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DIGITAL MODELING OF EARTH'S SURFACE RELIEF

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Abstract:

This paper presents the theoretical basis for the development of the Earth's surface relief digital models. The sources and structure of the modeling data, as well as the methods of their processing, are described, depending on the spatial-temporal properties of the Earth's surface. Digital relief models have great significance and wide application in modern geodetic works. Representation of the spatial structure of the relief can be done with different methods of interpolation. In this paper their mathematical foundations are also given. General indicators of the quality of digital relief models are briefly described.

Keywords: digital relief models, Earth's physical surface, interpolation.

ДИГИТАЛНО МОДЕЛОВАЊЕ РЕЉЕФА ПОВРШИ ЗЕМЉЕ

Резиме:

У раду су представљене теоријске основе израде дигиталних модела рељефа површи Земље. Описани су извори и структура података моделовања као и поступци њихове обраде, зависно од просторно-временских својстава површи Земље. Дигитални модели рељефа имају велики значај и широку примјену у савременим геодетским радовима. Приказивање просторне структуре рељефа може се вршити различитим методама интерполације. Њихове математичке основе такође су дате у овом раду. Кратко су описани општи показатељи квалитета дигиталних модела рељефа.

Кључне ријечи: дигитални модели рељефа, физичка површ Земље, интерполација.

1. INTRODUCTION

The relief of the Earth's crust is defined as the deviation of the physical surfaces of the Earth from the geoid, the level surface of the potential of the Earth's gravity acceleration, which can be approximated by the mean sea level [1]. Digital Relief Models (DRM) of Earth's physical surface are sets of digital data, about the spatial structure of the Earth's surface, expressed in vector or raster form. They describe the shape and metric properties of elements of the Earth's physical surface, by parts or as a whole. They represent elementary geospatial surfaces, or their union - sets of material points of different densities, positions, orientations and mutual relations. Often, as synonyms, the terms digital terrain model (DTM) and digital elevation model (DEM) are used, although there is a difference between them [2]. Digital elevation model describes the heights in general, not only of the terrain, but also of buildings and other artificial objects, whereas digital terrain model gives only the heights of the terrain itself.

Digital modeling of the relief includes the collection, processing, presentation, application and research of the accuracy of digital data on the physical surface of the Earth. Data on the Earth's physical surface can be obtained using various techniques and technologies, such as topographic survey, photogrammetry, laser or radar detection and ranging, etc.

Global, regional and national DRMs, as subsystems of the geographic information system, are of a special scientific and practical importance. In geodetic theory and practice, they are especially important data, because they are used for geodetic measurements planning, geodetic networks designing, for calculation of the gravitational influence of topographic masses in the process of geoid determination, for creation and improvement of various geodetic databases, during process of modeling and simulating spatial phenomena, etc. In addition, DRMs are also used in other related fields, e.g. during the design process of line (roads, railways, power lines, oil pipelines, etc.) and other infrastructure facilities (bridges, dams), for calculating the amount of groundworks, then for military purposes (artillery, aviation), etc. It can be said that for each country, relief and heights data, which are sufficiently accurate, with good geographical distribution and spatial resolution, are of key importance for design and construction of capital infrastructure facilities.

2. DATA FOR CREATION OF MODEL'S STRUCTURE

The ability to create DRM is decisively influenced by the sources, the collection, and structure of the relief data. In principle, DRMs are made by using five basic data sources: a) geodetic topographic survey; b) digitized cartographic material; c) photogrammetry; d) LiDAR (Light Detection And Ranging) and e) InSAR (Interferometric Synthetic-Aperture Radar) technology.

Photogrammetry for the purposes of making DRM involves the use of an aerial photographic camera, mounted on a hull of aircraft [3]. After processing, the obtained photo material is used to collect data on objects and phenomena on the physical surface of the Earth. InSAR is an advanced satellite method for DRM production, which uses a radar to measure the phase difference between two recordings. In this way, a sample (interferogram) is obtained, which contains data on the mutual relationship between geometric structures of the images, based on the distance between radar antennas and Earth's surface. LiDAR uses laser beams for measurement, and for the production of DRM, recordings from the airplane are applied. Among other things, combined with the technology of global navigation satellite systems, it enables the formation of precise DRM in a geocentric coordinate system. Compared to other methods, the entire process is highly automated, making LiDAR currently the most preferred source for DRM data.

