma(0)\}\\ \Rightarrow&\gamma(x) \geqslant \gamma(y).\end{aligned}$$

Hence γ is a fuzzy prime ideal of M. □

Definition 3.17. Let μ and γ be fuzzy subsets of X. The cartesian product of μ and γ is defined by

$$(\mu \times \gamma)(x, y) = \min\{\mu(x), \gamma(y)\},\ \text{for all } x, y \in X.$$

THEOREM 3.16. *Let μ and γ be fuzzy prime ideals of an ordered $\Gamma-$semiring M. Then $\mu \times \gamma$ is a prime ideal of an ordered $\Gamma-$semiring $M \times M$.*

PROOF. Let μ and γ be fuzzy prime ideals of ordered $\Gamma-$semiring M and $(x_1, x_2), (y_1, y_2) \in M \times M, \alpha \in \Gamma$. Then

$$\begin{aligned}(\mu \times \gamma)\Big((x_1, x_2) + (y_1, y_2)\Big) &= \mu \times \gamma\Big(x_1 + y_1, x_2 + y_2\Big) \\ &= min\{\mu(x_1 + y_1), \gamma(x_2 + y_2)\} \\ &\leqslant min\Big\{min\{\mu(x_1), \mu(y_1)\}, \min\{\gamma(x_2), \gamma(y_2)\}\Big\} \\ &= min\Big\{min\{\mu(x_1), \gamma(x_2)\}, \min\{\mu(y_1), \gamma(y_2)\}\Big\} \\ &= min\Big\{(\mu \times \gamma)(x_1, x_2), (\mu \times \gamma)(y_1, y_2)\Big\}\end{aligned}$$

$$\begin{aligned}(\mu \times \gamma)\Big((x_1, x_2)\alpha(y_1, y_2)\Big) &= \mu \times \gamma\Big(x_1\alpha y_1, x_2\alpha y_2\Big) \\ &= \min\{\mu(x_1\alpha y_1), \gamma(x_2\alpha y_2)\} \\ &= \min\Big\{max\{\mu(x_1), \mu(y_1)\}, \max\{\gamma(x_2), \gamma(y_2)\}\Big\} \\ &= \max\Big\{min\{\mu(x_1), \gamma(x_2)\}, \min\{\mu(y_1), \gamma(y_2)\}\Big\} \\ &= \max\Big\{(\mu \times \gamma)(x_1\alpha x_2), (\mu \times \gamma)(y_1\alpha y_2)\Big\}\end{aligned}$$

If $(x_1, x_2) \leqslant (y_1, y_2)$ then $x_1 \leqslant y_1$ and $x_2 \leqslant y_2$ and

$$\begin{aligned}(\mu \times \gamma)(x_1, x_2) &= \min\{\mu(x_1), \gamma(x_2)\} \\ &\geqslant \min\{\mu(y_1), \gamma(y_2)\} \\ &= (\mu \times \gamma)(y_1, y_2).\end{aligned}$$

Therefore $\mu \times \gamma$ is a fuzzy prime ideal of the ordered $\Gamma-$semiring $M \times M$. □

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DEFINITION 3.18. Let μ be a fuzzy subset of X and $\alpha \in [0, 1 - \sup\{\mu(x) \mid x \in X\}]$ The mapping $\mu^T_\alpha : X \to [0, 1]$ is called a fuzzy translation of μ if $\mu^T_\alpha(x) = \mu(x) + \alpha$.

DEFINITION 3.19. Let μ be a fuzzy subset of X and $\beta \in [0, 1]$. Then mapping $\mu^M_\beta : X \to [0, 1]$ is called a fuzzy multiplication of μ if $\mu^M_\beta(x) = \beta\mu(x)$.

DEFINITION 3.20. Let μ be a fuzzy subset of X and $\alpha \in [0, 1 - \sup\{\mu(x) \mid x \in X\}]$, $\beta \in [0, 1]$. Then mapping $\mu^{MT}_{\beta,\alpha} : X \to [0, 1]$ is called a magnified translation of μ if $\mu^{MT}_{\beta,\alpha}(x) = \beta\mu(x) + \alpha$, for all $x \in X$.

THEOREM 3.17. *A fuzzy subset μ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M if and only if μ^T_α is a fuzzy prime ideal of an ordered $\Gamma-$semiring M*

PROOF. Suppose μ is a fuzzy prime ideal of ordered $\Gamma-$semiring M and $x, y \in M, \gamma \in \Gamma$.

$$\begin{aligned}\mu_\alpha^T(x+y) &= \mu(x+y)+\alpha\\ &\geqslant \min\{\mu(x),\mu(y)\}+\alpha\\ &= \min\{\mu(x)+\alpha,\mu(y)+\alpha\}\\ &= \min\{\mu_\alpha^T(x),\mu_\alpha^T(y)\}.\\ \mu_\alpha^T(x\gamma y) &= \mu(x\gamma y)+\alpha\\ &= \max\{\mu(x),\mu(y)\}+\alpha\\ &= \max\{\mu(x)+\alpha,\mu(y)+\alpha\}\\ &= \max\{\mu_\alpha^T(x),\mu_\alpha^T(y)\}\end{aligned}$$

Let $x \leqslant y$. Then $\mu(x) \geqslant \mu(y) \Rightarrow \mu(x)+\alpha \geqslant \mu(y)+\alpha$ and $\mu_\alpha^T(x) \geqslant \mu_\alpha^T(y)$. Hence μ_α^T is a fuzzy prime ideal of the ordered $\Gamma-$semiring M.

