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HELMERT TRANSFORMATION ON THE CASE OF GAUSS-KRÜGER AND UTM

Abstract:

Globally, maps are our primary source of comprehensive information about the shape, size and arrangement of Earth features. Maps are the only way we can get a unique and comprehensive view of the world. Unfortunately, globally, all maps are somehow deformed, affecting our perception and understanding of the various geometrical properties of the world. Cartographic projection is a way of mapping points from an ellipsoid to a plane and as such is the basis for making a mathematical map basis. The two projections most used in our geospatial are the Gauss-Krüger projection and the UTM (Universal Transverse Mercator) projection. The paper deals with Helmert's transformation of these two projections.

Keywords: Helmert transformation, Gauss-Krüger projection, UTM projection

ХЕЛМЕРТОВА ТРАНСФОРМАЦИЈА НА ПРИМЕРУ ГАУС-КРИГЕРОВЕ И УТМ ПРОЈЕКЦИЈЕ

Сажетак:

На глобалном нивоу, карте су нам примарни извори свеобухватних информација о облику, величине и распореду карактеристика Земље. Карте су једини медијум са којег можемо добити јединствен и свеобухватан поглед на свет. Нажалост, на глобалном нивоу све карте су на неки начин деформисане, и утичу на нашу перцепцију и спознају различитих геометријских својстава света. Картографска пројекција представља начин пресликавања тачака са елипсоида на раван и као таква представља основ за израду математичке основе карата. Две пројекције које су се највише користиле на нашем геопростору су Гаус-Кригерова пројекција и UTM (Universal Transverse Mercator) - Трансферзална Меркаторова пројекција. У раду се обрађује Хелмертова трансформација ових двају пројекција.

Кључне ријечи: Хелмертова трансформација, Гаус-Кригерова пројекција, УТМ пројекција

1. INTRODUCTION

The acqusition and conceptualization of spatial knowledge are important topics in human spatial cognition. In general, maps are our primary graphic source of information. However, they deform characteristics of Earth, such as size and shape. The coordinate system is a set of parameters that uniquey identify the location of an any object in a given coordinate system. Transforming coordinates from one coordinate system to another is a common activity. In it's simplest terms, a map projections is transformation of spherical set of global data into plane. Mathematically, transformation from three dimensions into two is called „projection". Projection includes deformations in form of changes in shapes, length and angles. The three basic categories of map projection are equivalent, conformal and equidistant. Equivalent projections preserve equality of surface, conformal preserve similarity of shapes and equidistant preserve equality of lengths. For a relatively small geographical area, deformations can be negligible. However, for large geographical areas, especially at the global level, the distortions are significant and unavoidable. Through this article, our aim is to improve understanding in how to identify and interpret deformations, in hope this will help improving the study of deformations of the projections and restricting wrong decoding of informations derived from the maps.

2. HELMERT TRANSFORMATION

Most of us come across geographical maps every day. In order to correctly interpret information presented on these maps, especially when they includes large areas, we need to understand pattern of deformations presented on the map. Maps can be considered as a represented communication tool between the cartographer and the user who need to use his (her) knowledge to decode the cartographic and geographic symbols [1]. In order to understand the aspect of communication and interpretation of maps, we need to understand users perception of maps. Previous research on the impact of map projections has been limited [2]. Studies based on perception of projection properties on maps have mainly focused on the basic understanding of map properties, such as visual preference for projections and basic skills needed them to understand the projections [2]. Cartographic projection is a mathematically defined mapping of the surface of an ellipsoid or sphere on a map plane [3]. It establishes a coincidence between the geographical coordinates on the Earth's ellipsoids and the orthogonal coordinates of the same points on the plane. The number of possible projections is unlimited [4]. Although projections are a difficult topic to study, successful interpretation of geographic information – even at the basic level – requires the ability to visually access the accuracy of spatial features on a map. This does not mean, that the technical knowledge of map projections is unnecessary, but we should be able to determine deformations produced by projection [5].

When coordinates of geodetics points are calculated in different coordinate system, it is necessary to transform the coordinates from one coordinate system to another. If coordinates of some points are given on local coordinate system, then is necessary to determine their corresponding coordinate values in the state coordinate system. The transition from one coordinate system to another is called the transformation of coordinates.

