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## DYNAMIC BEHAVIOR OF A CABLE-STAYED FOOTBRIDGE OVER RIVER VRBAS IN BANJA LUKA

### *Abstract:*

This paper presents a dynamic behavior analysis of an old cable-stayed footbridge over river Vrbas in Banja Luka. Identification of modal parameters, of this prone to vibrations footbridge structure, was performed using Operational Modal Analysis with Frequency Domain Decomposition method. Experimental test setups and obtained results, compared to the numerical values obtained by FE model updating, are shown. Modal Assurance Criterion was used for the confirmation of the uniqueness of experimentally obtained mode shapes, and also for the comparison of FE model mode shapes to the experimentally obtained ones, in the locations of measurement.

*Keywords:* footbride vibration, Operational Modal Analysis, Modal Assurance Criteria, model updating

## ДИНАМЧКО ПОНАШАЊЕ ВИСЕЋЕГ ПЈЕШАЧКОГ МОСТА ПРЕКО РИЈЕКЕ ВРБАС У БАЊОЈ ЛУЦИ

### *Сажетак:*

У раду је приказана анализа динамичког понашања висећег пјешачког моста, преко ријеке Врбас у Бањој Луци. Идентификација модалних параметара овог моста, подложног осјетним вибрацијама, извршена је преко оперативне модалне анализе и *Frequency Domain Decomposition* методе. Приказане су поставке за експериментално испитивање, те добијени резултати, који су упоређени са вриједностима калибрисаног нумеричког модела. Процедура *Modal Assurance Criterion* је кориштена за потвду јединствености модалних облика добијених преко резултата мјерења, а такође при поређењу модалних облика добијених преко нумеричког модела са експерименталним резултатима, у тачкама у којима је извршено мјерење.

*Кључне ријечи:* вибрације пјешачких мостова, операциона модална анализа, калибрација модела

## 1. INTRODUCTION

When analyzing the dynamic characteristics of pedestrian bridges, attention is given to the analysis of the resonant phenomena occurrence and the maximum acceleration of the structure.

In regulations, nowadays, the dynamic problem of pedestrian bridges is solved by considering and limiting natural frequencies and accelerations in the structures' finite element models. Considered dynamic load cases for human walk take into account the basic parameters crucial for the resonance phenomena, such as the structure mode shapes, oscillation frequencies and corresponding damping, as well as the range of frequencies of a human walk. After applying defined load cases, the maximum acceleration of construction is considered, which is directly related to the perception of people, in terms of feeling of comfort or discomfort [1]–[4].

In this paper, the dynamic behavior of a lively cable-stayed footbridge over river Vrbas in Šeher, Banja Luka, is analyzed. Footbridge considered is prone to vibrations felt by pedestrians.

The bridge design dates from the seventies, when resonant phenomena occurrence and maximum accelerations were not considered, and it doesn't meet today's comfort criteria defined by the relevant norms.



Figure 1 *Figures and captions are centered. (use style*

To identify dynamic behavior of the footbridge structure, modal parameters were determined using Ambient Modal Identification, also known as Operational Modal Analysis (OMA), which is based on vibration data collected when the structure is under its operating conditions [5], [6].

Modal parameters identified by OMA are compared to modal parameters of the finite element numerical model, and manual model updating was performed.

## 2. FOOTBRIDGE STRUCTURE DATA

Footbridge structure data were given in the original design, found in the Archives of the Republic of Srpska. Design was made in 1961, and the bridge was constructed in 1963. Data obtained from the bridge design were verified by in-situ visual inspection and dimensional control.

The bridge is suspended, and the length of the main stiffening truss is 62,00 m. Stiffening truss consists of two vertical steel trusses, with upper and lower profiles [65x65x9 and L100x100x10, respectively. Profiles are connected with diagonal and vertical bars made of steel rails 700 kg/m'. The height of the vertical steel trusses is 900 mm. In the upper zone, horizontal bracing, connecting two trusses, is coupled with an 8 cm thick concrete deck. Horizontal bracings in lower and upper zone are also made of steel rails 700 kg/m'.

The main cable is steel wire  $\varnothing 52$  mm and suspenders are steel profiles  $\varnothing 18$ . Main cable relays on 10 m tall diamond-shaped steel pylons.

Bridge disposition and cross-section are shown in Figures 2 and 3.