The classical geodetic survey for the needs of DRM production is not economically justified, due to the unfavorable ratio between invested time and funds, and the achieved results (in this case, the amount of collected data). However, areas which for some reason are inaccessible to photogrammetric or LIDAR imaging must be measured by conventional methods. Also, the coordinates of photogrammetric control points are determined by geodetic methods.

Digitization of cartographic material, compared to other methods, can be considered as a secondary data source, along with the processing of already existing digital relief data. Basically, on the digitized map, the contour lines (isohypse) are selected, and height values are added to their corresponding pixels [4]. Other sources, such as e.g. different databases of altitudes, altimetric data, etc, represent significant supplemental datasets since they enable complete and accurate expression of spatial-structural properties of the physical surfaces of the Earth. Particularly important are leveling data, used to form the basis of DRM and significant improvment of the basic data structure properties.

3. MODEL STRUCTURE AND ORGANIZATION

In general, points forming a discrete terrain representation are not regularly arranged and can be arbitrarily selected. Therefore, the basis of DRM is usually made up of randomly distributed data, regardless of the conditions of collection, and that is essentially the statistical sampling principle. The density of sampled points can be increased in areas where terrain forms are more variable and more rugged, and especially in the vicinity of characteristic fracture lines, where the surface of the terrain changes orientation (steep slopes, bays). These data are then complemented, edited, gradually modeled, and in this way, the final set of digital relief data is created. The original data are gradually structured through the preparation and harmonization of data from different sources, interpolation, and approximation of lines and elementary surfaces of relief, and forming of the base (reference structure) of DRM. According to the distribution of the base reference points, DRMs are structured (as parts or as whole) as:

- Triangulated Irregular Network - TIN;
- Models of equidistant points, i.e. grid DRM;
- The reference points of the model represent both the vertices of scalene triangles and equidistant points (the so-called universal or TIN-grid DRM).

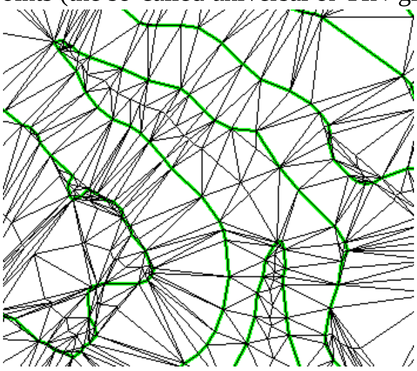

Figure 1. TIN DRM

TIN DRM (Figure 1) are sets of digitized vertices of scalene planar triangular surfaces. They are formed by choosing the characteristic relief points, whose properties are expressed by elementary surfaces, linear and point elements. In this way, it is possible to capture the most important morphometric features of the selected area. Problems can arise in determining the height of mountain pass points, steep sections (canyons, gorge), boundary areas, etc. TIN DRM models are applied for: geoinformation systems (GIS), cartography, geomorphology, geology, geophysics, the formation of reference structures of the grid and universal DRMs, etc. Although handling data is sometimes difficult, this form of structuring allows for better representation of the shape of the relief, with a smaller number of points. Also, if the DRM resolution is variable, TIN DRM provides better results than other methods, because of the size of the triangles changes with the resolution.

Grid DRM (Figure 2) is a matrix of height values of points located at equal distances. Each row or column of a matrix is a special digital record of the material points heights.

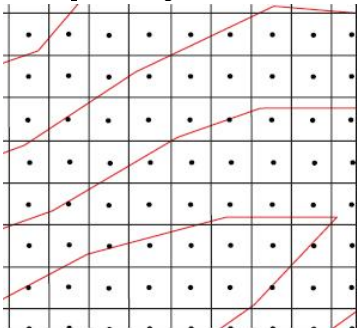

Figure 2. Grid DRM

Grid DRMs are formed directly by sampling data by coordinate lines of the adopted parametrization of the relief, or indirectly, by transforming TIN or universal DRMs. They are well-adapted to the way computers work, so processing large amounts of data is facilitated. The problem of the concurrence of the DRM content and the spatial-structural properties of the relief is solved exclusively by the appropriate density of equidistant points, depending on the morphometry and the required accuracy of the DRM functionals.

