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PROFESSIONAL PAPER

Classification of vehicle routing problem

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Abstract: The vehicle routing problem is a general name for a whole set of problems in which a group of vehicles located in a depot needs to visit all customers with the condition that only one vehicle visits each customer and that all vehicles return to the depot. Vehicle routing problems are a generalization of the traveling salesman problem. The purpose of present survey in the field of vehicle routing problem (VRP) is to classify different well known variants of VRP according to the type of objects with previously defined system performance was applied . These variants include a whole class of problems where a set of routes for a car fleet located in one or more depots should be determined for some geographically distant nodes or customers, at that, the purpose of routing is the optimization of the series of customers requests (number of vehicles, distance traveled, time spent in vehicles or user waiting time).

Keywords: Vehicle routing problems (VRP), vehicle routing, optimization

INTRODUCTION

Road transport is the most used form of transport in the world. Since it is the most flexible form of transport, today, in the era of globalization and the emergence of internet commerce, its importance is greater than ever before. Road traffic, due to its characteristics and due to the fact that the road infrastructure has the best geographical coverage, enables exactly that.

On the other hand, road transport is, after air transport, the second most expensive form of transport and therefore for suppliers, especially for those who have a large number of daily deliveries with a fleet, the process of optimizing transport routes is extremely important, in order to reduce their costs, but at the same time maintained the quality and speed of service to its customers.

Vehicle routing problem (VRP) is standard name that refers to combinatorial optimisation problems where a set of customers are to be serviced by available vehicles, the solution of which aims to reduce the total costs of transportation and storage. Some of the most common parameters that are optimized are:

- vehicle number,
- the distance traveled by the vehicle,
- vehicle time spent,
- customer waiting time.

Fewer vehicles reduce overall costs. Vehicles depreciate whether they are used or not, so if possible, it is better to do the same job with fewer vehicles. A smaller distance traveled reduces vehicle depreciation costs, fuel costs and the total number of vehicles required for transportation. Less time spent means less expenses on driver's wages, etc.

The term 'vehicle routing' was coined by Golden et al. [1]. Actually, the solution to the classical VRP consists in the construction of a Hamiltonian cycle for a connected weighted graph whose vertices are the customers, and the edges show the cost (time and distance) of the route. In the general case, the task is to determine the number and locations of service facilities in such a way as to minimize the sum of the operating costs of locations and routing costs.

In fact, routing problems are located at the intersection of two well-studied problems:

Travelling Salesman Problem (TSP) - if the carrying capacity C of each vehicle is infinite (or rather, sufficient C=i ∈di), then VRP falls into the category of the Multiple Travelling Salesman Problem;

Bin Packing Problem (BPP) - the solution to this problem resembles the solution of VRP, provided that all the boundary costs are assumed to be zero (thus, the effectiveness of all feasible solutions will be identical).

Routing issues are key issues in the fields of transportation, handling and logistics [2]. In many areas of the market delivery of goods adds a certain sum to its value, and this sum is comparable to the cost of the product itself. Nevertheless, the use of computer optimization techniques of delivery is often expressed in saving about 5-20% of its total value.

MOTIVATION AND SURVEY METHODOLOGY

One of the basic steps in any theory is the description and classification of the research subject according to the chosen criteria. In this paper, the classification of the location model according to the type of objects with previously defined system performance was applied. Specifically, it is a locational routing problem that allows us to make three important decisions: where to locate the plant, how to allocate consumers, and how to route vehicles.

VRP is one among the most widely studied topics in the field of operations research. The Vehicle Routing Problem is a general name for a whole set of problems in which it is necessary for a group of vehicles located in a depot to visit all customers with the condition that only one vehicle visits each customers and that all vehicles return to the depot.

Most works on VRP is available in the form of journal articles, conference proceedings, thesis, technical reports/books etc. Inspite of such a large amount of literature, not much effort seems to have been made on reviewing VRP. Gendreau and Potvin [3] discussed stochastic VRP and solution methodologies. Laporte and Osman [4] is the most cited contribution in the field of review of VRP. Review of VRP provided by Eksioglu et al. [5] revealed more than a thousand articles published on VRP till that date. However, most of the recent surveys/reviews of the available literature mainly focus on the specific variants of VRP and/or specific solution techniques. Pillac et al. [6] and Laporte [7] presented surveys on CVRP. Vidal et al. [8] focus on VRPTW and so on.

