A COMPARATIVE APPROACH TO ANALYZING UNIVERSITY ENTRANCE MATHEMATICS EXAMS

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Abstract

The authors analyzed results of entrance exams for University of Banja Luka in the Republic of Srpska, B&H. Twelve exams were administered from 2012 to 2015 and designed to assess the mathematical proficiency of candidates for admission to the Faculty of Mechanical Engineering. The authors conducted further examination of the 280 tasks using Bloom's [5] and MATH [28] taxonomies, and Thompson's multi-dimensional approach [4, 30]. The analysis is aimed at understanding the mathematical proficiencies of the tested candidates, as an almost-representative population of high school graduates in the Republic of Srpska. Currently, no data or means of obtaining data on the mathematical proficiency of high school graduates in the Republic of Srpska exists. Thus, the results are a critical first step towards conducting research that would provide this information. Such information has implications for secondary and post-secondary mathematics instruction in the Republic of Srpska.

Key words: [assessment, advanced mathematics knowledge, taxonomies, mathematical tasks]

ZDM Subject Classification: **B40, B70, D60**

1. Introduction

Every state in the U.S. has either adopted the Common Core State Standards for Mathematics or has aligned their curriculum so that their standards meet or exceed the standards [7]. On the contrary, in the Republic of Srpska (an entity of Bosnia & Herzegovina) mathematics standards do not exist.

Instead of informatively developing standardized resources and assessment tools, the Minister of Education in the Republic of Srpska has determined mathematics curricula for primary and secondary schools. The basis of the standards is not clearly communicated because these documents are only two to three pages. Furthermore, the guidelines are very modest, and the expectations for teaching are only briefly described. Thus, any evaluation of the

mathematical proficiency or the effectiveness of teaching for this population is challenging. University entrance exams are one of the only opportunities for evaluating and recording the state of mathematics education in this region. This paper reports results from university entrance exams and discusses how these results can be used to gain understanding of the mathematical proficiencies of students and the current state of mathematics education in the Republic of Srpska. The authors believe these data will provide insight about the effectiveness of teaching in the Republic of Srpska.

The aim of this research is to understand the different types and levels of mathematical knowledge that students demonstrate on the entrance exams. Before analyzing tasks with this goal in mind, what is being measured—mathematical proficiency—and how mathematical proficiency will be measured must be defined. The following review is a brief overview of research that supports answering these questions.

2. Theoretical Background

This study uses a multi-dimensional analysis synthesizing the *MATH* (*Mathematical Assessment Task Hierarchy*) taxonomy [28] and the *SPUR* (*skills, properties, uses,* and *representations*) framework [4, 29] to gain an understanding of the specific mathematical proficiencies of the tested population.

Mathematical tasks are defined as activities that focus students' attention on particular mathematical concepts, ideas, or skills, but the authors acknowledge that mathematical tasks are open to interpretation and thus evolve throughout the implementation process. Similarly, Henningsen and Stein [12] define mathematical tasks using a conceptual framework, which explicates the entire process of task implementation. They argue that tasks pass through three phases: as written by curriculum developers, as set up by the teacher in the classroom, and as taken up by students during the lesson. Furthermore, they describe tasks as having two dimensions: task features and cognitive demand.

On a related note, Kaur [14] focused on the nature and source of mathematical tasks, and specified that tasks evolve as they pass through phases of implementation. The authors adhere to Henningsen and Stein's [12] and Kaur's [14] conceptualizations of tasks, and argue that through a multi-dimensional analysis students' responses to assessment tasks can be investigated while taking into account the students' perceptions of the tasks, and thereby attending to the interpretive nature of tasks.

In the last sixty years, educators have created a variety of guidelines and frameworks for assessment, ranging from content specific guidelines (e.g., [19, 20]) to general cognitive levels (e.g., [5]). One of the most popular tools is Bloom's *Taxonomy of Educational Objectives: Cognitive Domain,* which is a hierarchical system, consisting of descriptive levels of cognitive processes. The six levels—knowledge, comprehension, application, analysis, synthesis and evaluation, range from concrete to abstract. The classification system is designed to evaluate students' knowledge in academic content areas. Educators have applied Bloom's taxonomy to a wide range of topics, including science [8], computer science [13], English language learning [22], music [11], and physical fitness [27] education. Studies [10] designed to apply Bloom's taxonomy in the context of mathematics have shown the framework does not support assessment writers in anticipating potential student responses. Specifically, the framework does not provide task designers with an "accurate model" of the cognitive processes that students will use when solving assessment tasks [10, p. 30].