Universal DRMs represent sets of points given as elements of rectangular matrices or vertices of scalene triangles. They are reliably expressing the spatial-structural properties of relief, such as elementary surfaces, the unions of elementary surfaces, curves and straight lines, bending points, etc. They are made by mixed data sampling or by matching grid and TIN DRM content. Also in these models, there is a problem of singular points, but it can be exceeded by increasing the density and changing the distribution of data, with the appropriate software support.

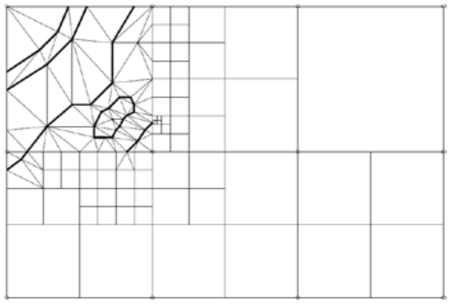

Figure 3. Universal DRM

The interval ($\Delta x = \Delta y = const$) of equidistant points is, in principle, determined if the linear interpolation of heights can be applied to the reference points of the model. If they comprise the necessary area and structural points of the physical surface of the Earth, the TIN-grid DRMs represent the most acceptable and useful digital data sets on relief of the Earth's surface.

4. MATHEMATICAL TREATMENT AND DESIGNING OF DIGITAL RELIEF MODELS

The design process, depending on the scope, purpose, and application of the DRM, can conditionally be divided into "related intervals", but nevertheless significantly different according to content, methods of data interpolation, an approximation of lines and elementary relief surfaces, etc. In the first part of the process, the reference/basic allocation (grid, TIN, TIN-grid) and density of data are formed, based on various and harmonized original data. The main and final part includes a) processing of source data using appropriate software; b) the formation of DRM, until the necessary data sets are created. Then subdivision into subgroups can be performed, along with statistical quality indicators calculation, model adaptation, etc.

In general, using the interpolation and approximation procedure, it is necessary to preserve the quality and accuracy of the original data and to achieve the required DRM quality. The interpolation of lines of the Earth's physical surface is most commonly performed by applying [5]:

- cubic polynomials and spline functions of the third degree;
- linear polynomials;
- Fourier's functional series;
- method of the least squares;
- least squares collocation, etc.

In the approximation of the elementary surfaces of the relief, regardless of the adopted parametrization, the methods of interpolation usually are:

- bilinear;
- bicubic polynomials;
- bicubic spline functions;
- finite element method;
- 2D covariance functions, etc.

4.1. APPROXIMATION OF RELIEF SPATIAL STRUCTURE LINES

The approximation of the relief lines is mainly based on interpolations of continuous functions by polynomials and functional series. Interpolation polynomials, regarding the calculation method, are suitable for analytic expressions of numerical methods for determining the approximate values of continuous functions. The approximations of the continuous lines of the spatial relief structure in DRM construction are mostly based on interpolations of cubic and spline polynomials, and Fourier's trigonometric series. The functions below will be denoted by $H(x)$ since heights are to be determined.

4.1.1. Interpolation using cubic polynomials

If a function $f(x)=H(x)$ has n derivations in point $x_0 \in [a,b]$, it can be approximated by Taylor (Taylor, B.) polynomial [6]:

$$T_n(H,x_0;x)=H(x_0)+\frac{H'(x_0)}{1!}(x-x_0)+\frac{H''(x_0)}{2!}(x-x_0)^2 + \frac{H'''(x_0)}{3!}(x-x_0)^3+\ldots \tag{1}$$

Continuous function $H(x)$, up to $n+1$st derivation, gives in point $x_0 \in [0,L]$ polynomial:

$$H(x)=M_n(x)+R_n(x). \tag{2}$$

The approximation error ε, of function $H(x)$, by Taylor polynomial $T_3(x)$, depends on the degree of the polynomial and the regularity class of the function, and it follows that the cubic Taylor polynomial is:

$$\begin{aligned} &H(x)\approx T_3(x);\ \left|H(x)-T_3(x)\right|\le\varepsilon;\ n=3;\ x\in[0,L],\\ &H(x)\approx T_3(x)=a_0+a_1x+a_2x^2+a_3x^3, \end{aligned} \tag{3}$$

where:

- $a_n,\ n\in[0,3]$, coefficients of polynomials;
- $x_n,\ n\in[0,3]$, length value at an interval $x\in[0,L]$;
- $H(x)$, point height value at distance $x\in[0,L]$.