The purpose of present survey is to classify different well known variants of VRP. Keeping in view the exhaustive review of Eksioglu et al. [5], here, we have tried to review the works available on VRP and its variants from 2009 onwards[14] [15] [16].

In real problems there are numerous constraints, which actually define numerous variations of this problem, which are known to us under different names of VRP. In this paper, the researchers systematized and presented the classification of all relevant vehicle routing problems according to the used (adopted) optimization criteria (number of vehicles, distance traveled, time spent in vehicles or user waiting time).

VEHICLE ROUTING PROBLEM (VRP)

The vehicle routing problem is a general name for a whole set of problems in which a group of vehicles located in a depot needs to visit all customers with the condition that only one vehicle visits each customer and that all vehicles return to the depot. In doing so, the goal is to determine the routes along which the total costs will be minimal. Costs are primarily affected by the number of engaged vehicles, so minimization of the number of vehicles is the primary goal, while minimization of the total distance traveled or time spent is usually a secondary goal of optimization.

In case we have only one vehicle and if there are no additional restrictions, then VRP turns into the oldest and one of the most studied routing problems, the well-known Traveling Salesman Problem, when it is necessary to visit all points of the graph with a single vehicle at minimum cost (distance, time) (Figure 1).

This problem boils down to visiting the nodes by the shortest possible path. Let the nodes and their mutual distances be given. The goal of solving the traveling salesman problem is to visit all nodes starting from the first to the last and back, so that all cities except the starting one are visited exactly once and the total distance traveled is minimal. Solving every vehicle routing problem also requires solving the Traveling Salesman Problem as its subproblem.

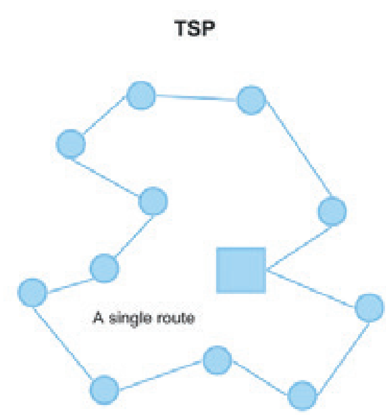

Figure 1. Illustration of the traveling salesman problem (TSP)

Vehicle routing problems are a generalization of the traveling salesman problem. If the limitation of the ca-

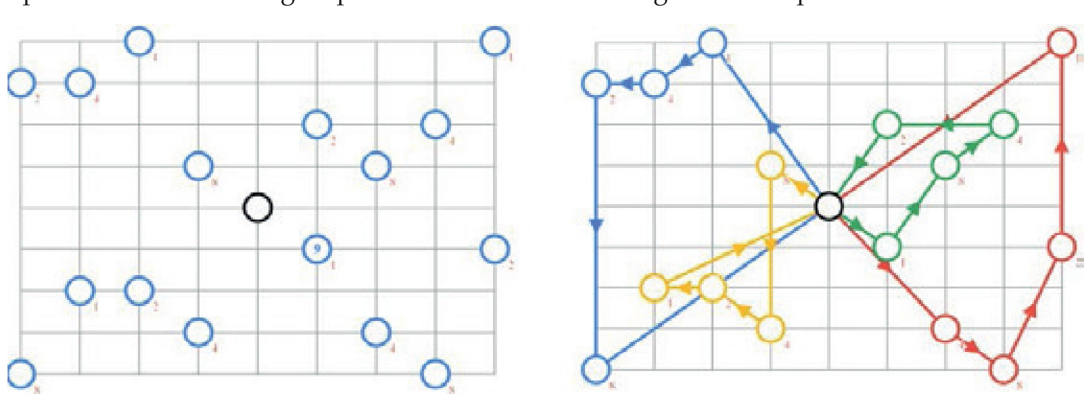

Figure 2. A typical input and one of the possible output for VRP

pacity, that is, the amount of goods he can carry, is added to the traveling salesman, it is clear that in some cases one traveling salesman is not enough to solve the problem. A similar problem arises if the commercial traveler has to arrive at each node in a predetermined time interval. Additional restrictions introduced in TSP may require the parallel engagement of several salesmen (vehicles), multiple depots (warehouses), interaction between salesmen (transshipment of goods from vehicle to vehicle) and the like (Figure 2).

