EMIPRIJSKO ODREĐIVANJE OSNOVNOG PERIODA SLOBODNIH VIBRACIJA AB ZGRADA

Andrija Radovic, *andrijaradovic1@gmail.com*
University of Belgrade, Faculty of Civil Engineering
Predrag Mitkovic, *predrag.mitkovic@pr.ac.rs*
University of Pristina Faculty of Technical Sciences

Rezime:

Određivanje vrednosti osnovnog perioda vibracija armiranobetonskih zgrada predstavlja složen zadatak. Kako bi i u najranijoj fazi projektovanja imali određene podatke o osnovnom periodu vibracija inženjeri se služe raznim približnim metodama pomoću kojih se veoma brzo i jednostavno, ali ipak dovoljno tačno može izvršiti procena tražеnog parametra.

U radu su prikazani najčešće primenjivani empirijski izrazi koji opisuju zavisnost osnovne periode u funkciji broja spratova, ili visine objekta. Svi ovi izrazi uz veću ili manju tačnost daju procenu tražene veličine što je pokazano kroz veći broj primera. Izvršen je grafički prikaz svih izraza i njihovo upoređivanje sa eksperimentalno i numerički dobijenim vrednostima.

Ključne reči: emirijski izrazi, osnovni period vibriranja, AB zgrade

EMPIRICAL DETERMINATION OF THE FUNDAMENTAL PERIOD OF FREE VIBRATION OF RC BUILDINGS

Abstract:

Determining the value of the fundamental period of vibrations of reinforced concrete structures is a complex task. In order to have certain data on the fundamental period of vibration in the earliest design stage, engineers use various approximate methods, which can quickly and simply, but still sufficiently accurate, provide an estimate of the required parameter.

The paper presents the most frequently used empirical expressions describing the dependence of the fundamental period in the function of the number of storeys, or the height of the building. All of these expressions, with greater or lesser accuracy, give an estimate of the required value, as shown by a number of examples. A graphic representation of all expressions and their comparison with experimental and numerically derived values has been performed.

Keywords: empirical expressions, fundamental period of vibration, RC buildings

1. INTRODUCTION

For most buildings in seismic areas, earthquake represents the strongest load, which is also relevant for structural design. In order to perform the correct analysis, knowledge of the characteristics of earthquakes, but also the object itself, is necessary. Based on the research of seismologists, geologists and geomechanics, in practice designers receive data on potential earthquake characteristics at a given location. However, in the initial design phase, designers do not know the dynamic or any other characteristics of the building. Knowing these characteristics is of primary importance for assessing earthquake effects on buildings.

Since the first tone of oscillation is most commonly dominant in relation to others, it can be assumed that the total mass of the system oscillates only in this tone, and thus the seismic regulations of most countries are based on this tone. Determining the fundamental period of vibration of RC constructions is one of the most important steps in aseismic design and assessment of buildings' behaviour during the earthquake. In order to have certain data on the fundamental period of vibration at the earliest design stage, engineers use various approximate methods, and in recent years simplified empirical expressions are commonly used to quickly and simply, yet still sufficiently reliable, estimate the value of the fundamental period of vibration.

2. EMPIRICAL DETERMINATION OF THE FUNDAMENTAL PERIOD OF FREE VIBRATIONS

The value of the fundamental period of vibration depends on a large number of parameters, such as the choice of the constructive system, dimensions of the building in the base, the height of the building and the number of storeys, the foundation ground on which the object is founded, the nonconstructive elements, the mass of the building, the damping ... Since it depends on many parameters, the value of the fundamental period of vibration can be accurately estimated only after a more detailed numerical analysis has been performed. Despite great possibilities provided by modern software tools, experience has shown that even some numerical models may have some drawbacks, so the final values of the period of vibration can be determined only by experiments.