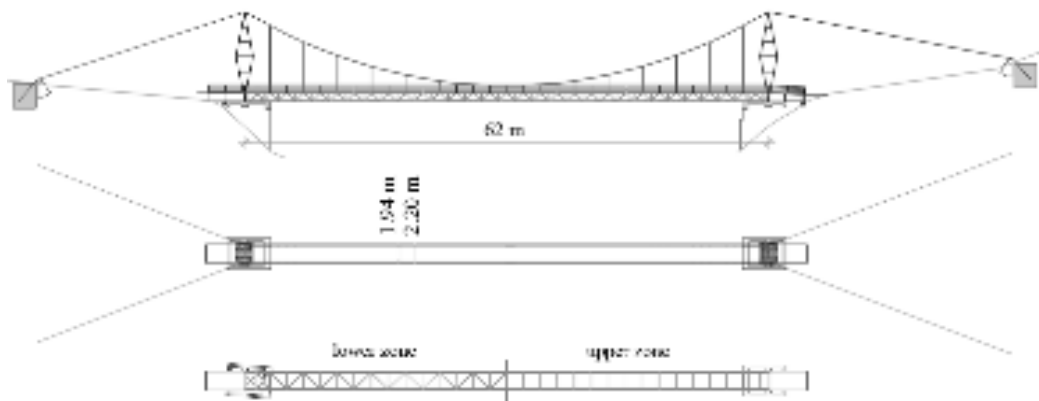


Figure 2 Footbridge layout

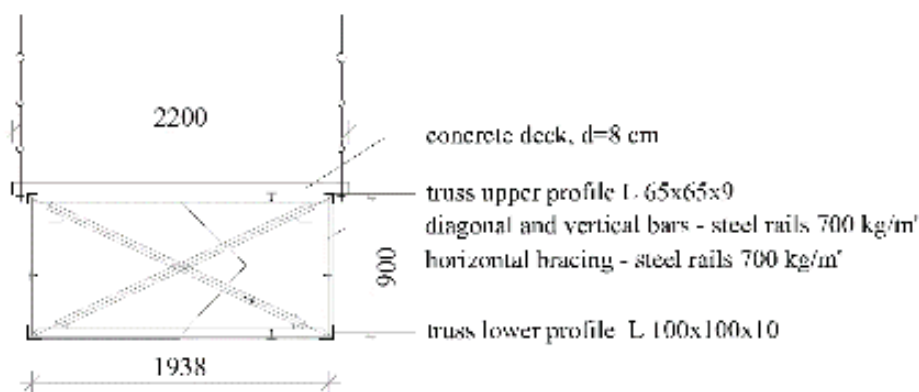


Figure 3 Footbridge cross section

### 3. IDENTIFICATION OF DYNAMIC PROPERTIES

Dynamic tests aimed to evaluate the modal properties of the bridge structure, and also the effects of pedestrian actions by measuring maximum accelerations under its operating conditions.

For dynamic identification of structural characteristics OMA was used, as for large structures, it is complicated and expensive to apply controlled exterior stimulation. In these cases, ambient excitation is a practical solution. Construction can be aroused by wind, traffic or human activities, and the excitation is not measured, only the response is [5].

As a method of identification, Multiple Test Setups Measurement Procedure was used, meaning that the applied sensors were moved from one set of positions to another, in several test setups [6]. Sensors were placed in the positions where the modes of interest are having a good response level, regarding the mode shapes. These positions were determined using the initial FE model of the footbridge, based only on the main design and visual inspection.

To get quality information about the structure vibrations, two main parameters, which significantly affect the ambient vibration measurement results, must be considered. These are the total sampling time and the time interval between the two samples - sampling frequency. In this case, the duration of the continuous measurement for each setup was 1 hour, which gave low frequency resolution. Used sampling frequency was 600 Hz, which is much more than the minimum recommended regarding the expected structure frequencies, and it contributes to the quality of the sampled signal.

#### 3.1. Measuring equipment

Vibration responses were measured using universal measuring amplifier of the QuantumX series - MX840A, produced by Hottinger Baldwin Messtechnik – HBM with 24-bit resolution and simultaneous sampling, connected to Catman Easy software for visualization of signals during measurements. As sensors, high-sensitive accelerometers were used, model-2240, produced by the company Silicon Designs, with measuring range  $\pm 2g$ .

For signal processing, a Butterworth low pass filter was applied, with a cut-off frequency of 50 Hz.