Therefore, the TSP problem has as its primary task the determination of the route, and the VRP problem simultaneously contains the assignment problem (assigning an individual customer to a specific vehicle/route and the problem of determining the route of each individual vehicle).

VRP can be modelled as a directed weighted graph $G(V, E)$ where $V = \{v_0, v1, \ldots, vn\}$be the set of nodes i.e. customers to be visited from the central depot v_0. Also $E = [\{vi, vj\}, (i, j) = 0, 1, 2, \ldots, n, i \neq j)]$ is the set of arcs interlinking two locations i, j. Furthermore, a set of vehicle having homogeneous capacity is available to serve all the customers.

Mathematically, VRP can be represented as:

$$Min\ F = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{K=1}^{V} c_{ij}\, x_{ij}^{v} \quad (1)$$

Subject to:

$$\sum_{v=1}^{V} \sum_{J=1}^{N} x_{ij}^{v} \leq V \text{ for } i = 0 \quad (2)$$

$$\sum_{v=1}^{V} x_{ij}^{v} = \sum_{j=1}^{N} x_{ji}^{v} \leq 1 \text{ for } i = 0 \text{ and } v \in \{1, \ldots, V\} \quad (3)$$

$$\sum_{v=1}^{V} \sum_{J=0}^{N} x_{ij}^{v} = 1 \text{ for } i \in \{1, \ldots, N\} \quad (4)$$

$$\sum_{v=1}^{V} \sum_{i=0}^{N} x_{ij}^{v} = 1 \text{ for } j \in \{1, \ldots, N\} \quad (5)$$

$$\sum_{i=0}^{N} c_i \sum_{j=0}^{N} x_{ij}^{v} \leq q_v \text{ for } v \in \{1, \ldots, V\} \quad (6)$$

$x_{ij} = 1$ if customer j is served after serving customer i and 0 otherwise ($i \neq j$; $i,j = 0, 1, \ldots, N$).

Here:
V is total fleet size
N number of locations/customers to be visited
c_i customer i ($i = 1, 2, \ldots, N$)
c_0 central depot
d_{ij} travelling distance between customer i and customer j
q_i total servings for customer i
q_v upper limit for capacity of vehicle.

Here the objective function given by (1) is to be optimised satisfying constraints (2) to (6).

Objective function (1) corresponds to minimisation of total travelled distance. The first constraint (2) ensures that all of the tour must be completed with at most V vehicles. Beginning and completion of tour at central depot is ensured by (3). Constraints (4) and (5) restrict the partial servings i.e. every location must be visited by exactly one vehicle. Constraint (6) ensures that the net demand on every route must be within vehicle's capacity.

In practice, the basic VRP can be associated with constraints, for instance, maximum allowed capacity of the vehicle, length of route, arrival/departure time at each location and service time, collection or delivery of goods.

Usually, when solving real optimization problems, a number of constraints arise, from which the most important variants arose:

- each vehicle has a limited capacity and load capacity (Capacitated VRP - CVRP);
- each customer must be serviced at a certain time (VRP with Time Windows - VRPTW);
- some car should pick up the goods from the customer after all clients are served (VRP with Backhauls - VRPB);
- the customer can return some of the goods at the depot (VRP with Pick-Ups and Deliveries -VRPPD);
- delivery can be made within a few days (Periodic VRP - PVRP);
- any member of the task can have random behavior (Stochastic VRP - SVRP);
- the customer can be served with a variety of vehicles (Split Delivery VRP - SDVRP);
- the company uses several transport vehicles from the depot to service customers (Multiple Depot VRP - MDVRP).