The great uncertainty that occurs during the determination of the period of free vibration, as well as the need to calculate this value as quickly and as easily as possible, has led researchers from around the world to explore in greater detail the possibilities for estimating this value, and many empirical expressions have been imposed as particularly suitable. Although they are very simple, they can generally include a large number of different buildings and, what is very important, give satisfactory solutions for practical application. By collecting a high volume of experimentally obtained data with numerous numerical studies performed, as well as by a correct interpretation of the results, many authors have proposed their own expressions based on which the values of the fundamental periods of vibration for different types of buildings are obtained. All these expressions give the estimate of the required value, more or less accurately. Some of these expressions also have found their place in the most up-to-date standards, such as the EC or ASCE. All expressions mostly connect parameters such as the constructive system, the height of the building and the number of storeys with the fundamental period of free vibrations. Using empirical expressions, designers can, at the earliest stage of the design, make a choice of certain input data, and then, based on them, perform an assessment of the dynamic

characteristics of the construction. It is recommended to use them for buildings up to 30 storeys or 60 (80) m in height. Some of the best-known expressions describing the relation between the number of storeys and the size of the fundamental period will be shown below. These terms generally present a linear dependence between these parameters and can be written in the following form:

$$T_1 = \alpha \cdot N \tag{1}$$

where:

T_1 is value of the fundamental period of free vibrations

α is coefficient of direction and it is determined experimentally

N is number of storeys

Kobayashi et al. (1987), based on experimental measurements performed on 20 RC moment resisting frame structures with 5 to 30 storeys, proposed the following expression for the area of Mexico:

$$T_1 = 0.105 \cdot N \tag{2}$$

Aničić (2002) proposes a very similar expression for RC moment resisting frame constructions:

$$T_1 = 0.1 \cdot N \tag{3}$$

Expressions proposed by Navarro et al. for the area of Spain and Portugal, which have been modified several times (2002, 2004, 2009), are also represented in practice. These expressions mainly refer to RC constructions with shear walls. It should be noted that practically identical expressions were proposed much earlier by many researchers such as Midorikawa (Chile, 1990), Kobayashi et al. (Spain, 2002), Enomoto et al. (Spain, 1999), Sanchez et al. (Spain, 2002) ...

Most of these expressions range within the following limits:

$$T_1 = 0.048 \div 0.05 \cdot N \tag{4}$$

When expressions that relate the height of the building and the size of the fundamental period are considered, they can generally be presented in the following form:

$$T_1 = C_t \cdot H^x \tag{5}$$

where:

C_t is coefficient that takes into account the type of constructive system

x is determined experimentally

Dunand et al. (2002), based on tests carried out on 26 RC constructions in France, proposed the following expression:

$$T_1 = 0.015 \cdot H \tag{6}$$

Later, the same expression was proposed by Satake et al. (Japan, 2003) by analysing a significantly larger number of buildings, 205 RC buildings with shear walls. Also, Galliopi et al. (Italy, 2009) suggest an identical expression.

After the publication of Eurocodes 8 (EC8), as a comprehensive and one of the most up-to-date regulations in this field, the terms proposed by this Code gain significance. According to the EC, the value of the fundamental period of vibration can be roughly determined using the following expression:

$$T_1 = C_t \cdot H^{3/4} \tag{7}$$

where:

$C_t = 0.075$ for moment resisting space RC frames

$C_t = 0.050$ for " other " constructions (RC constructions with shear walls)

For RC constructions with shear walls, and if there is more input data available, EC alternatively proposes that the aforementioned coefficient can be determined as follows:

$$C_t = \frac{0.075}{\sqrt{A_c}} \tag{8}$$

$$A_c = \sum \left[A_i \left(0.2 + \left(\frac{l_{wi}}{H} \right)^2 \right) \right] \tag{9}$$

where:

A_c is the total effective area of the shear walls in the first storey of the building in m^2

A_i is the effective cross-sectional area of the shear wall "i" in the observed direction in the first storey in m^2

l_{wi} is the length of the shear wall "i" in the first storey in the direction of the forces expressed in $m \left(\frac{l_{wi}}{H} \leq 0.9 \right)$

Since this term introduces the largest number of parameters affecting the value of the fundamental period of vibration, it can be expected that its application will give the most approximate and, in practice, the most useful results. Due to all this, additional attention should be given to it. In order to simplify the calculation, diagrams (Figure 7) have been constructed, from which the values of the fundamental period can be very simply read.

The last expression considered is proposed by the American Society of Civil Engineers (ASCE) and can be written in the form (5), where:

$C_t = 0.047$, $x = 0.90$ for moment resisting space RC frames

$C_t = 0.049$, $x = 0.75$ for RC structures with shear walls

Below is a graphic interpretation of all previous expressions. This method of interpretation is very suitable for comparing the above expressions, and, on the other hand, it can also be used to graphically determine the fundamental period of free vibrations.