### 3.2. Test setups for mode shapes identification

To get clear mode shapes and to estimate a large number of natural modes, using Multiple Test Setups Measurement Procedure, dynamic behavior was recorded with 8 accelerometers, differently arranged in 5 different test setups.

In each test setup response was measured in five locations. On locations on the upstream part of the footbridge, accelerometers were positioned in two directions Z and Y, and on the downstream part of the bridge, only in the Z direction. In every test setup, 5 accelerometers measured response in the vertical, and 3 in the lateral direction. To sum up, in 5 test setups, vibrations were measured on 21 location, where the vertical response was measured at all locations, and horizontal at 11 locations, as shown in Figure 4.

On reference position, 2 accelerometers were placed, measuring response in both Z and Y direction. Reference accelerometers were not moved in test setups and they measured response in their positions during every measuring time interval. The purpose of reference accelerometers is to adjust the mode shape values obtained in different test setups. The reference position should be chosen so that the reference sensors have a good response during all setups, meaning that the mode shapes of interest should have a good amplitude in reference points. Since several first vertical, torsional and horizontal mode shapes could be predicted by the initial FE model, it is decided that one reference position for the planned measurements is enough, and it was placed in one-quarter of a span. The rest of the sensors were moved from one location to another in different setups.

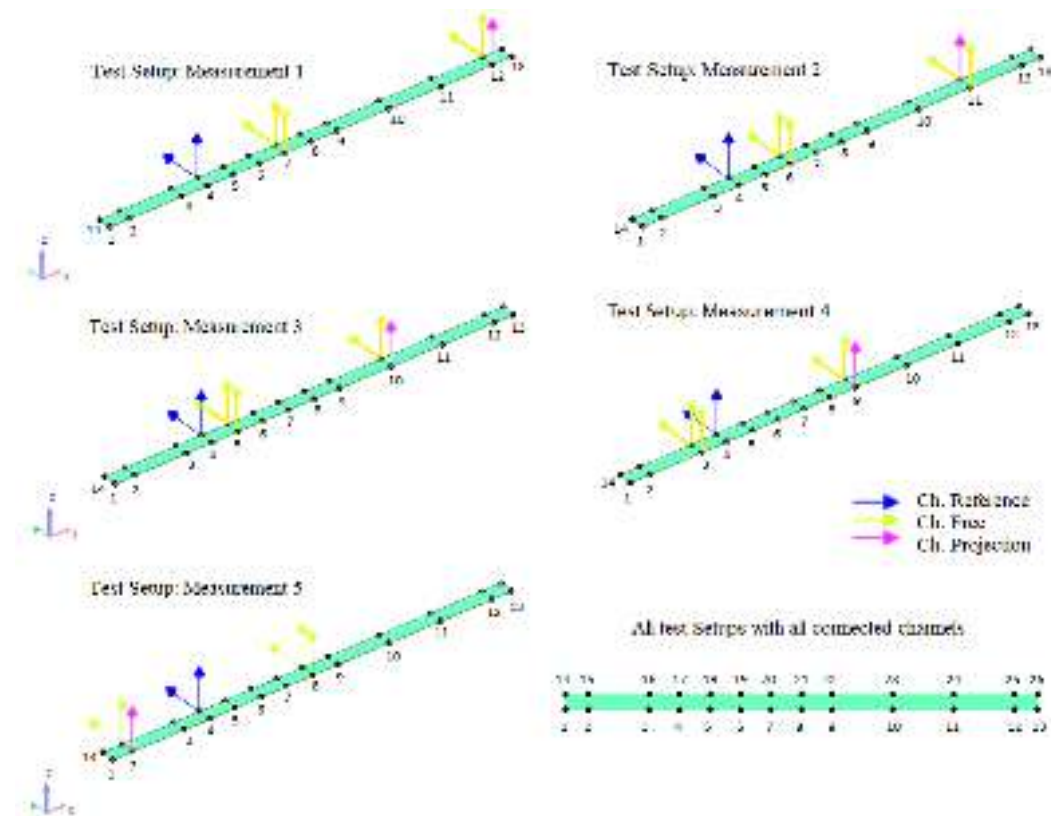


Figure 4 Test setups disposition

### 3.3. Data processing

For identification of modal parameters, Frequency Domain Decomposition (FDD) was performed, using ARTEMIS software package.