Likewise, Thompson [29] studied mathematics teachers' use of Bloom's taxonomy and found that teachers struggle to translate Bloom's thinking skills to actual student practices and thus find the taxonomy difficult to use when creating assessment tasks. In response, Bloom's taxonomy was revised with a goal to incorporate practical approaches based on research in

learning psychology, instruction, and assessment [2, 6, 18]. The revised taxonomy has two dimensions, a *knowledge dimension* and a *cognitive process dimension,* designed to account for both domain knowledge and cognitive demand [2].

While some researchers critique Bloom's taxonomy for lacking in terms of applicability, others [28] describe the taxonomy as falling short when applied in a mathematics context in general. For example, Smith et al. [28] argue that Bloom's taxonomy is actually productive for structuring assessment tasks, but is limited when applied to mathematics assessments specifically. In response, they proposed further modification of Bloom's taxonomy, designing the *MATH* taxonomy, to aid in the structuring of assessment tasks, specific to mathematics. In the *MATH* taxonomy, the authors identified eight categories of mathematical knowledge and skills. For example, "reciting factual knowledge" or "justifying and interpreting" are categories of knowledge and skills in the *MATH* taxonomy. These examples are both actions because the authors organize the taxonomy by the type of activity involved in the task, rather than by the difficulty level of the task.

Researchers and educators have proposed several frameworks to use when thinking about what it means to "understand" mathematics. For example, Kilpatrick, Swafford, and Findell [15] suggested five interrelated strands of mathematical proficiency: *conceptual understanding, procedural fluency, strategic competence, adaptive reasoning,* and *productive disposition.* These frameworks have the potential to help us identify various components of mathematical proficiency.

The work of Thompson and Kaur [29] and Bleiler and Thompson [4] is another example of a framework designed to assess the strands of mathematical proficiency. Thompson, Kaur and Bleiler adapted a model originally used for curriculum development (e.g., [31]), to assess students' mathematical understanding across four dimensions. The first dimension, *skills,* represents those procedures that students should master with fluency; they range from selecting and comparing algorithms to discovering or inventing algorithms, and include procedures with technology. The second dimension, *properties,* is identifying the principles underlying the mathematics, ranging from the naming of properties used to justify conclusions to derivations and proofs. The third dimension, *uses,* is the application of concept to the real world or to other concepts in mathematics and range from routine "word problems" to the development and use of mathematical models. Lastly, the fourth dimension, *representations,* includes graphs, pictures, and other visual depictions of the concepts, including standard representations of concepts and relations to the discovery of new ways to represent concepts.

This multi-dimensional approach, known as *SPUR* provides teachers with useful information about the depth of their students' mathematical understanding. Furthermore, assessments encompassing all four dimensions will give teachers insight on the strengths and weaknesses of their students' conceptual knowledge and the robustness of students' current mathematical understanding and potential problem solving abilities. Thompson and Kaur [29] and Bleiler and Thompson [4] are examples that illustrate how to use *SPUR* to create a balanced assessment of students' conceptual understanding.

This work draws upon the frameworks described above to analyze students' assessments and develop standards based on the analysis. The authors view these frameworks as tools to support us in conducting a multi-dimensional analysis that will address questions about the mathematical proficiency of students in the Republic of Srpksa.

3. Methodology

At the beginning of each academic year from 2012 to 2015, applicants for admission to the University of Banja Luka are assessed on their mathematical proficiency (e.g., [9, 17, 21, 23, 24, 25, 26]). Entrance exams contained 10 items. All items were short answer open response

questions, and students were given 120 minutes to solve them. The tasks that appeared on the exams were categorized into the following topics: Arithmetic/Early Algebra (30%), Set-Theoretic (5%), Logic (7.5%), Algebra (42.5%), and Geometry and Analytical Geometry (15%). The percent indicates the frequency of that topic.

In the analysis, the authors designed and/or selected about 280 tasks to test the mathematical literacy of over 1,300 candidates. Some tasks were designed for the assessment, whereas other tasks were modifications of tasks used in prior research. For example, the second task (Figure 1) is similar to those used by Zener and Duncker in Germany in the 1920's on an assessment of Einstellung effect [1].