In the expressions proposed in the EC and ASCE, two values of storey height were adopted, representing the largest number of common buildings, h = 2.8 and h = 3.3 m.

Figure 1 shows the difference between the formulas proposed for moment resisting frame structures and those proposed for structures with shear walls. The difference between the formulas that include the same type of constructive system is also evident, which is a consequence of the size of the statistical sample, as well as many other specific characteristics of the location and building.

The largest difference obtained by using empirical expressions for constructions with shear walls moves within the boundaries from 0.08s for single-storey building and

gradually increases to 0.15s for buildings up to 30 storeys. With moment resisting frame constructions, this difference is much higher and ranges from 0.08s, but increases with the increase of the number of storeys even up to a maximum value of 1.16s, which suggests a significantly greater uncertainty in the use of these expressions.
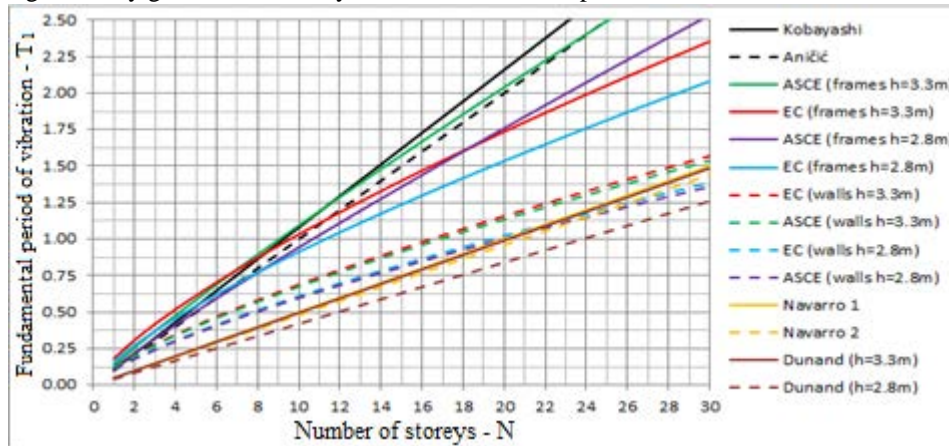

Figure 1. Dependence of the fundamental period of vibration from the number of storeys

3. VERIFICATION OF RESULTS BY NUMERICAL AND EXPERIMENTAL ANALYSIS

This section presents two selected models of buildings with shear walls that were analysed using different softwares. In this way, it is possible to compare the experimental and numerical results with the results obtained using the empirical expressions. In addition, an additional comparison of the experimentally and empirically obtained values for another seven tested buildings was performed.

The first example is a seven-storey building with shear RC walls that was examined on a seismic platform in Japan [7]. The dimensions of the building in the basis are 17x16 m, and the total height is 21.75 m.

The results of the first four experiments are presented:

- No damage - elastic behavior $T_{1,1} = 0.43s$,

- The appearance of flexural cracks $T_{1,2} = 0.55s$,

- Expansion of existing and appearance of new cracks $T_{1,3} = 1.15s$,

- The construction retains integrity with the appearance of major damage $T_{1,4} = 1.36s$.

Numerical analysis performed in the Tower software, which is the most prevalent in our area and is most used in design practice. Four models of different characteristics were analysed, which included the most common approximations that are carried out in practice.

- The first model - the closest to a real construction,
- The second model - increased soil rigidity in modal analysis,

579

- The third model - introduction of effects of cracks by reducing the shear and flexural stiffness of beams and columns by 50%, according to the requirements of EC8,
- The fourth model - a significant reduction in the stiffness of all elements in the modal analysis

In all models, the variable value of stiffness of the soil was adopted, and the effect of changing this parameter on the results of the modal analysis was analysed.
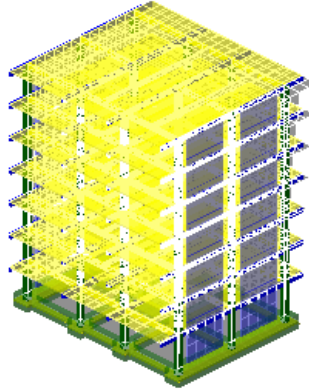