A schematic representation of the approach to the classification of vehicle routing problems is shown in the Figure 3.

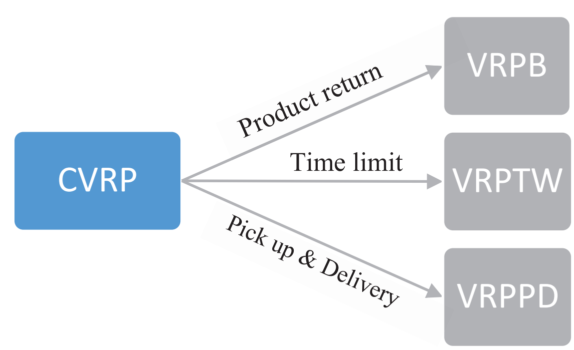

Figure 3. Scheme classification of VRP

CAPACITATED VRP (CVRP)

Capacitated Vehicle Routing Problem (CVRP) is a basic type of vehicle routing problem. In the CVRP, all the customers correspond to deliveries and the demands are deterministic, known in advance, and may not be split [17]. The vehicles are identical and based at a single central depot. Only the capacity restrictions for the vehicles are imposed. The objective is to minimize the total cost (length or travel time) to serve all the customers. Solving CVRP is the determination of routes (each route is connected to only one vehicle) where the total cost of the route should be minimal. The total cost is obtained as the sum of the costs of the arcs belonging to the route. The solution should satisfy these conditions:

- each route should start and end at the depot;
- each client vertex visited exactly once;
- the sum of the demands by customers who are served in one route must not exceed the capacity of the vehicle.

VRP WITH TIME WINDOWS (VRPTW)

The VRP with Time Windows is the extension of the CVRP in which capacity constraints are imposed and each customer is associated with a time interval called a time window. In addition to these times, the time when the vehicles leave the depot, the travel time that the vehicles spends on each of the arches and an additional service time for each customer is also specified [18].

The solution to the VRPTW problem consists in finding routes with minimum cost and satisfying the following conditions:

- each route should start and end at the depot;
- each client vertex visited exactly once;
- the sum of the demands by customers who are served in one route must not exceed the capacity of the vehicle;
- for each customer, the service starts within the time window and the vehicle stops for time instants.

VRP WITH BACKHAULS (VRPB)

The VRP with Backhauls is the extension of the CVRP in which the customer set is partitioned into two subsets. The first subset L contains n Linehaul customers each requiring a given quantity of product to be delivered. The second subset B contains m Backhaul customers, where a given quantity of inbound product must be picked up. In the VRPB problem, there is a restriction priority where all customers in set L must be served before customers in set B. When the cost matrix is asymmetric, the problem is called Asymmetric VRP with Backhauls (AVRPB).

The solution to the VRPB and AVRPB problem consists in finding routes with minimum cost and satisfying the following conditions:

- each route should start and end at the depot;
- each client vertex visited exactly once;
- the total demands of the linehaul and backhaul customers visited by a route do not exceed, separately, the vehicle capacity;
- in each route all the linehaul customers precede the backhaul customers, if any.

VRP WITH PICK-UP AND DELIVERY (VRPPD)

The pickup and delivery problem (VRPDP) deals with delivery as well as collection of items from the customers, aiming to minimise the total travelled distance. Each location is associated with the items either to be recollected or delivered or both. There is also a precedence associated with each of the location to be visited. Moreover, the pairing constraints bound the set of routes so that one vehicle has to do both the pickup and the delivery of the load of one transportation request [19].

The solution should satisfy these conditions:

- each route should start and end at the depot;
- each client vertex visited exactly once;
- the current load of the vehicle along the route must be nonnegative and may never exceed the vehicle capacity;
- for each customer *i*, the customer O_i, when different from the depot, must be served in the same route and before customer *i*;
- for each customer *i*, the customer D_i, when different from the depot, must be served in the same route and after customer *i*.