The study of deformations and some other important property of projection is based on a comparison of corresponding elements of ellipsoidal surface and the elements in projection [6]. One of the basic characteristics of all spatial data is their spatial connection. If it is necessary to represent point in other coordinate system, it is necessary to perform datum transformation. Geodetic datum defines the size and the shape of Earth's ellipsoid, the coordinate origin and orientation of the system relative to the Earth. The parameters that define datum transformation are called transformation parameters. For datum transformation, seven parameters need to be known: 3 translation parameters, 3 rotation parameters and the scale parameter. This transformation is also known as Helmert transformation [7, 8].

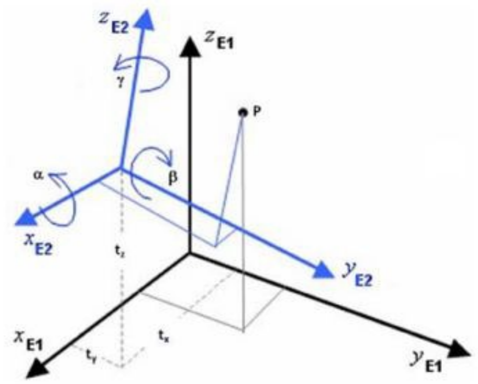

Figure 1. *Helmert transformation [7]*

For two points from two different coordinate systems, it is possible to share data if coordinate systems have an associated spatial reference. In Gauss-Krüger projection points are obtained by projecting the Bessel ellipsoid onto the intersecting transverse cylinder, while the coordinates in the UTM projection are obtained by projecting the WGS84 ellipsoid onto intersecting transverse cylinder. In addition to the fact that the parameters of these two ellipsoids differ, the coordinate systems in which these two ellipsoids are placed are also different [8].

The Bessel ellipsoid was created in 1841 by the German Wilhem Bessel. This ellipsoid is very accurate for the territories of Europe and Euroasia, despite the fact that its axes are about 700 meters shorter then the axis of the ellipsoid WGS84, which are calculated from satellite measurements. In Serbia, it represented the official datum until October 2009, when it become the ellipsoid WGS84. WGS84 (World Geodetic System 1984) is a realization of the conventional terrestrial reference system and as such is the official terrestrial reference of the US Departmen of Defense, both for positioning and navigation, as well as for all cartographic and surveying activities. The advent of Global Positioning System (GPS) has significantly contributed to the promotion of the WGS84 ellipsoid as a global datum standard. WGS84 contains a geocentric Cartesian coordinate system, which is defined by the station coordinates of control GPS segment [9].

3. GAUSS-KRÜGER PROJECTION

Gauss-Krüger projection belongs to group of conformal cylindrical projection. The mapping from the ellipsoid is made to a transverse cylinder, so that it touches the ellipsoid along meridian. The basic characteristics of projection are [10]:

- projection is conformal,
- scale along the central line is equal to one and does not depend on latitude and
- the central line is mapped as a straight line and adopted for the X-axis of the Cartesian coordinate system, and the equator is mapped as a straight line perpendicular on central line and adopted as Y-axis of Cartesian coordinate system; other meridians are mapped as curved lines symmetrical with the respect to central line; the other parallels are mapped as curved lines symmetrical with the respect to the equator.

The projection is called Gauss-Krüger, because its basic theory was given by a German scientist and professor, Karl Friedrich Gauss, and the working formulas where presented by German professor and surveyor, Luis Krüger [6].

The essence of mapping points from the ellipsoid to a plane in a Gauss-Krüger projection is to calculate Cartesian coordinates of those points based on geographical coordinates. The equations used to perform this calculation are derived under the following conditions [11]:

- on the Earth's ellipsoid, a cylinder is conceived so that rotational axis of that cylinder lies in the equatorial plane, and the cylinder touches the ellipsoid by one meridian, as shown in the figure 2 and
- points from the surface of the ellipsoid should be mapped directly onto the surface of the cylinder so that, after developing the cylinder into a plane by cutting one derivate from a plane containing the equator, a conformal projection of the mapped surface is obtained.