FDD is an output-only system identification technique, and by its implementation, a decomposition of the system response into a set of independent single degree of freedom (SDOF) systems is performed, one for each mode. After obtaining singular values of spectral density matrices, since multiple test setups were available, corresponding curves of all test setups are averaged, and modal information is presented in one display [6].

Candidate modes were estimated by the “Peak picking” method [6] and later verified by the Modal Assurance Criteria (MAC) method [6]–[8].

Since suspension footbridges can exhibit closely spaced modes of vibration, to estimate candidate modes, it is recommended to perform “Peak picking” on the average second, and third singular value curve.

To confirm the uniqueness of the obtained mode shapes, at extracted modal frequencies, mode shapes were compared using MAC procedure, and their similarity is represented by the MAC value. For two mode shapes, the scalar MAC value is defined as a normalized dot product of the complex modal vector ( $\psi_r$ ,  $\psi_s$ ), at each common node, and the resulting scalars are arranged into the MAC matrix [7]:

$$MAC(\{\psi_r\}, \{\psi_s\}) = \frac{|\{\psi_r\}^t \{\psi_s^*\}|^2}{\{\psi_r\}^t \{\psi_r^*\} \{\psi_s\}^t \{\psi_s^*\}}, \quad (1)$$

MAC matrix is filled with values in the range of 0 to 1. For identical mode shapes, MAC will have a value of 1 (100%). If modes shapes are different, they are orthogonal, so the dot product of their modal vector will be 0, so as the MAC value [6]–[8].

### 3.4. Measurement results

Following procedure stated above, obtained mode shapes in ARTEMIS software and maximum accelerations are presented herein.

#### 3.4.1. Mode shapes

As the preliminary FE model analysis indicated, measurement results confirmed that the first vertical and first lateral mode are closely spaced, so, as recommended in [6], in the process of estimation in ARTEMIS, the peaks on the first and second singular value curve are considered.

Figure 5 shows the singular values of spectral densities and the “Peak Picking” of dominant frequencies for all test setups, for frequencies ranges 0-15 Hz and 1-5 Hz.

