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PRELIMINARY COMMUNICATION

Modelling of the Interdependence Between Speed and Traffic Flow Density. A Neuro – Fuzzy Logic Approach

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Abstract. The speed-traffic flow density interdependence diagram has a number of variations, starting with the theoretical model, through various empirical models that were developed and models based on actual research done on traffic flow. The functional interdependence is obtained using the Sugeno fuzzy logic system, where representative values proposed in HCM 2010 have been adopted as parameters of output association functions. Subsequently the neural network is trained based on actual traffic flow data, which by adjusting the association function of the fuzzy logic system yields an output form of the basic traffic flow diagram. It was noticed that this hybrid expert system produces better output results by applying the "subtractive clustering" method on data that are used for training a neural network. Finally, the model was tested on several input data groups, and the interdependence between speed and traffic flow density is shown in graphical form.

Keyword. Basic traffic flow diagram, traffic flow theory, neural networks, fuzzy logic, subtractive clustering, hybrid expert systems.

INTRODUCTION

Earlier attempts to estimate the road capacity led to the establishment of the first relations between speed and flow, thus several papers on this topic were published in the 1920- and 1930-ies, most of them in Britain, Germany and the USA. It is difficult to determine which paper was the first to be published, but according to references, that honor would go to Schaar [15]. A wider preview of theoretical curves, which were considered at the time, and also some more contemporary approaches are set out in papers [4, 6, 7, 8, 9, 14, 21], while the paper [1] contains basic theoretical principles regarding the relation between speed and traffic flow density.

Theoretical interdependence between speed and traffic flow density in automatic traffic flow control conditions on highways is another among a series of problems regarding the interdependence of the basic traffic flow parameters [2]. The driver's decisions are replaced by a computer-based control system which carries out the optimization of the traffic flow.

The use of a queuing system, which is based on traffic counting and measuring traffic behavior as a function of relevant limitations, is another model that was used to illustrate the interdependence between speed and traffic flow density [22]. The road was divided into segments of equal length, where each segment is considered a service station where vehicles enter (λ) and are queued (μ) and served according to the parameters of the queuing system. The output result is shown in the form of analytically obtained curves of the traffic flow diagram.

The result of fixed- and floating point Langevin equations for traffic flow is a set of relations which can be linked to the basic traffic flow diagram [11].

The new model of the basic traffic flow diagram as a function of homogenous states, which was confirmed by research conducted on German highways, is one of the most important papers in the recent history of traffic flow theory studies [24]. According to the model, the basic traffic flow diagram can entirely be defined using the following five key parameters: desired speed, sequence distance between vehicles in the flow, average traffic flow speed, average forced flow sequence distance and forced flow vehicle density.

Figure 1. [24] shows a diagram of interdependence between speed and traffic flow density on German highway A43, for a saturated state of flow. The author has determined that the form of the curve shown in the figure is identical for highways and other categories of motorways.

The basic diagram of traffic flow has its specific properties in big urban areas. The paper [5] contains experimental results concerning this interdependence, which were obtained using a combination of detectors in the network and a vehicle fleet equipped with sensors.

The authors of this paper propose a model based on fuzzy logic, where the Sugeno fuzzy logic system is used, implemented in the programming language "MATLAB" [18, 19]. As guiding output (reference) values for the design of the system we took the ones proposed in HCM 2010[20].

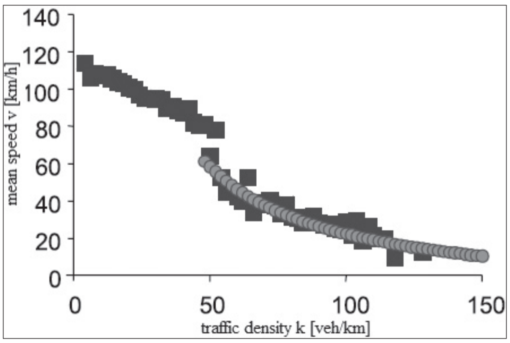

Figure 1. Form of speed-density interdependence for forced flows on German highway A43