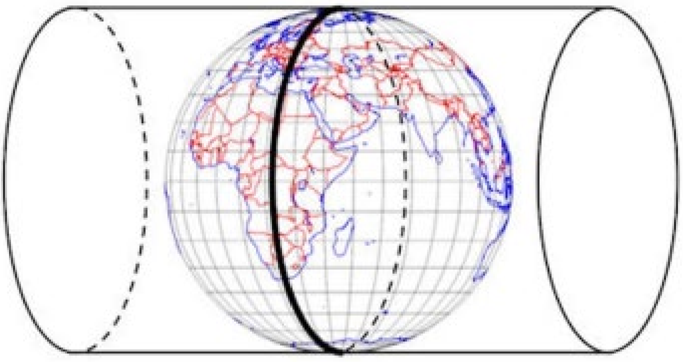

Figure 2. *Mapping from a rotating ellipsoid to a cylinder [12]*

When defining projection, the condition is set that the deformations of the length must not exceed 0.0001, that is, the maximum deformation over a length of one kilometer may be one decimeter. After setting this condition, it is determined how wide the meridional zone can be, and that the deformations remain within the limits of the set condition. The calculation showed that for the given condition the maximum width of the zone could be $2°31'$, and at the end of the zone, the scale factor would be $m = 1 + 0.0001$. If the cylinder is positioned so that it cuts the ellipsoid symmetrically with respect to the central meridian of the zone, the scale factor in the central meridian would be $m_0 = 1 - 0.0001$. By moving away from the central meridian of the zone, the scale factor increases, first becoming equal to the one at the contact meridians, and at the ends of the mapping zone reaches $m = 1 + 0.0001$. Following this condition, states north of 40th parallel may have a zone width of $3°$. It has been defined that the mapping will be performed on a cutting cylinder and that the width of the zone is $3°$, because the Serbia is between the 42nd and 46th northern parallels [10].

4. UTM PROJECTION

UTM (Universal Transverse Mercator) – a transverse Mercator projection, an adaptation of the Mercator projection, is a projection in which an ellipsoid intersects a cylinder whose axis lies in the equatorial plane. Although this projection is under this name, it is essentially Gauss-Krüger projection applied in a special way. The meridians and parallels of the UTM projection are not straight lines, as in the case of Mercator projection, except for the equator and the central meridian of the zone, as well as any meridian that is 90° from the central. Deformations grow east and west from the central meridian. In order to increase the surface in which deformations can be tolerated, a cutting cylinder is applied. In this case, we have two meridians on which the scale factor is equal to one [13, 14].

The UTM is conformal projection. For this reason, surface and lengths are mapped with some deformations. When defining projection, the condition was set that the deformations of the lengths should not exceed 0.0004, that is, the maximum deformation over a length of one kilometer could be four decimeters. After setting this condition, it is determined how wide the mapping zone can be so that the deformations remain within the limits of set condition. If the cylinder is positioned that it cuts the ellipsoid symmetrically with respect to the central meridian of the zone, the scale factor at the central meridian of the zone is $m_0 = 1 - 0.0004$. By moving away from the central meridian of the zone, the scale factor increases, first becoming equal to one on the conctat meridians, and at the ends of the mapping zone is $m_0 = 1 + 0.0004$ [10, 14].

5. TRANSFORMATION FROM GAUSS-KRÜGER PROJECTION INTO UTM PROJECTION

A complete transition from the Gauss-Krüger projection to the UTM projection is shown on the following scheme:

$$(y, x)_{GK} \rightarrow (B, L)_{Bessel} \rightarrow (X, Y, Z)_{Bessel} \rightarrow (X, Y, Z)_{WGS84} \rightarrow (B, L)_{WGS84} \rightarrow (E, N)_{UTM}$$

This algorithm implies that the geodetic datum transformation is performed on the basis of Helmert transformation and that the ellipsoidal height (*h*) of the point whose coordinates are transformed is known [11].