The polynomial coefficients are determined on the basis of known point heights and their mutual distances. Also, in the interpolation of the functions by Taylor polynomials, significant quantities are the degree of the polynomial n, the length of sampling L, the sampling interval Δx and the error of height approximation ε. They are evaluated during the process of development and application of DRM and may be the initial indicators for the selection and design of the appropriate DRM data structure.

4.1.2. Interpolation using a spline function

The essence of the spline interpolation is that the interpolation polynomial of a higher degree is replaced with more polynomials of the lower degree. If the function $H(x,y)$ is given on an interval $y\in[0,L]$, which is divided by nodes of interpolation [6]:

$$[0, L]: 0 = y_0 < y_1 < y_2 < ... < y_m = L, \tag{4}$$

it can be approximated by the spline function $H_{sp}(y)$ of a different degree m. A set of polynomials $P_m(y)$ and functions $C^{(m)}[0, L]$, of real variables determined on an interval $y \in [0, L]$, is connected by spline polynomial $S_{sp}(y)$, with a defect $k(1 \le k \le m)$ and with nodes (4).

Spline function $S_3(y)$ is cubic interpolation function, of function $H_{sp}(y)$, on an interval $y \in [0, L]$, if on every segment $y_i \le y \le y_{i+1}$ has a value equal to polynomials $S_m(y) = H_{sp}(y)$, or if the function has corresponding third-degree spline polynomial:

$$\begin{aligned} S_3(y) \approx H_{[i,i+1]}(y) = a_{0[i,i+1]} + a_{1[i,i+1]}(y - y_i) \\ + a_{2[i,i+1]}(y - y_i)^2 + a_{3[i,i+1]}(y - y_i)^3. \end{aligned} \tag{5}$$

Coefficients a_m, $m \in [0, 3]$ for each individual segment, are determined on the basis of known height points. In addition, equivalences must be satisfied in all nodal points:

$$\begin{aligned} S'(y) = H'_{[i-1,i]}(y) = H'_{[i,i+1]}(y), \\ S''(y) = H''_{[i-1,i]}(y) = H''_{[i,i+1]}(y). \end{aligned} \tag{6}$$

4.1.3. Interpolation using Fourier trigonometric series

If function $H(x)$ satisfies the conditions of the Dirichlet theorem (Dirichlet, G. P.), then, in general, the corresponding Fourier series [5]:

$$H(x) = \frac{a_0}{2} + \sum_1^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right), \; x \in [-L, L], \tag{7}$$

has coefficients:

$$\begin{aligned} a_0 &= \frac{1}{L} \int_{-L}^{L} H(x) dx, \\ a_n &= \frac{1}{L} \int_{-L}^{L} H(x) \cos \frac{n\pi x}{L} dx, \\ b_n &= \frac{1}{L} \int_{-L}^{L} H(x) \sin \frac{n\pi x}{L} dx. \end{aligned} \tag{8}$$

For the finite values and wavelengths l_k, the frequency is expressed as the reciprocal value of the wavelength $f_k = \frac{1}{l_k} = \frac{k}{l}$, $k \in [1, m]$. Thereby, the amplitude A_1 has a corresponding frequency $1/L$, the amplitude A_k has a frequency k/L, etc. Fourier series, limited by the finite value $k \in N$:

$$H(x)=\sum_{1}^{m}\left(a_k\cos\frac{2\pi kx}{L}+b_k\sin\frac{2\pi kx}{L}\right),\ a_k=b_k=0,\ \text{за } f_k>f_{\max},$$
$$\mathrm{H}(x)=\sum_{1}^{m}A_k\cos\frac{2\pi kx}{L},\ A_k=0,\ f_k>f_{\max},\tag{9}$$

enables the representation of spatial-structural properties of the Earth's surface relief. Coefficients a_k, b_k and $A_k^2=a_k^2+b_k^2$ are determined from point heights, during sampling process or through improvements of DRM content with heights of specified points, using, for example, geometric leveling. Because of its specific application, Fourier series are suitable for interpolation of relief lines with diverse morphometry (steep slopes, canyons), but wavelengths must be carefully selected, due to possible mismatch with the spatial-structural properties of the relief.