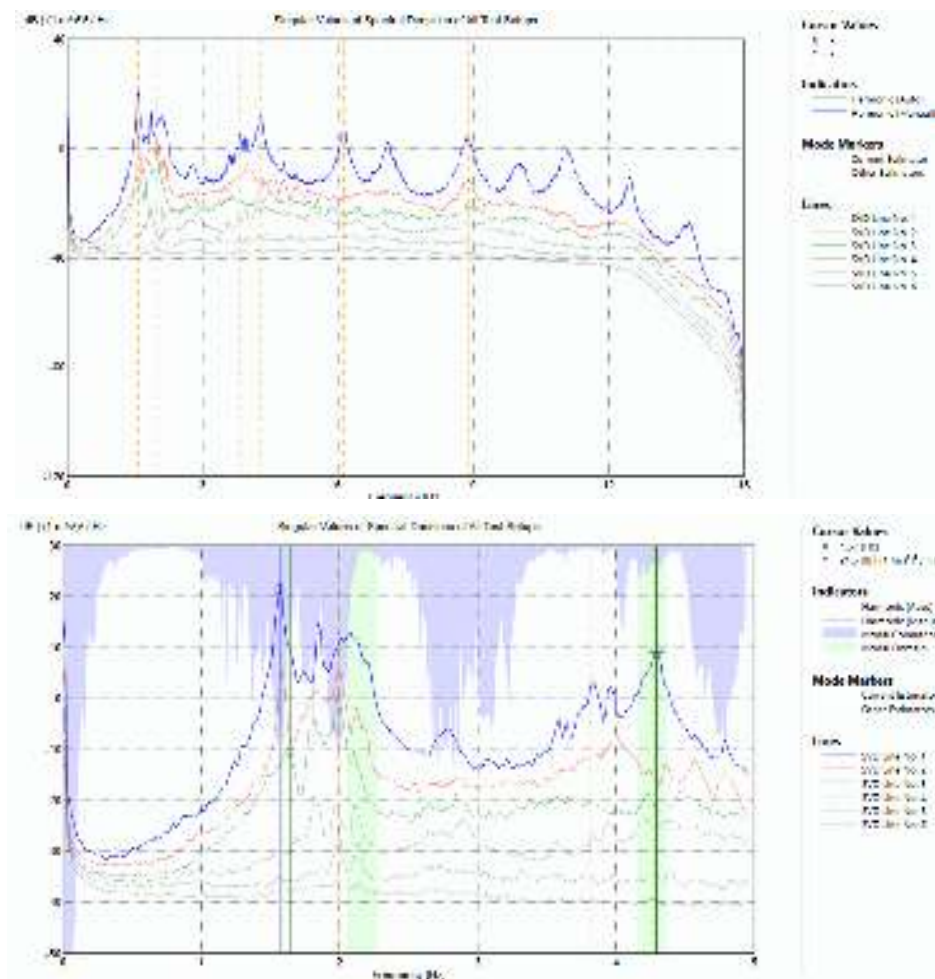


Figure 5 Singular values of spectral densities and “Peak Picking” of dominant frequencies for all test setups and frequencies ranges 0-15 Hz and 1-5 Hz

Graphical overview of the mode shapes of the bridge deck in ARTEMIS software, for the first nine modes, is given in Figure 6, along with measured modal frequency values. As shown, a clear mode shapes estimation is obtained.

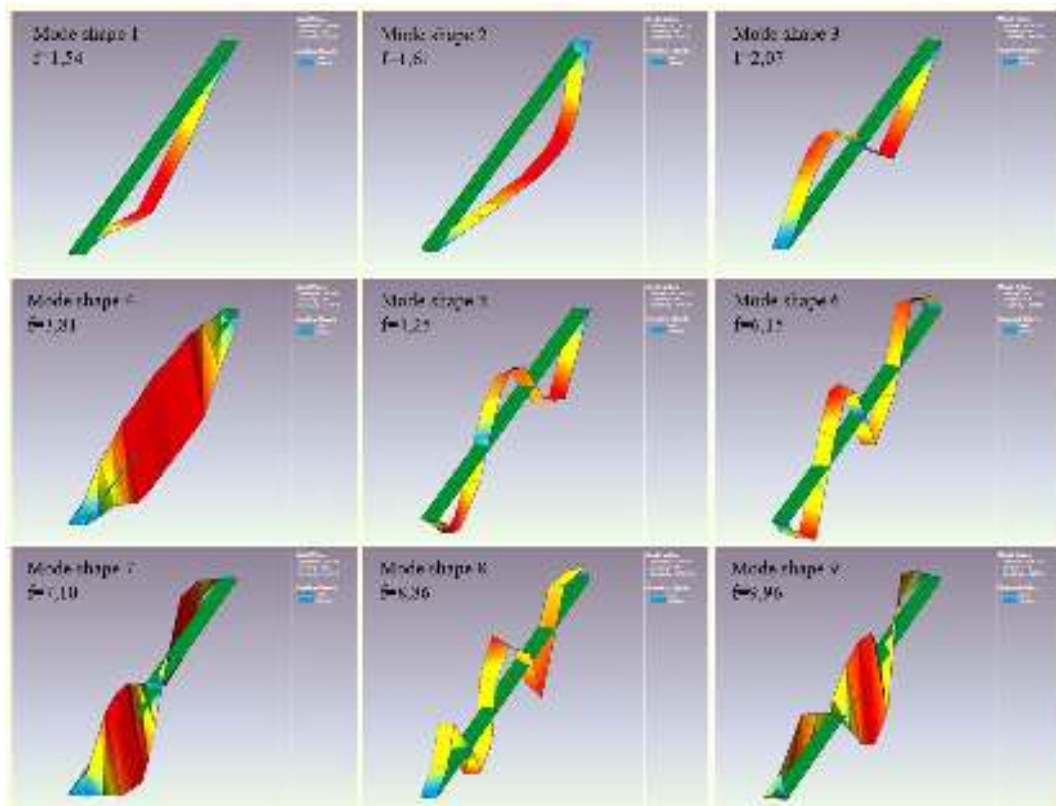


Figure 6 Experimentally determined first nine mode shapes of vibration

ARTEMIS has the option of tabular and graphical presentation of the values obtained by the MAC procedure. The diagonal elements of the matrix are a comparison of each mode shape with itself, and the value "1" is an indicator of the complete match of the shape. The elements outside of the matrix diagonal are a result of the comparison of the different modes, and their low values show the uniqueness of the observed modes.

MAC matrix for performed measurements on the footbridge in Šeher is shown in Figure 7.