The course of transformation itself based on a given scheme follows next steps [11, 15]:

- the Cartesian coordinates *y* and *x* of some point in the plane in Gauss-Krüger projection is converted to the corresponding geodetic coordinates *B* and *L* (latitude and longitude) on the Bessel ellipsoid;
- coordinates *B* and *L* with the associated ellipsoidal height *h* are converted to the spatial Cartesian coordinates *X, Y* and *Z* on the Bessel ellipsoid;
- then the spatial Cartesian coordinates *X, Y* and *Z* on the Bessel ellipsoid are transformed into the spatial Cartesian coordinates *X, Y* and *Z* related to the WGS84 ellipsoid;
- coordinates *X, Y* and *Z* related to the WGS84 ellipsoid are converted to the geodetic coordinates *B, L* and *h* on the WGS84 ellipsoid and
- coordinates *B* and *L* are converted to coordinates *E and N* in the UTM projection.

In the first step, the Cartesian coordinates *y* and *x* of some point in the Gauss-Krüger projection are transformed into the corresponding geodetic coordinates *B* и *L* (latitude and longitude) on the Bessel ellipsoid. The following applies in this step:

- $a = 6377397{,}155$ - the large axis of the Bessel ellipsoid,
- $b = 6356078{,}96325$ - the small axis of the Bessel ellipsoid,
- $m_o = 0{,}9999$ - scale along the central meridian of the Gauss-Krüger,
- $y_o = 6500000$ and $L_o = 18°$ for points in the zone 6 of the Gauss-Krüger projection or
- $y_o = 7500000$ and $L_o = 21°$ for points in the zone 7 of the Gauss-Krüger projection.

First, the unmodulated coordinates are calculated using the following:

$$y' = \frac{y - y_o}{m_o}$$

$$x' = \frac{x}{m_o} \tag{1}$$

After that, following parameters are calculated:

$$e = \sqrt{1 - \frac{b^2}{a^2}}\ ;\ e' = \sqrt{\frac{a^2}{b^2} - 1} \tag{2}$$

$$m_1 = \frac{x'}{a\left(1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256}\right)} \tag{3}$$

$$e_1 = \frac{1 - \left(1 - e^2\right)^{1/2}}{1 + \left(1 - e^2\right)^{1/2}} \tag{4}$$

$$B_1 = m_1 + \left(\frac{3}{2}e_1 - \frac{27}{32}e_1^3\right)\sin 2m_1 + \left(\frac{21}{16}e_1^2 - \frac{55}{32}e_1^4\right)\sin 4m_1 +$$
$$+ \frac{151}{96}e_1^3 \sin 6m_1 + \frac{1097}{512}e_1^4 \sin 8m_1 \tag{5}$$

$$V_1 = \frac{a}{\left(1 - e^2 \sin^2 B_1\right)^{1/2}} \tag{6}$$

$$M_1 = \frac{a\left(1 - e^2\right)}{\left(1 - e^2 \sin^2 B_1\right)^{3/2}} \tag{7}$$

$$T_1 = \tan^2 B_1 ; C_1 = e'^2 \cos^2 B_1 \text{ и } D = \frac{y'}{V_1} \tag{8}$$

Using the given parameters, the required coordinates are obtained using the following formulas:

$$B = B_1 - \frac{V_1 \tan B_1}{M_1} \left[\begin{array}{l} \frac{D^2}{2} - \left(5 + 3T_1 + 10C_1 - 4C_1^2 - 9e'^2\right)\frac{D^4}{24} + \\ + \left(61 + 90T_1 + 298C_1 + 45T_1^2 - 252e'^2 - 3C_1^2\right)\frac{D^6}{720} \end{array} \right] \tag{9}$$

$$L = L_o + \frac{1}{\cos B_1} \left[\begin{array}{l} D - \left(1 + 2T_1 + C_1\right)\frac{D^3}{6} + \\ + \left(5 - 2C_1 + 28T_1 - 3C_1^2 + 8e'^2 + 24T_1^2\right)\frac{D^4}{120} \end{array} \right] \tag{10}$$

In the second step, the coordinates *B* and *L* , obtained from the previous step, with assigned ellipsoidal height *h* are converted into Cartesian coordinates *X, Y* and *Z* relating to the ellipsoid of the Bessel. The procedure is as follows:

$$V = \frac{a}{\left(1 - e^2 \sin^2 B\right)^{1/2}} \tag{11}$$

$$X = \left(V + h\right)\cos B \cos L$$
$$Y = \left(V + h\right)\cos B \sin L$$
$$Z = \left[V\left(1 - e^2\right) + h\right]\sin B \tag{12}$$

The third step is the transformation of Cartesian coordinates *X, Y* and *Z* related to the Bessel ellipsoid into *X, Y* and *Z* coordinates related to the WGS84 ellipsoid. For the transformation itself between two datum, it is necessary to know the transformation parameters. These parameters are derived from points whose coordinates are known in both systems [11, 15].