The candidates were applying for admission to the Faculty of Mechanical Engineering at Banja Luka University between 2012 and 2015. In analyzing the tasks, the authors examined and assessed over 10,000 solutions, provided by the tested candidates, to these tasks to answer the following research questions: (1) What can be learned about the tested population by analyzing their performance on individual tasks? Specifically, in what topics (e.g., Arithmetic/Early Algebra, Set-Theoretic, Logic, Algebra, Geometry and Analytical Geometry) is the tested population proficient? (2) How proficient is the tested population in these topics? (3) In what ways does the success of the tested population in one topic vary or differ across tasks? (4) How do the tasks on which students demonstrate variation differ from the tasks on which students perform consistently**?**

Tasks were evaluated using both qualitative and quantitative analysis. With an aim to mutually compare tasks, the authors organized student responses according to the level of complexity of sophistication and coded them using an ordered discrete set ({∅, 0, 1, 2, 3, 4, 5}). Here, the code ∅ means that the tested candidate did not respond to the task. Code 0 means that the response provided to a given task was unacceptable. Responses that were not coded as ∅ or 0, were sorted into the five remaining categories according to their completeness.

4. Findings

In this section, findings are presented and used to answer the research questions. As previously stated, the entrance exams consisted of five topics. Table 1 shows the results of students' success in each of the five topics. Note % is computed as (number of questions or part questions in the topic)/(total number of questions or part questions in the assessment)·100 for Table 1.

Table 1

Candidates' success in Arithmetic /Early Algebraic tasks								
Success	∅	**0**	**1**	**2**	**3**	**4**	**5**	
Percent (%)	20.67	27.53	4.5	5.0	2.25	3.25	36.8	**100.0**
		48.2					**40.05**	
Candidates' success in Algebraic tasks								
Success	∅	**0**	**1**	**2**	**3**	**4**	**5**	
Percent (%)	21.17	22.98	8.32	14	5.97	7.9	19.66	**100.0**
		44.15				**27.56**		
Candidates' success in Set-Theory tasks								
Success	∅	**0**	**1**	**2**	**3**	**4**	**5**	
Percent (%)	31.73	33.73	7.63	4.42	1.2	2.1	19.28	**100.0**
		65.46				**21.38**		
Candidates' success in Geometric tasks								
Success	∅	**0**	**1**	**2**	**3**	**4**	**5**	

Percent (%)	34.95	23.12	19.92	12.01	3.58	3.94	4.48	**100.0**
		58.07					**8.42**	
Candidates' success in Logical tasks								
Success	∅	**0**	**1**	**2**	**3**	**4**	**5**	
Percent (%)	59.59	26.83	0.0	0.0	13.58	0.0	0.0	**100.0**
		86.42					**0.0**	

In addition to the candidates' overall success in each topic, tasks from each topic were selected to provide an example of the tasks that determine the candidates' success in these topics. Through content analysis, the authors hypothesize that by identifying which content strands each task addresses, tasks can be compared within and across topics. In turn, candidates' strengths and weakness can be compared according to content strands, and the authors are able to glean a specific understanding of their mathematical abilities. For instance, the authors analyzed 5 arithmetic/early algebra tasks and 7 algebra tasks. On which tasks did students' performance vary significantly? And, what are the differences between these tasks? Figure 1 shows two arithmetic/early algebra tasks. On the first task 72.86% of students received a score of 5 (complete and correct), whereas on the second task only 17.42% of students received a score of 5 (complete and correct). These two tasks were chosen because they are both categorized as arithmetic/early algebra, and, compared to other assessment tasks, these tasks are considered the most accessible. In spite of any assumptions, the candidates' success on these two tasks varied significantly. Thus, the authors wonder how other tools for analyzing assessments can alter one's perspective and offer insight about students' mathematical proficiency.

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(1) A river divides into two branches. The first branch has 1/3 of the water. The remaining water flows into the second branch, which is divided into two. One of the two branches has 3/4 of the water, and the remaining water flows into the other branch. What part of the whole river flows through each of the branches?

(2) If you have a 10 liter and a 3 liter container, measure exactly 8 liters of liquid.

Figure 1

5. Conclusion

This research aims to define mathematical proficiency according to the tested candidates, whom are graduates of high school in the Republic of Srpska. Developing an understanding of the mathematical proficiency of the tested population gave insight about the state of mathematics education in the Republic of Srpska. Moving forward the authors ask, how can existing frameworks help to further analyze this data with an aim to gain understanding about the mathematical proficiencies of high school students in the Republic of Srpska? What does it mean for students to be proficient in mathematics? How can educators informatively assess mathematical proficiency? Lastly, how can teachers, policy makers, and curriculum developers use this knowledge to inform instruction?

Of course, this approach provides partial and incomplete data. Despite the shortcomings, analysis of this assessment offers insight into the difficulties that incoming students might encounter at the university, given their prior mathematical skills and abilities. Furthermore, this information serves as an helpful resource for university mathematics teachers, as they are able to design an approach to teaching mathematics that might result in higher student performance.

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