The second part of the paper will be about neuro - fuzzy logic. The third part of the paper is dedicated to the basics of subtractive clustering. In the fourth part of the paper, the proposed model will be presented. This model is based on a neuro-fuzzy logic system and also on subtractive clustering of data used for training a neural network. The data for the training of the neural network have been taken from the paper [24]. That data has been obtained from measurements carried out on German highways. In the last part of the paper, the model will be tested on a group of input data sets (values of traffic flow speed), where as a result values of traffic flow density and a corresponding curve of this interdependence will be obtained. The data sets that are used to test the model, are independent from the data used to train the neural network and have been obtained on Serbian highways. It is, of course, possible to test the model with available data sets from any highway in the world. The model was designed in the programming language MATLAB.

NEURO - FUZZY LOGIC

In real life, one cannot answer all questions with just yes or no. For example, is there really a strict line of distinction between tall and short people, or can we say that a person is absolutely good or absolutely bad. The answer to these questions is certainly negative.

According to the classical set theory one can belong to the set of tall people or not, which means that elements of a set can absolutely belong, or absolutely not belong to that set. If we, for example, have a classical Fibonacci set A = {1,1,2,3,5,8,13,21...}, a set element {8} belongs to the Fibonacci se with a probability of 100 %.

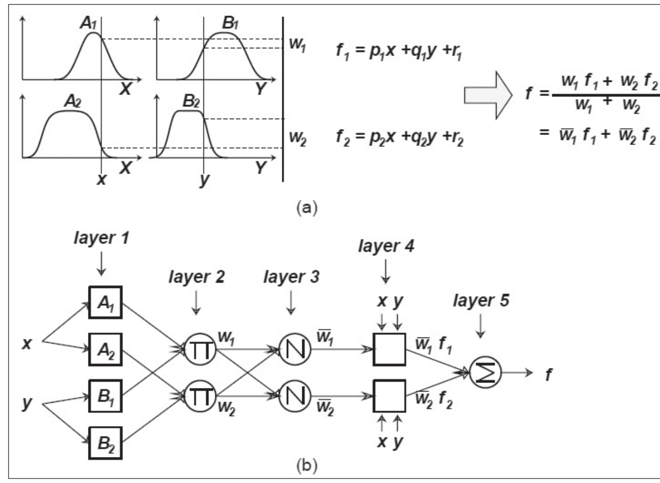

Figure 2. (a) First-order Sugeno fuzzy model; (b) Corresponding ANFIS architecture

Inspired by this way of thinking, in 1965 Zadeh [28] proposed a modified set theory (fuzzy sets) where the association to a set can be expressed through different percentages, and not only through absolutely belongs to (1) or absolutely does not belong to (0). Therefore, according to this modified set theory, one can belong to the set of tall people with 70 %, and belong to the set of short people with 30 %. Take, for example, a fuzzy logic system of the Sugeno type containing two rules, where the output functions are given in linear form [16, 17].

Rule 1: IF X is A_1 AND Y is B_1 THEN $f_1 = p_1x+q_1y+r_1$
Rule 2: IF X is A_1 AND Y is $B2$ THEN $f_1 = p_1x+q_1y+r_1$

Figure 2. [16, 17] (a) graphically illustrates the mechanism of fuzzy conclusion in order to obtain output function f based on input values $[x, y]$. Weight factors ω_1 and ω_2 are usually derived from level of belonging in the premise, whereas the output function f is the weighted average of each of the rules in the THEN part. To improve the Sugeno fuzzy model, i.e. to improve its performance, the fuzzy model has been put into the frame of a neural network. As a result we have the ANFIS architecture shown in figure 2. (b), where the role of each network layer is explained in detail. All the network nodes in the same layer execute a function of the same type. O_i^j marks the outputs of the i-th node in the j-th layer.