Figure 2. Numerical 3D model of the analyzed building

The results of the modal analysis are shown in the following Figures.
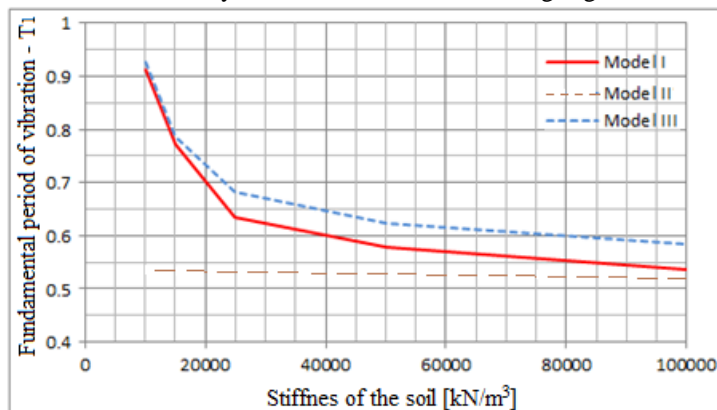
Figure 3. Comparing the values of fundamental period for the first three models

From the previous analysis for the first and third models, the influence of the type of local soil on the value of the fundamental period of vibration is clearly visible. As the effects of the cracks are taken into account in the third model, somewhat higher values of the fundamental period of vibration were expected. In the second model, given the large magnification, soil rigidity is no longer of practical significance to the result of modal analysis.
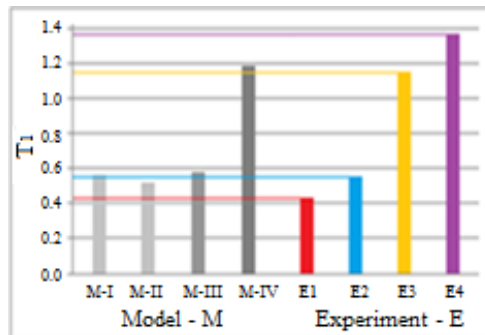
Figure 4. Comparison of values obtained by numerical and experimental analysis

From Figure 4, it can be seen that the values of the fundamental period of vibration in the first three numerical models coincide very well with the results of the first and second experiments. These deviations are practically negligible. The results of the fourth model simulating the appearance of damage coincide well with the results of the third experiment. It is evident that in this case, a significant concurrence was achieved between numerical and experimental results.

For the same building, the values of the fundamental period of vibration were determined using the empirical expressions shown in the previous chapter, and then they were compared with the experimental values. All calculated values are presented in the following figure.
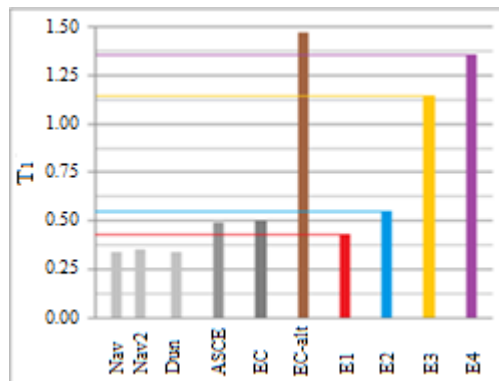

Figure 5. Comparison of empirical and experimental results

Figure 5 shows a good match between most empirically determined values with experimental ones (elastic area E1 and E2). The expressions proposed in the EC and ASCE give the nearest values. For practical application, Navarro's and Dunand's expressions which give slightly lesser value as results, but they are on the safe side. The first impression is that these expressions acceptably describe the value of the fundamental period of vibration. Although the most accurate solution was expected from the alternative (expanded) expression (equations 7-9) proposed by the EC, since it takes into account the largest number of parameters, in this case, it did not provide a satisfactory result and its result deviated too much from the experimental values.

The second example is the eleven-storey building built in New Zealand, which has been tested by ambient and forced vibrations [9]. The structural system consists of columns together with a central RC core. The height of the building is 33.8 m.
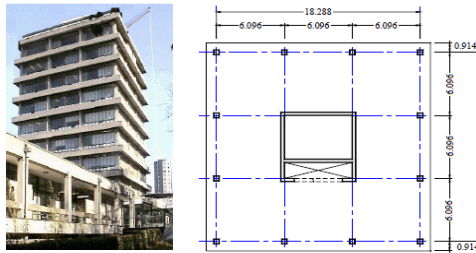
Figure 6. The appearance of the examined building with typical floor plan

Figure 7 illustrates the process of graphic reading of the fundamental period for RC constructions with shear walls according to EC - alternative expression (equations 7-9). The accuracy of the method depends on the precision in operation and reading of the obtained values.