To calculate transformation parameters it is necessary to know the coordinates of at least two corresponding points and the height of the third point in both coordinate system (datums) [16]. The transformation parameters are than evaluated through the adjustment by the method of the least squares. This method is based on the principle that the sum of the squares of the measurement result corrections (residuals) is minimal. In this case, the differences between the known coordinates of identical points and their coordinates obtained by transformation are minimized [17]. This model is in literature often referred to the model Bursa-Wolf. If this model is applied to the geodetic networks of a smaller scope, the translation and the rotation parameters are highly correlated. An alternative model is Molodensky-Badekas model which is a more general transformation model in which rotations are relative to the grid centroid in the new datum [16]. When it comes to the final result, the two models give different parameters of translation, but the rotation parameters and the scale factor remain the same [17]. Higher number and better distribution of this points in the observed territory are prerequisite for the set of parameters that result in better fit. Parameters can be calculated by the user himself or downloaded by national surveyor or mapping agency or any other appropriate institution. As an example, in table 1 are shown transformation parameters given by national surveyor agency of Serbia – Republic geodetic authority. Parameters are determined by 1217 points of the state trigonometric network. For military use in Serbia, parameters are provided by Military Geographic Institute [15].

Table 1. *Transformation parameters (Republic Geodetic Authority, 2017) [15]*

Transformation parameters	Value	Standard deviation
Translation x	574.040907 m	$\sigma(t_x)$=0.015 m
Translation y	170.129711 m	$\sigma(t_y)$=0.015 m
Translation z	401.553949 m	$\sigma(t_z)$=0.015 m
Rotation x	-4.88790271″	$\sigma(\varepsilon_x)$=0.032″
Rotation y	0.66492609″	$\sigma(\varepsilon_y)$=0.049″
Rotation z	13.24674576″	$\sigma(\varepsilon_z)$=0.044″
Scale	6.88937746 ppm	$\sigma(k)$=0.106 ppm

From the calculated coordinates from the previous step, a vector r_{Bessel} is formed, from parameters of transition a vector $t = \begin{bmatrix} X_0 & Y_0 & Z_0 \end{bmatrix}$, and from the three rotation parameters around the coordinate axes *X, Y* and *Z* for the angles α, β and γ respectively, a rotation matrix *R* is formed. The scale factor between the two coordinate system is indicated by dm. The vector of Cartesian coordinates related to the WGS84 ellipsoid is calculated by the following expression:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS84} = \left(1+dm\right)R^T \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{Bessel} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{13}$$

Matrix *R* is:

$$R = \begin{bmatrix} \cos b\cos g & \cos a\sin g + \sin a\sin b\cos g & \sin a\sin g - \cos a\sin b\cos g \\ -\cos b\sin g & \cos a\cos g - \sin a\sin b\sin g & \sin a\cos g + \cos a\sin b\sin g \\ \sin b & -\sin a\cos b & \cos a\cos b \end{bmatrix}$$

Matrix *R* can also be written in the developed form:

$$R = R_1(\varepsilon_X)R_2(\varepsilon_Y)R_3(\varepsilon_Z) \text{, wherein:}$$

$$R_1\left(e_x\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos e_x & \sin e_x \\ 0 & -\sin e_x & \cos e_x \end{bmatrix}$$

$$R_2\left(e_y\right) = \begin{bmatrix} \cos e_y & 0 & -\sin e_y \\ 0 & 1 & 0 \\ \sin e_y & 0 & \cos e_y \end{bmatrix}$$

$$R_3\left(e_z\right) = \begin{bmatrix} \cos e_z & \sin e_z & 0 \\ -\sin e_Z & \cos e_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\varepsilon_X, \varepsilon_Y$ and ε_Z are the rotation parameters for corresponding axes.