The case of VRPPD in which time windows are present has been studied in the literature and is called the VRP with Pickup and Deliveries and Time Windows (VRPPDTW).

OTHER VARIANTS VRP

Many other variants of VRP were considered, which represent a combination of some of the previously mentioned variants. These include multi-depot VRP, stohastic VRP, periodic VRP and split delivery VRP [20] [21].

Periodic VRP (PVRP)

With classic VRP, the planning period is usually one day. In the case of periodic VRP, classic VRP is used by extending the planning period to several days [9].

The goal is to minimize the vehicle fleet and the total travel time required to serve all customers. The restrictions are the same as for the classic VRP with the additional conditions that the vehicle cannot be returned to the depot on the same day it leaves, and that during a multi-day period, each customer must be visited at least once. Each customer's needs must be met in one visit to one vehicle. Periodic VRP can be viewed as the problem of generating a group of routes for each day such that the involved constraints are satisfied.

Stochastic VRP (SVRP)

In this version no exact information about customer's actual demands is available before starting the tour. Moreover, the travels times are also stochastic due to varying traffic, accidents etc. This is in a way stochastic VRP where routes need to be planned based on the assumed probability distribution of the demand of the customer. In such problems, a strategy needs to be specified as to what would happen when a vehicle fails to follow its committed route due to shortage of goods to be delivered.

The constraints for the task are connected with the fact that when some data are random, it is no longer possible to require that all constraints will be satisfied for all realizations of the random variables. So the decision maker may either require the satisfaction of some constraints with a given probability, or the incorporation into the model of corrective actions when a constraint is violated.

Some articles dealing with this variant are Marinakis et al. [10] and Juan et al. [11].

Split Delivery VRP (SDVRP)

Classic VRP expands, allowing one customer to be serviced with different types of vehicles, if it reduces overall costs. This case is typical of a situation where the size of the customer's order is equal to the capacity of the vehicle. As a rule, the optimal solution for the routing problem with different types of transport [12] is more difficult to achieve than for the classic VRP.

With this VRP, the goal is to minimize the vehicle fleet and the total travel time required to supply all customers.

Restrictiveness, unlike the classic VRP, consists in the absence of a requirement to serve the customer with only one vehicle. In addition, the fleet includes cars with different payloads.

VRP is modified into Spit Delivery VRP by splitting each customer order into several indivisible orders.

Multiple Depot VRP (MDVRP)

A company can have several depots with whose help it serves its customers. If the customers are clustered around depots, then the distribution problem can be modelled as a set of independent VRPs. However, if the customers and the depots are intermingled then a Multi-Depot Vehicle Routing Problem should be solved [13].

A MDVRP requires the assignment of customers to certain depots. A fleet of vehicles is based at each depot. Each vehicle leaves its depot, services the customers assigned to that depot, and then returns to the same depot.

The objective of the problem is to minimize the number of vehicles and the total travel time. The constraint for the task consists in the demand for each route to satisfy the standard VRP constraints and also to begin and end at the same depot.

CONCLUSION

Due to the practical importance of VRP in real life, the problem has attracted the attention of numerous researchers in the past. Most of the works are dedicated to classic goals, e.g. minimizing total distance traveled, fleet size or time. Some survey also has been done considering multiple objectives such as load balancing, distance/time balancing etc. The papers dealing with profit maximisation, penalty consideration, customer's satisfaction etc. usually tend to optimise multiple objectives simultaneously.

This paper provides a classification of VRP through certain of its most popular variants. In research, we provide a defined classification of the basic problems of the vehicle routing class. These problems, which received considerable attention in the scientific literature, were studied and classified in accordance with the optimization criteria used. Capacitated VRP is introduced first, which is the simplest and most studied member of the family, then we present time-windowed VRP, backhaul VRP, VRP with pick ups and Deliveries, then we present Stohastic VRP, Split Delivery VRP and finally Multiple Depot VRP.

Moreover, it was also noted that there are no benchmarks available for a more realistic classification of VRP versions. As a result, there is still room for further work in the field. Certain subvariants of the vehicle routing problem in the available literature may motivate some of the researchers in this field.

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