Layer 1. Each nod in this layer generates one linguistic variable association function (low, medium, etc.). For example, the i-th node generates a bell-shaped association function which can be mathematically expressed in the following way:

$$O_i^1 = \mu A_i(x) = \frac{1}{1 + \left|\frac{x - c_i}{a_i}\right|^{2b_i}} \tag{1}$$

Where x is the input value for node i; A_i is the linguistic variable association function relating to the node, $\{a_i, b_i, c_i\}$ is a group of parameters for the change of association function shape and refers to the rules premise (part IF).

Layer 2. Each node in this layer calculates weight coefficients through multiplication, which can be expressed in the following way:

$$O_i^2 = \omega_i = \mu A_i(x) \cdot \mu B_i(y), \; i = 1,2 \tag{2}$$

Layer 3. Node i in this layer calculates the ratio of the weight coefficient of i-th rule and the sum of all weight factors. This ratio can be expressed as follows:

$$O_i^3 = \overline{\omega_i} = \frac{\omega_i}{\omega_1 + \omega_2}, \; i = 1,2 \tag{3}$$

Layer 4. Node i in this layer calculates the participation of the i-th rule in the total system output. This can be mathematically described as follows:

$$O_i^4 = \overline{\omega_i} \cdot f_i = \overline{\omega_i} \cdot (p_i x + q_i y + r_i) \tag{4}$$

Where $\overline{\omega_i}$ is the output from the third network layer, i $\{p_i, q_i, r_i\}$ is a group of parameters which refers to the rule consequence (part THEN).

Layer 5. A single node in this layer calculates the total output as a sum of each rule's contributions:

$$O_i^5 = \sum_i \overline{\omega_i} \cdot f_i = \frac{\sum_i \omega_i f_i}{\sum_i \omega_i} \tag{5}$$

Structure of an adaptive network, shown in figure 2. (b) is functionally equivalent to the fuzzy system shown in figure 2. (a). This leads to the conclusion that the output function from the neuro - fuzzy system is linearly dependent on the parameters from the consequences of fuzzy rules:

$$\begin{aligned} f = \overline{\omega_1} \cdot f_1 + \overline{\omega_2} \cdot f_2 = (\overline{\omega_1} \cdot x) \cdot p_1 + \\ + (\overline{\omega_1} \cdot y) \cdot q_1 + (\overline{\omega_1}) \cdot r_1 + (\overline{\omega_2} \cdot x) \cdot p_2 + \\ + (\overline{\omega_2} \cdot y) \cdot q_2 + (\overline{\omega_2}) \cdot r_2 \end{aligned} \tag{6}$$

Backpropagation is used as an algorithm for training, which calculates the error (derives the square error taking into consideration the output function of each of the nodes) recursively, starting with the output layer and then back to the input layer. Further improvements of fuzzy hybrid systems can be found in [25].

SUBTRACTIVE CLUSTERING

Grouping numeric data is a base for many types of classification and system modelling algorithms. The purpose of grouping is to identify the subgroup of data of the same nature from a big group of data, in order to give a summary of system behavior.

Data clustering, also known as cluster analysis, groups an unallocated data set into homogenous clusters based on similar properties. Several varieties of clustering are proposed, like fuzzy clustering, hierarchical clustering or subtractive clustering, etc.

A simple cluster center determination method estimates the center of the cluster based on lowering and lifting the objective function with a "greedy" algorithm [26]. However, this method is efficient for a small group of data, but in case of a large group of data, the algorithm becomes unusable. To reduce the complexity on a large group of data, the subtractive clustering method is proposed, which will be described in the following text [3].

Fuzzy clustering is a "monitored" algorithm because it is necessary to determine the number of clusters in advance. If it is not possible to determine the number of clusters in advance, a "non-monitored" algorithm is used. Subtractive clustering is based on the measurement of data density in relation to the data space characteristics. Thus, the idea is to find the maximum data density in the data area over which the cluster analysis is carried out. Data with the largest number of neighbors is selected as the cluster center. The objective function has the following form [10]:

$$M(x_i) = \sum_{j=1}^{n} e^{-\alpha \|x_i - x_j\|^2} \tag{7}$$

Where α is a positive constant, and $\|x_i - x_j\|^2$ is the square of distance between x_i and x_j.