Linear interpolation: $T_1 = 0.98 + (1.21 - 0.98) \cdot \dfrac{33.8 - 30}{40 - 30} = 1.08s$
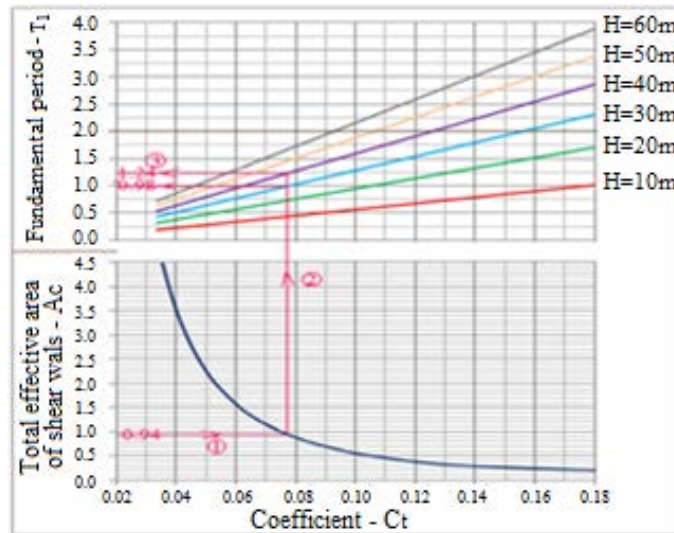

Figure 7. Graphical determination of the fundamental period according to EC

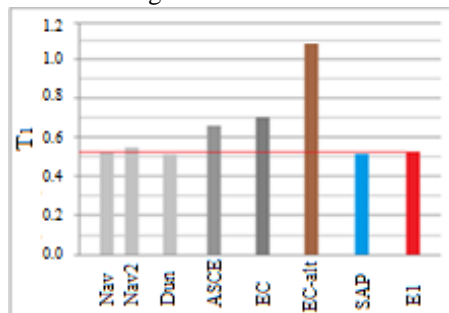All calculated values are presented in Figure 8.

Figure 8. Comparison of empirical, numerical and experimental values

In this case, it should be noted that the values obtained by using the Navarro's and Dunand's expressions were the most approximate, while expressions from the EC and ASCEs gave results which were somewhat higher. Once again, the alternative expression in the EC did not produce satisfactory results. It is important to emphasize that the EC does not make any recommendations in the case of walls of complex cross-sections or walls grouped in the core, as in this case. Whether the effective flange width should be taken into account in the mentioned cases, is up to the designer. It is certainly desirable to do this, but in practice these flanges are most often ignored.

As shown in the previous two examples, the extended expression from the EC, the expectations from which were the highest, did not produce satisfactory results. An additional analysis has reached the same conclusion, suggesting correction of expressions (8) and (9). As a more convenient variant, the correction of the expression (8) is suggested. Two possible corrected curves are shown in Figure 9. Bearing in mind the potential significance of this expression, a more detailed numerical and experimental analysis of all the parameters represented in these expressions should be carried out, and only then can the correct correction be made.
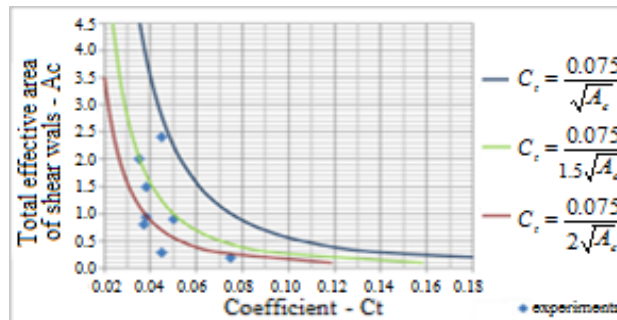

Figure 9. Two possible suggestions for the correction of expression (8)