4.2. APPROXIMATION OF SURFACES OF RELIEF'S SPATIAL STRUCTURE

The physical surface of the Earth is complex and time-varying, due to enduring endogenous and exogenous geodynamic phenomena, processes and forces. Mathematically, it can not be fully expressed, so the approximation of the elementary surfaces of the relief by analytic functions is realized by interpolating the original data in the process of forming the DRM [7].

4.2.1. Interpolation with polynomials

The study of the Earth's surface relief spatial structure can be performed by interpolation using 2D cubic polynomials [5]. Continuous function $H(x,y)$, with continuous partial derivatives of degree $n+1$, can be expanded near point P_0 into functional series:

$$H(x,y)\approx P_n(P)=H(P_0)+\frac{H'(P_0)}{1!}+\frac{H''(P_0)}{2!}+\ldots+\frac{d^nH(P_0)}{n!}+R_n,\tag{10}$$

where R_n is series residual. Cubic 2D polynomial is an approximation function:

$$\begin{aligned}H(x,y)&=a_0+a_1x+a_2y+a_3x^2+a_4y^2+a_5xy\\&+a_6x^3+a_7y^3+a_8x^2y+a_9xy^2,\end{aligned}\tag{11}$$

the coefficients of which a_n are determined on the basis of at least ten known heights of the material points. Interpolation of the point heights is done within the basic DRM field or using the known height of points of arbitrary distribution.

In the interpolation of point heights and the approximation of the physical surfaces of the Earth, the significant application also has:

- bisquare polynomials $H(x,y)=a_0+a_1x+a_2y+a_3x^2+a_4y^2+a_5xy$;
- bilinear polynomials $H(x,y)=a_0+a_1x+a_2y+a_3xy$;
- linear polynomials $H(x,y)=a_0+a_1x+a_2y$.

4.2.2. Interpolation using finite element method

The basis of interpolation with the finite elements is the division of the physical surfaces of the Earth into the elemental surfaces, or the finite quadruple and triangular surfaces [6]. In the network of i points, $k = i - 2$ triangles can be formed, for which three scale factors are determined, and that makes $3i - 6$ unknown variables. A function that approximates the triangular surface:

$$H\left(x,y\right)=\sum_{1}^{3}s_i d\left(PP_i\right);\; H\left(x,y\right)=\sum_{1}^{3}s_i\sqrt{\left(x-x_i\right)^2+\left(y-y_i\right)^2}, \qquad (12)$$

in the area $\left(x,y\right)\in D\subset R^2$, has joint distances for two triangles, from which follows a condition $d\left(PP_i\right)=0$.

The accuracy of the approximation depends on the spatial structure, density and accuracy of the original data, which can be improved by applying the heights determined by more precise methods, e.g. geometric or GNSS/geometric leveling.

4.2.3. The spherical harmonic relief model

The Earth's global relief, derived from the sums of the mean values of altitudes of restricted areas, can be represented by spherical functions [5]:

$$Y_n\left(\bar{\varphi},\lambda\right)=a_{n0}P_n\left(\sin\bar{\varphi}\right)+\sum_{m=1}^{n}\left(\bar{a}_{nm}\cos m\lambda+\bar{b}_{nm}\sin m\lambda\right)\bar{P}_{nm}\left(\sin\bar{\varphi}\right), \qquad (13)$$

applying spherical geocentric coordinates $\bar{\varphi}$ and λ. Orthonormal spherical harmonic coefficients can be expressed by the unit sphere:

$$\begin{Bmatrix}\bar{a}_{nm}\\ \bar{b}_{nm}\end{Bmatrix}=\frac{1}{4\pi}\iint_{\sigma}Y_n\left(\bar{\varphi},\lambda\right)\begin{Bmatrix}\cos m\lambda\\ \sin m\lambda\end{Bmatrix}\bar{P}_{nm}\left(\sin\bar{\varphi}\right)d\sigma, \qquad (14)$$

where $d\sigma=\cos\bar{\varphi}d\bar{\varphi}d\lambda$ is an element of a sphere σ, with radius $R=1$.