If M_1^* is the maximum found value of the objective function, and x_i^* is the data for which the maximum objective function value was found (this data will be selected as the first cluster center), the following modified objective function for the determination of the next cluster is proposed:

$$\hat{M}^{j}(x_i) = \hat{M}^{j-1}(x_i) - M_{j-1}^* \sum_{j=1}^{n} e^{-\beta \|x_i - x_{j-1}^*\|^2} \tag{8}$$

Where x_{j-1}^* is the newly determined centroid, and β is a positive constant. This method is, among others, also used to define the initial number of clusters in fuzzy clustering.

Contrary to the classic (sharp) classification of data, fuzzy clustering offers the possibility to assign a data not only to one, but multiple, cluster groups, so that each data belongs to one of the clusters with a certain level of association. Each cluster group is represented by a prototype, which consists of the cluster center and probably of an additional information regarding the size and shape of a cluster. The level of association which determines which data will be associated to the cluster, is obtained from the distance of the data from the cluster center in relation to the size and shape of the information.

Further improvements of the subtractive clustering method can be found in [12, 13, 23]. It is also possible to use the well-known entropy equation as an objective function, under the approach that data with a minimum entropy value becomes the candidate for the next cluster center [27].

APPLICATION OF ANFIS ON A V-G TRAFFIC FLOW DIAGRAM

The idea of this paper regarding the use of fuzzy logic is to propose values related to the traffic flow density from input data (traffic flow speed values). The justification of such an approach lies in the fact that flow density is a parameter that is hard to measure or calculate. Based on this assumption, an input to the Sugeno-type fuzzy logic system is formed, while guiding reference output values are adopted based on the values recommended in HCM 2010. Figure 3. shows fuzzy set used with this model.

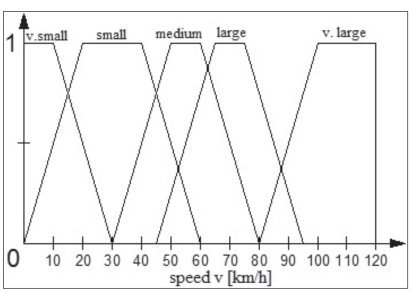

Figure 3. Fuzzy sets as an input value of a Sugeno fuzzy logic system

Before forming an ANFIS model it is necessary to define the initial structure of the Sugeno-type FIS model. That can be achieved using the technique of network division or the data clustering technique. In the first case, the author has applied the technique of network division, whereas in the second case data clustering was used, where in the following text speed-flow density dependence curves will be shown, which represent an output for each of the mentioned cases.

Models based on fuzzy logic consist of fuzzification, forming of a base of "if – then" rules, and as a final step, the selection of the output variable value, and of defuzzification. The fuzzy rules base together with the output variable association function values, presented in the form of constants, are shown in the following table 1. Output functions can be linear or constant. In this paper, the output functions are assumed to be constant. Figure 4. shows a curve which illustrates the dependence between speed and flow density. FIS was tested on 212 input data which represent traffic flow speed values on Serbian highways. This table shows fuzzy logic system output results without the use of neural network.

Table 1. Base of FIS model fuzzy rules

No.	and V(km/h) is	THEN g (veh./km) is		Weight fact.
1.	very small	mf_1	100	1
2.	small	mf_2	70	1
3.	medium	mf_3	60	1
4.	large	mf_4	30	1
5.	very large	mf_5	10	1

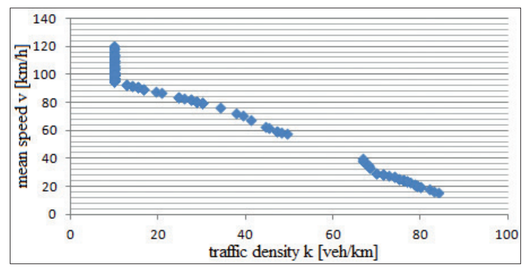

Figure 4. Speed-flow density curve as an output from FIS

The data for the training of the neural network have been taken from the paper [24]. That data has been obtained from measurements carried out on German highways. Data for the learning of the neural network consisted of 411 data pairs (speed and flow density values). A combination of the backpropagation algorithm and the minimum square method was used as an algorithm for network learning. Other parameters of the applied ANFIS model are:

- Number of linear parameters: 5
- Number of nonlinear parameters: 20
- Total number of parameters: 25
- Number of fuzzy rules: 5
- Number of epochs: 800

The dependence of the average test error on the number of epochs is shown in figure 5., while figure 6. shows the association functions of the input variable of a corresponding ANFIS model, which are automatically adjusted based on the neural network test. The base ANFIS model fuzzy rules model is shown in table 2., where output parameter values have been changed.