The four step is the conversion of Cartesian coordinates from the WGS84 ellipsoid to the geodetic coordinates *B, L* and *h* on the WGS84 ellipsoid. The following parameters of the WGS84 ellipsoid are used for this conversion:

- a = 6378137 – the large axis of WGS84 ellipsoid and
- b = 6356752,31425 – the small axis of WGS84 ellipsoid.

Following parameters are used for this step:

$$e = \sqrt{1 - \frac{b^2}{a^2}};\ e' = \sqrt{\frac{a^2}{b^2} - 1};$$

$$p = \sqrt{X^2 + Y^2} \text{ and } q = \arctan\frac{Za}{pb} \tag{14}$$

Geodetic coordinates on the WGS84 ellipsoid are calculated using the following formulas:

$$B = \arctan\frac{Z + e'^2 b \sin^3 q}{p - e^2 a \cos^3 q}$$

$$L = \arctan\frac{Y}{X}$$

$$h = \frac{p}{\cos B} - N \tag{15}$$

where N is calculated by the formula:

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 B}} \tag{16}$$

The geodetic latitude B and ellipsoidal height h are usually calculated by a more complex iterative procedure. However, the solution obtained by the above formulas for our territory are in accordance with iterative procedure.

The fifth step is the conversion of geodetic coordinates B and L on the WGS84 ellipsoid into Cartesian coordinates E and N in the UTM projection. The following parameters are used:

- $m_o = 0,9996$ - scale factor among the central meridian of UTM zone and
- $L_o = 21°$ - for points in the 34th UTM zone.

Parameters that need to be calculated before the final step are:

$$A = \left(L - L_o\right)\cos B$$

$$T = \tan^2 B$$

$$C = e'^2 \cos^2 B$$

$$V = \frac{a}{\left(1 - e^2 \sin^2 B\right)^{1/2}} \tag{17}$$

$$M = a\left[\begin{array}{l}\left(1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 - \frac{5}{256}e^6\right)B - \left(\frac{3}{8}e^2 + \frac{3}{32}e^4 + \frac{45}{1024}e^6 \sin 2B\right) + \\ + \left(\frac{15}{256}e^4 + \frac{45}{1024}e^6 \sin 4B\right) - \frac{35}{3072}e^6 \sin 6B\end{array}\right] \tag{18}$$

$$y' = V\left[A + \left(1 - T + C\right)\frac{A^3}{6} + \left(5 - 18T + T^2 + 72C - 58e'^2\right)\frac{A^5}{120}\right] \tag{19}$$

$$x' = M + V\sin B \left[\frac{A^2}{2} + \left(5 - T + 9C + 4C^2\right)\frac{A^4}{24} + \\ + \left(61 - 58T + T^2 + 600C - 330e'^2\right)\frac{A^6}{720} \right] \tag{20}$$

and finally:

$$E = y'm_o + 500000$$
$$N = x'm_o \tag{21}$$

6. TRANSFORMATION FROM UTM PROJECTION INTO GAUSS-KRÜGER PROJECTION

The transformation from UTM to Gauss-Krüger projection is done analogous to the transformation from Gauss-Krüger to UTM projection. The transformation process is shown in the following scheme:

$$(E, N)_{UTM} \to (B, L)_{WGS84} \to (X, Y, Z)_{WGS84} \to (X, Y, Z)_{Bessel} \to (B, L)_{Bessel} \to (y, x)_{GK}$$

The flow of the transformation itself based on the given scheme is as follows [11, 15]:

- Cartesian coordinates *E* and *N* of some point in the UTM projection are converted to the corresponding geodetic coordinates *B* and *L* (latitude and longitude) on the WGS84 ellipsoid;
- the coordinates *B* and *L* with the associated ellipsoidal height *h* are converted to the spatial Cartesian coordinates *X, Y* and *Z* related to the WGS84 ellipsoid;
- then the spatial Cartesian coordinates *X, Y* and *Z* on the WGS84 ellipsoid are transformed into the spatial Cartesian coordinates *X, Y* and *Z* that refer to the Bessel ellipsoid;
- the coordinates *X, Y* and *Z* relating to the ellipsoid of the Bessel are converted to geodetic coordinates *B, L* and *h* on the Bessel ellipsoid and
- coordinates *B* and *L* are converted to *y* and *x* coordinates in the Gauss-Krüger projection.