In the spherical approximation of the Earth, the physical surface $S\left(R+H\left(\bar{\varphi},\lambda\right),\bar{\varphi},\lambda\right)$ can be expressed as a spherical harmonic function:

$$H_{nm}\left(\bar{\varphi},\lambda\right)=R\sum_{n=0}^{N}\sum_{m=0}^{n}\left(\bar{g}_{nm}\cos m\lambda+\bar{h}_{nm}\sin m\lambda\right)\bar{P}_{nm}\left(\sin\bar{\varphi}\right), \qquad (15)$$

which represents the spherical harmonic expansion of the Earth's surface relief, in whole, or part by part. For the relief of the Earth's continental surface, the orthonormal spherical harmonic coefficients are calculated using the expression:

$$\begin{Bmatrix}\bar{g}_{nm}\\ \bar{h}_{nm}\end{Bmatrix}=\frac{1}{4\pi R}\iint_{\sigma_1}\bar{H}\left(\bar{\varphi},\lambda\right)\begin{Bmatrix}\cos m\lambda\\ \sin m\lambda\end{Bmatrix}\bar{P}_{nm}\left(\sin\bar{\varphi}\right)d\sigma_1, \qquad (16)$$

where σ_1 is a spherical approximation of the continental part of the surface.

The choice of methods of interpolation and approximation of lines and surface elements of the Earth's relief is conditioned by the requirements of application and accuracy of the model. In principle, the formation of the reference structure and the formation of DRM is done using more accurate interpolation procedures (e.g. spline functions), while during

DRM utilization, linear interpolation methods are used. In the formation of DRM reference structure, interpolation and approximation must ensure the preservation of the original data accuracy.

5. INTERPRETATION OF DIGITAL RELIEF MODELS

Since digital relief models are sets of data on the relationships between spatial and structural properties and relief functionals, these data can be studied by analyzing graphical representations (visualization) and by statistical analysis of data sets on the relief or functionals of the DRM application. Interpretation of DRM, for application in geodetic theory and practice, is expressed by indicators [5]:

- completeness;
- reliability;
- susceptibility;
- economic aspect;
- the accuracy of DRM.

The first four elements of DRM interpretation can be considered as general quality indicators, and accuracy estimators express the most significant properties of DRM, indirectly and directly determined by general indicators of the qualitative specificities of digital sets of data on the Earth's relief.

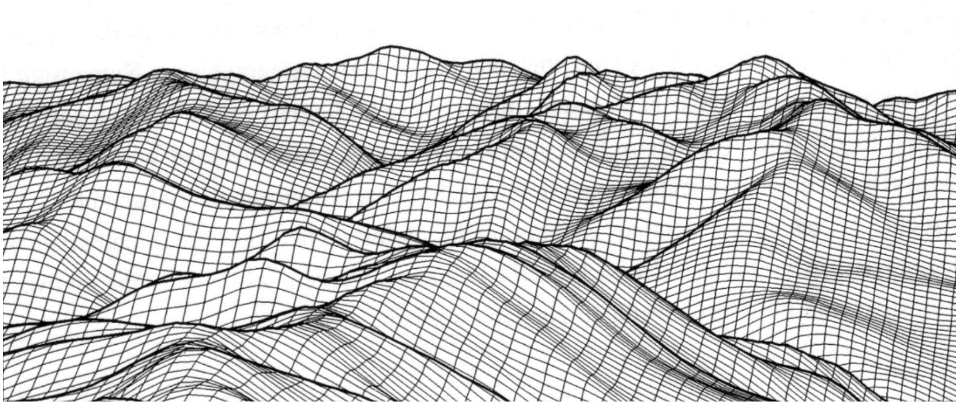

Figure 4. An example of grid DRM

5.1. GENERAL INDICATORS OF DIGITAL RELIEF MODELS QUALITY

The completeness of the DRM is expressed through the level of coverage of the spatial structure of the relief: structural points, lines and surface elements, average angles of inclination and wavelengths of relief variability in horizontal directions. With the appropriate data sampling, interpolation method and approximation functions, the necessary data on the morphological properties of the spatial-temporal structure of the relief are successfully obtained. By statistical analysis, visualization and comparison of source data and created DRMs, the indicators of DRMs completeness are obtained.