Conversely suppose that μ_α^T is a fuzzy prime ideal of an ordered $\Gamma-$semiring M, $x, y \in M$ and $\gamma \in \Gamma$.

$$\begin{aligned}\mu(x+y)+\alpha &= \mu_\alpha^T(x+y)\\ &\geqslant \min\{\mu_\alpha^T(x),\mu_\alpha^T(y)\}\\ &= \min\{\mu(x)+\alpha,\mu(y)+\alpha\}\\ &= \min\{\mu(x),\mu(y)\}+\alpha\end{aligned}$$

Therefore $\mu(x+y) \geqslant \min\{\mu(x),\mu(y)\}$.

$$\begin{aligned}\mu(x\gamma y)+\alpha &= \mu_\alpha^T(x\gamma y)\\ &= \max\{\mu_\alpha^T(x),\mu_\alpha^T(y)\}\\ &= \max\{\mu(x)+\alpha,\mu(y)+\alpha\}\\ &= \max\{\mu(x),\mu(y)\}+\alpha\end{aligned}$$

Therefore $\mu(x\gamma y) = \min\{\mu(x),\mu(y)\}$.

$$\begin{aligned}&\text{Let } x \leqslant y.\text{Then } \mu_\alpha^T(x) \geqslant \mu_\alpha^T(y).\\ &\Rightarrow \mu(x)+\alpha \geqslant \mu(y)+\alpha\\ &\Rightarrow \mu(x) \geqslant \mu(y).\end{aligned}$$

Hence μ is a fuzzy prime ideal of the ordered $\Gamma-$semiring M. □

THEOREM 3.18. *A fuzzy subset μ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M if and only if μ_β^M is a fuzzy prime ideal of an ordered $\Gamma-$semiring M.*

PROOF. Suppose μ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M and $x, y \in M, \gamma \in \Gamma$. Then

$$\begin{aligned}\mu_\beta^M(x+y) &= \beta\mu(x+y)\\ &\geqslant \beta\min\{\mu(x),\mu(y)\}\\ &= \min\{\beta\mu(x),\beta\mu(y)\}\\ &= \min\{\mu_\beta^M(x),\mu_\beta^M(y)\}.\\ \mu_\beta^M(x\gamma y) &= \beta\mu(x\gamma y)\\ &= \beta\max\{\mu(x),\mu(y)\}\\ &= \max\{\beta\mu(x),\beta\mu(y)\}\\ &= \max\{\mu_\beta^M(x),\mu_\beta^M(y)\}.\end{aligned}$$

Let $x \leqslant y$. Then $\mu(x) \geqslant \mu(y) \Rightarrow \beta\mu(x) \geqslant \beta\mu(y)$ and $\mu_\beta^M(x) \geqslant \beta_\beta^M\mu(y)$.

Hence μ_β^M is a fuzzy prime ideal of the ordered $\Gamma-$semiring M. Conversely, suppose that μ_β^M is a fuzzy prime ideal of an ordered $\Gamma-$semiring M and $x, y \in M, \gamma \in \Gamma$. Then

$$\begin{aligned}\mu_\beta^M(x+y) &\geqslant \min\{\mu_\beta^M(x),\mu_\beta^M(y)\}\\ \Rightarrow\ \beta\mu(x+y) &\leqslant \min\{\beta\mu(x),\beta\mu(y)\}\\ &= \beta\min\{\mu(x),\mu(y)\}\end{aligned}$$

Therefore $\mu(x+y) \geqslant \min\{\mu(x),\mu(y)\}$ and

$$\begin{aligned}\mu_\beta^M(x\gamma y) &= \max\{\mu_\beta^M(x),\mu_\beta^M(y)\}\\ &= \beta\max\{\mu(x),\mu(y)\}\\ \beta\mu(x\gamma y) &= \beta\max\{\mu(x),\mu(y)\}\end{aligned}$$

Therefore $\mu(x\gamma y) = \max\{\mu(x),\mu(y)\}$. Let $x \leqslant y$. Then

$$\begin{aligned}\mu_\beta^M(x) \geqslant \beta_\beta^M\mu(y) &\Rightarrow \beta\mu(x) \geqslant \beta\mu(y)\\ &\Rightarrow \mu(x) \geqslant \mu(y).\end{aligned}$$

Hence μ is a fuzzy prime ideal of ordered $\Gamma-$semiring M. □

THEOREM 3.19. *A fuzzy subset μ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M if and only if $\mu_{\beta,\alpha}^{MT} : X \to [0,1]$ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M.*

PROOF. Suppose: μ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M. It is equivalent with the following assertion: μ_β^M is a fuzzy prime ideal of an ordered $\Gamma-$semiring M, by Theorem 3.18. Further on, the last sentence is equivalent with: $\mu_{\beta,\alpha}^{MT}$ is a fuzzy prime ideal of an ordered $\Gamma-$semiring M, by Theorem 3.17. Hence the theorem. □

4. Conclusion:

We introduced the notion of ideal, prime ideal, fuzzy ideal, fuzzy prime ideal in an ordered $\Gamma-$semiring and studied their properties and relations between them. We characterize the prime ideals in an ordered $\Gamma-$semiring with respect to fuzzy ideals. In continuous of this paper we propose to study fuzzy soft prime ideals over ordered $\Gamma-$semirings.

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Received by editors 18.01.2017; Available online 20.02.2017.

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