Figure 10 shows the results of the analysis of additional seven RC buildings of different type.
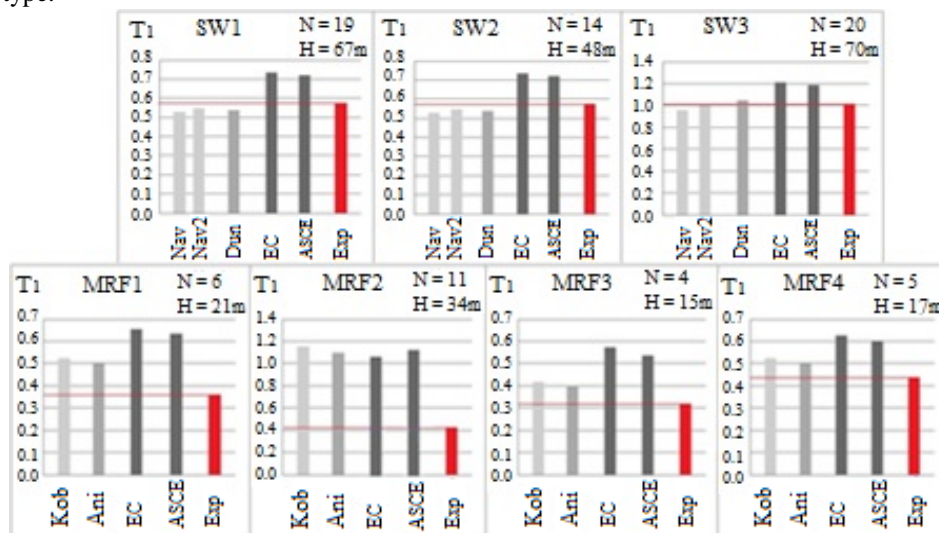

Figure 10. Comparison of the values of the fundamental periods for seven buildings

583

4. CONCLUSION

Determining the value of the fundamental period of vibrations of reinforced concrete structures is a complex task. Previous analysis has shown that for constructions with RC shear walls the nearest values are obtained using the Navarro's and Dunand's expressions, while the expressions proposed in the EC and ASCE give slightly higher results. It seems that the actual value of the fundamental period is within the limits obtained by using these expressions. It can be said that the value of the fundamental period of vibration for this type of construction can be predicted with great certainty through the application of the aforementioned expressions.

When it comes to moment resisting space RC frames, the scattering of results is considerably higher, so the reliability of these expressions is significantly lower. For buildings up to seven storeys, all of these terms give similar values. For constructions of this type that have over 10 storeys, the expressions proposed by Aničić and Kobayashi represent the upper limit. It is unlikely that the actual value of the fundamental period of the moment resisting frame structures will really be found beyond this limit.

In general, it can be said that most of these expressions adequately describe the value of the fundamental period of vibration. It should be remembered that despite the high reliability of individual expressions, they all represent only rough approximations, and they should be used accordingly.

In the construction practice of our region, RC constructions with shear walls are predominantly present; the RC moment resisting frames are used for buildings with a small number of storeys, so it can be said that the presented expressions are a good engineering aid in the initial design phase. In some cases, they can also serve as a control

of numerical models. If a large difference between these values is obtained, this may indicate potential errors in modelling.

REFERENCES

[1] EN 1998-1: Eurocode 8: Design of structures for earthquake resistance - Part 1: General rules, seismic actions and rules for buildings, Brusseles, December 2004.

[2] Aničić D. i saradnici, Zemljotresno inženjerstvo - visokogradnja, Građevinska knjiga, Beograd, 1990.

[3] ASCE, Minimum design loads for buildings and other structures, Published by the American Society of Civil Engineers, Virginia, USA, 2006.

[4] Salatić R., Dinamika konstrukcija i zemljotresno inženjerstvo - priručnik, Akademska misao, Beograd, 2016.

[5] Draganić H., et al, Comparison of RC frames periods with the empiric expressions given in Eurocode 8, Technical Gazette 17,1(2010), 93-100.

[6] Alendar V., Projektovanje seizmički otpornih AB konstrukcija kroz primere, Građevinski fakultet Univerziteta u Beogradu, Beograd, 2004.

[7] U.S. - Japan Cooperative research program, Construction of the full scale reinforced concrete test structure, A report on research sponsored by National Science Foundation Grant No. CEE-81-21843, Michigan, 1983,

[8] Kabeyasawa T. et al, Analysis of the full-scale seven-story reinforced concrete test structures, The Facultz of Engineering University of Tokyo, 1983.

[9] Butterworth J. et al, Experimental determitanion of modal damping from full scale testing, 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada, August 1-6, 2004.