

π -THE TRANSCENDENTAL NUMBER

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ABSTRACT. In this paper π is expressed in a new way, rather than, a series converging to get the exact value. This formula relates the mathematical constant π with trigonometric tan.

1. Introduction

The number π is a mathematical constant, the ratio of a circle's circumference to its diameter. It is represented by the Greek letter π since the mid-*XVIII*th century. It was calculated to seven digits using geometrical techniques in Chinese mathematics and to about five in Indian mathematics in the fifth century AD. π has a history of more than 1800 years. Hundreds of mathematicians had discovered many formulas to calculate the value of π . The historically first exact formula for π is based on infinite series, was not available until a millennium later, when in the 14th century the Madhava - Leibniz series was discovered in Indian mathematics [1]. Few years later, mathematicians and computer scientists discovered new approaches that, when combined with increasing computational power, extended the decimal representation of π to many trillions of digits after the decimal point [2].

It is an irrational number, so π cannot be expressed exactly as a fraction and it is also a transcendental numbers- A number that is not the root of any non-zero polynomial having rational coefficients. Its decimal representation never ends. Fractions such as $22/7$ are commonly used to approximate. The digit sequence of π is conjectured to satisfy a specific kind of statistical randomness, but to date no proof for this has been discovered. The ubiquity of π makes it one of the most widely known mathematical constants both inside and outside the scientific community Due to its relation with the circle π is found in many formulae in trigonometry and geometry, and in ellipses and spheres. Because of its special role as an eigenvalue, π appears in areas of science and mathematics such as number theory and statistics. It is also found in cosmology, thermodynamics, mechanics and electromagnetism.

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2. Derivation for obtaining formula for π

Let us consider a circle of radius 1 unit (Figure 1).

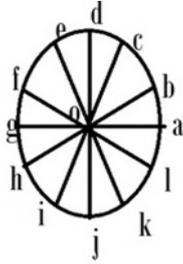


FIGURE 1

Divide the circle into 12 parts (Figure 2).

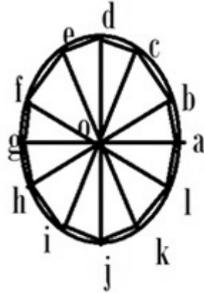


FIGURE 2

Consider triangle OAB (Figure 3).

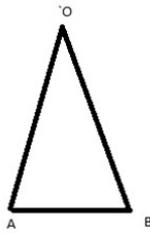


FIGURE 3

Draw a straight line from point O which touches the line AB named T (Figure 4).



FIGURE 4

Consider ΔAOT (Figure 5).



FIGURE 5

From ΔAOT ,

$$\tan \theta = \frac{AT}{OT}$$

The radius of the circle is 1. So

$$OT = 1. \quad \tan \theta = \frac{AT}{1}$$

.Here the circle is divided into 12 parts. The circle is divided into 12 triangles and each triangle is divided into 2 parts So totally 24 triangles. So

$$\Delta AOB = 30.$$

By which

$$\Delta AOT = 15. AT = \tan 15.$$

For all 24 parts

$$(2.1) \quad \text{Circumference} = 24 * \tan 15$$

Now, increase the number of triangles (Figure 6).

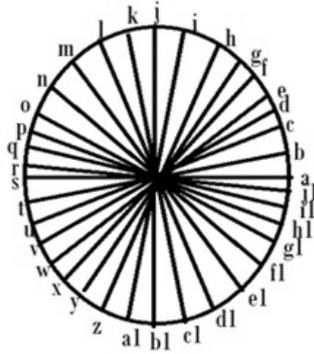


FIGURE 6

Now, the triangles looks like a circle. So increasing the number of triangles will increase the accuracy of circumference of circle.

Consider $\triangle AOB, \angle AOB = 10^\circ$.

From $\triangle AOT$ (Figure 7), $\tan 5^\circ = \frac{AT}{OT}$

Here $OT=1$.



FIGURE 7

$$AT = \tan 5^\circ$$

Now The circle is divided into 36 triangles and each triangle is divided into 2 parts totally 72 triangles.

$$(2.2) \quad \text{Circumference} = 72 * \tan 5^\circ$$

Now, split the circle into 72 triangles.

$$(2.3) \quad \text{Circumference} = 144 * \tan 2.5^\circ$$

From (2.1), (2.2) and (2.3) we can get that the product of angle and the number of triangles is equal to 360. So, if the circle is divided into 720 triangles, the angle is 0.5 degrees. *If*

$$\theta = 0.1, \text{No. of triangles} = 3600.$$

$$\text{If } \theta = 0.01 \quad N = 36000$$

Rewrite as

$$\theta = 10^{-1} \quad N = 360 * 10^1$$

$$\theta = 10^{-2} \quad N = 360 * 10^2$$

$$\theta = 10^{-3} \quad N = 360 * 10^3.$$

So

$$\theta = 10^{-x} \quad N = 360 * 10^x$$

The reason why we are decreasing the value of angle is , By increasing the no of triangles and decreasing the value of angle will increase the accuracy of circumference of circle.

$$\text{Circumference} = 2\pi r$$

$$\pi = \frac{C}{2r}$$

Here radius =1.

$$\pi = \frac{360 * \tan 10^{-x} * 10^x}{2}$$

$$\pi = 180 * \tan 10^{-x} * 10^x$$

$$X = 0, 1, 2, 3, 4, \dots\dots\dots$$

Increasing the value of x will increase the accuracy of π .

3. Comparison with original value of π

π	3.1415926535897932384626433832
X=1	3.14159 5843539840620602610260
X=2	3.1415926 85489255232394644222
X=3	3.141592653 908787854553979195
X=4	3.1415926535 92983184623171942
X=5	3.141592653589 825137924248630
X=6	3.141592653589793 5574572594
X=7	3.1415926535897932 41652589543
X=8	3.1415926535897932384 94542844
X=9	3.141592653589793238462 962377
X=10	3.14159265358979323846264 6573
X=11	3.141592653589793238462643 415
X=12	3.141592653589793238462643383 5
X=13	3.1415926535897932384626433832

FIGURE 8

4. Conclusion

π is expressed in a new way, rather than, a series converging to get the exact value. This formula relates the constant π with the trigonometric tan. In this formula there is tan function so we can differentiate it and use that in many areas. We can apply this formula in physics and in many areas where π is used. The results obtained from the formula is compared with the original value and found it to be correct.

References

- [1] G. E. Andrews, R. Askey and R. Roy *Special Functions* (p. 58), The University Press, Cambridge, ISBN 0-521-78988-5.
- [2] A. J. Yee and S. Kondo. 5 *Trillion Digits of π - New World Record*, Available on line at www.numberworld.org/misc-runs/pi-5t/details.html

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