The reliability of DRM is conditioned by data distribution and density, completeness, accuracy, geodetic reference system, DRMs application, etc. Reliability indicators are: a) appropriate geodetic reference system, in particular, the height system; b) the ability to access data and to apply DRM; c) compatibility of data distribution and density, and DRM structure with available software and intended application; d) the possibility of calculating small but statistically significant values of DRM functionals; e) the applicability of the control procedure with more accurate data (e.g. precise leveling) during DRM creation and design [1].

The susceptibility of DRM is expressed by the possibilities of calculating statistically significant values of the relief spatial structure and functionals (inclination angles, relief masses, gravitational potential and isostatic compensation of relief masses, etc.). Especially important susceptibility indicator is the RTMs (Residual Terrain Model) influence, on the physical parameters of the local gravity field, or on the functionals of anomalistic gravity field, e.g. vertical deflections.

The economic aspect of DRMs production and application can be represented by functional models of the required time or cost for making DRM. The time model is:

$$T\left(DRM\right) = a_{DRM}^{t}, \tag{17}$$

where: $T\left(DRM\right)$ - the time period of DRM production; t - time scale; a - digital relief modeling parameter, $0 < a < 1$. The cost model of DRM production is as follows:

$$S = \sum_{1}^{n}\left(k_{0i} + W_i k_i\right), \tag{18}$$

where: S - total cost of production; k_{0i} - costs for the i-th part of the production process; W_i - the number of actions in the i-th part of the production process; k_i - the cost of i-th action of the production process; n - the total number of segments of the DRM production process.

The longest period of time and the highest costs of DRM production are in the procedures of collecting, forming the basis, estimating the quality of DRMs and their functionals. Improving economic aspect can be achieved through standardization of data distribution, density and geodetic datum, timely maintenance of data, application of appropriate mathematical models and software, production of multiuser purpose DRM and use of global, regional and local DRMs.

5.2. THE ACCURACY OF DIGITAL RELIEF MODELS

The accuracy of representation, in the process of digital modeling of Earth's physical surface, is the most important indicator of the qualitative properties of DRM. It is evaluated by mathematical analysis and experimentally, using sets of control points. From a theoretical and practical point of view, a significant advantage is given to statistical analysis [8]. The accuracy of the completed DRM depends significantly on the accuracy of the original data, sampling procedures, interpolation method, approximation function, intended DRM purposes, etc. Therefore, sources of DRM errors and the problem of DRM accuracy estimation, are a specific field of study.

6. CONCLUSION

Digital relief models of the Earth's physical surfaces are digital data sets that describe the metric properties of the spatial-structural relationships of Earth's physical surface elements. They have wide application in all fields of study of geodesy, but also in related scientific fields. The basic data sources for modeling are the classic geodetic survey, photogrammetric images, InSAR and LiDAR technologies and digitized cartographic

material. According to the reference points allocation structure, these digital models can be TIN, grid or universal models.
Presenting a relief with digital models requires the application of a mathematical apparatus that includes different methods of interpolation and approximation of lines and surfaces of the relief spatial structure. It is customary that the DRM reference structure is formed by applying more accurate interpolation procedures, such as spline functions, while linear interpolations are used for the actual application of the model. Interpretation of digital models is done by analyzing the accuracy and general quality indicators: completeness, reliability, susceptibility and economic aspect.
For the successful and economical solution of scientific and practical geodetic and other engineering tasks, as well as for production of geoinformation systems, an organized and institutionalized digital modeling of Earth's surface relief is recommended. It is also necessary to standardize the DRM in terms of geodetic reference systems, structure, data density, and distribution, in order to enable the production of digital relief models suitable for different uses.

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