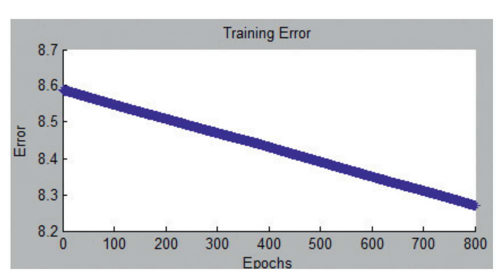

Figure 5. Average test error-number of epochs curve

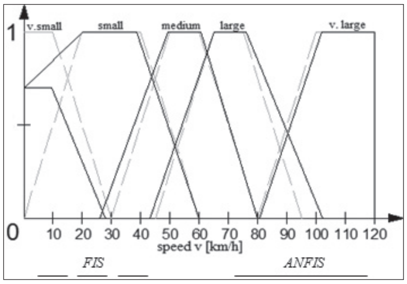

Figure 6. Adjusted ANFIS model association functions

Table 2. Base of ANFIS model fuzzy rules

No.	and V(km/h) is	THEN g (veh./km) is		Weight fact.
1.	very small	mf_1	97,13	1
2.	small	mf_2	55,79	1
3.	medium	mf_3	53,51	1
4.	large	mf_4	32,22	1
5.	very large	mf_5	19,96	1

Figure 7. shows the speed-traffic flow density curve as an output from a hybrid system. The ANFIS model was tested for the same input data as the FIS model (identical flow speed values).

The final system enhancement measure is the clustering of input data, where the association functions will change one more time (their second change). The principles of data clustering and an application method are more detailed described in the third chapter of this paper.

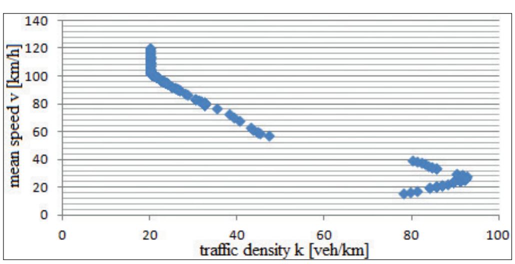

Figure 7. Speed-flow density curve as an output from ANFIS

After the input data clustering is completed using the subtractive clustering method, a significant change in the form of the association function (figure 8) occurred, fuzzy rules, where the output parameters finally take a linear form (table 3). Average test error-Number of epochs curve is shown on figure 9., whereas the speed-traffic curve is shown on figure 10.

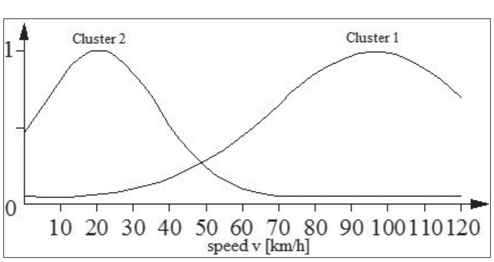

Figure 8. ANFIS model association function after completed clustering

Table 3. The fuzzy rules base after clustering

No.	and V(km/h) is	THEN g (veh./km) is		Weight fact.
1.	input cluster 1	output cluster 1	[-0.422; 65.26]	1
2.	input cluster 2	output cluster 2	[-0.5451; 101.3]	1