In the first step, Cartesian coordinates *E* and *N* of a point in UTM projection are transformed in appropriate geodetic coordinates *B* and *L* (latitude and longitude) on the ellipsoid WGS84. In this step, the following applies:

- $a = 6378137$ - the big semi-axis of the WGS84 ellipsoid,
- $b = 6356752,31425$ - the small semi-axis of the WGS84 ellipsoid,
- $m_o = 0,9996$ - scale along the central meridian of the UTM zone and
- $L_o = 21°$ for points in the 34th UTM projection zone.

First, the unmodulated coordinates are calculated:

$$y' = \frac{E - 500000}{m_o}$$
$$x' = \frac{N}{m_o} \tag{22}$$

After that, the calculations are performed according to formulas (2) to (10).

The second step is conversion of the coordinates *B* and *L* along with the associated ellipsoidal height *h* into Cartesian coordinates *X, Y* and *Z* that are refer to the WGS84 ellipsoid. Formulas (11) and (12) are used in this step.

In third step, the Cartesian coordinates *X, Y* and *Z* on the WGS84 ellipsoid are transformed into Cartesian coordinates *X, Y* and *Z* that refer to the Bessel ellipsoid. The procedure is analogous as in the third step of the transformations of coordinates from Gauss-Krüger projection to UTM projection. From the calculated coordinates from the previous step, a vector r_{WGS84} is formed, from the translation parameters vector $t = \begin{bmatrix} X_o & Y_o & Z_o \end{bmatrix}$, and from three rotation parameters around the coordinate axis *X, Y* and *Z* for the angles α, β and γ respectively, a rotation matrix *R* is formed. The vector of Cartesian coordinates related to the Bessel ellipsoid is calculated by the following expression:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{Bessel} = (1 + dm) R^T \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS84} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{23}$$

The fourth step in the transformation is a conversion of the coordinates *X, Y* and *Z* relating to the Bessel ellipsoid in a geodetic coordinates *B, L* and *h* on the same ellipsoid. The ellipsoidal parameters used in this step are:

$$a = 6377397{,}155$$

$$b = 6356078{,}96325$$

In the following, formulas (14) - (16) are used.

The fifth and the final step in this transformation is the conversion of coordinates *B* and *L* into the coordinates *y* and *x* in the Gauss-Krüger projection. The following parameters are used in this step:

- $m_o = 0{,}9999$ - scale along the central meridian of the Gauss-Krüger projection,
- $y_o = 6500000$ and $L_o = 18°$ for points in zone 6 of the Gauss-Krüger projection or
- $y_o = 7500000$ and $L_o = 21°$ for points in zone 7 of the Gauss-Krüger projection.
- Formulas (17) - (20) are used for calculation, after which the following formulas are used:

$$y = y' m_o + y_o$$

$$x = x' m_o \tag{24}$$

The transformation of coordinates requires specialized geodetic knowledge, especially in part which refers to the selection of models and parameters of transformation, because it affects on accuracy of transformed coordinates. However, if the procedure and formulas are cleared, users can transform coordinates without such knowledge.

7. CONCLUSION

Cartographic projections represent the way points from an ellipsoid are mapped to a plane. Due to the mapping between such surfaces, there is a deformation of the projection sizes. There are many map projections, each with its own purpose, each with its own advantages and disadvantages. When choosing a map projection, it is very important to keep in mind which area is being mapped and what sizes we want to preserve. The two most significant projections used in our country are the Gauss-Krüger and UTM (transverse Mercator projection) projections.

After 85 years of using Gauss-Krüger projection in our country, there is a need to standardize geospatial data with other countries. In this regard, the Government of the Republic of Serbia made a decision in October 2009 to switch to a UTM map projection, the datum of which will be the WGS84 ellipsoid. The transition to a new map coordinate system is a long process.

There is no indication that the map projection that will serve as the basis for the national survey will change in the near future. With the growing representation of UTM projection in various applications and systems, there is a growing range of users interested in the procedures and methods of transitioning from Gauss-Krüger to UTM projection, and vice versa. The question of choosing the transformation method and transformation parameters requires specialized geodetic knowledge, since the accuracy of the transformed coordinates depends on these choices.

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