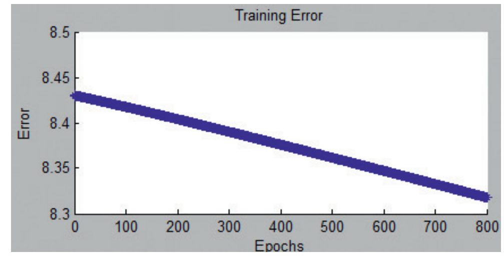

Figure 9. Average test error-number of epochs curve after clustering

Other parameters of the applied ANFIS model with data clustering are:

- Number of linear parameters: 4
- Number of nonlinear parameters: 4
- Total number of parameters: 8
- Number of fuzzy rules: 2
- Number of epochs: 800

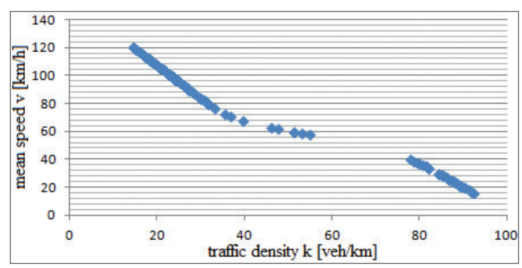

Figure 10. Speed-flow density curve as an output from ANFIS with data clustering

It should be noted that the data used for learning of the neural network were different from the ones the models were tested with. Therefore, clustering was carried out on the data used for learning of the neural network (411 pairs), and the models were tested o independent data (212 traffic flow speed values). Figure 10. illustrates an output from the proposed model and shows the speed-traffic flow density curve.

DISCUSSION OF THE RESULTS

The speed-traffic flow shown in figure 4. represents a set of output solutions which is obtained by applying FIS not including a neural network as part of a hybrid system. It is an approximate density value that was obtained using the fuzzy rule base and parametrical values which were adopted as input to the FIS.

The curve in figure 7., in the part where speed is lower than 20 km/h, i.e. under conditions of oversaturated (forced) traffic flow, shows the differences in regard to theoretical models, and also in regard to the model obtained from a Sugeno fuzzy logic system. It should be noted again that the ANFIS model was trained based on actual data obtained on German highways. The previously mentioned paper [24] is one of the most significant in the recent history of traffic flow theory studies. One of the conclusions which the author has come to in that paper, is that the sequence distance is not a function of speed, which explains the paradox shown in the above figure.

The curve shown in figure 10. was obtained by data clustering and is similar to the one shown in figure 1. Although the curve in figure 7., which is an output from the ANFIS model without clustering, has its specific properties, the speed-traffic flow density curve shown in figure 10. is adopted as an output result from the entire model. As can be seen, this model shows that the zone of saturated flow emerges for speeds of 40 km/h to 50 km/h, which are expected values regarding high level motorways for which this model gives an approximation.

CONCLUSION AND DIRECTIONS OF FUTURE RESEARCH

The approximation of a continuous, nonlinear function specified through an input/output ta set is a widely present problem. Because of the good results it showed, ANFIS is more often used as a neuro-fuzzy tool for the solution of the mentioned problem.

Using FIS on a system for which input/output data (which should be used for modeling) already exists, is realized by using ANFIS, because it is not always possible to see how association functions should look like, just by simple observation of input/output data. The use of a neuro-adaptive learning technique secures that the selected association functions and their parameters best fit the input/output data.

Although it is a less known and not very often used technique, neuro-fuzzy conclusion may also be used for forecasting, for basic approach and/or for checking results obtained by standard forecasting methods.

Applying ANFIS to the basic traffic flow diagram has yielded the model shown in this paper and it can be used for approximation of traffic flow density, being the parameter which is most difficult to determine. The interdependence was obtained using data which was collected on German highways, so that this model is best to use for the approximation of density on highest level motorways.

The logic used in this paper can also be used on the q-V traffic flow diagram, in which case flow would be an input value. Of course it would be necessary to also train the model on data from low category motorways or urban areas, and compare the differences that this model is able to offer. Especially interesting for further research would be to study the behavior of the basic traffic flow diagram in case of bottlenecks or objects on the road (tunnels, bridges, etc.) when the model proposed in this paper is applied to these cases.

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