	1.538 Hz	1.611 Hz	2.051 Hz	3.809 Hz	4.248 Hz	6.152 Hz	7.105 Hz	8.862 Hz
1.538 Hz	1	0	0.06	0	0.009	0.019	0	0.027
1.611 Hz	0	1	0.005	0.052	0.001	0	0.004	0.003
2.051 Hz	0.06	0.005	1	0	0.01	0.011	0	0.004
3.809 Hz	0	0.052	0	1	0.011	0	0.056	0.002
4.248 Hz	0.009	0.001	0.01	0.011	1	0.008	0.004	0.04
6.152 Hz	0.019	0	0.011	0	0.008	1	0.001	0.017
7.105 Hz	0	0.004	0	0.056	0.004	0.001	1	0.002
8.862 Hz	0.027	0.003	0.004	0.002	0.04	0.017	0.002	1

Figure 7 MAC matrix for experimentally estimated modes

### 3.4.2. Acceleration measurements

Results of acceleration measurements show that maximal vertical accelerations are about 50 mg in all measured setups.

In Figure 8, vertical and lateral accelerations induced by the pedestrians in ambient conditions are shown for one test setup and one measurement location.

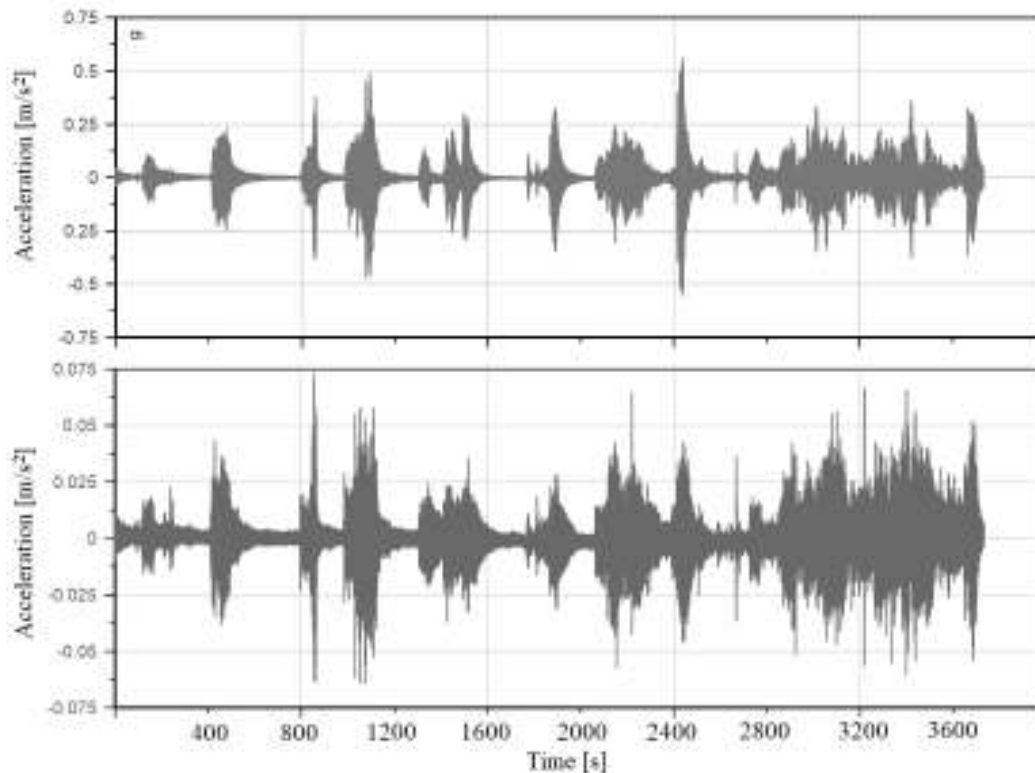


Figure 8 Vertical and lateral acceleration for test setup 1, in the reference position

Accelerations in many norms are highly related to the comfort criteria. As shown in Figure 8, the footbridge is very lively in both, vertical and horizontal direction, and these acceleration values under everyday conditions are unneglectable.

#### 4. NUMERICAL FE MODEL

The FE model was developed in software CSI SAP2000 Version 20.0.0. Bridge structure data - material characteristics, profiles, and geometry were obtained from the original main design.

The cables were modelled by "cable" elements, and the suspenders were modelled as simple rods. The weight of the cables and suspenders was cancelled to avoid local vibrations of these elements. 8 cm thick concrete plate was modelled as an "area" section, with shell element. Horizontal bracings coupled with concrete plate in the upper zone, horizontal bracings in the lower zone, and main truss beam profiles, were modelled as "frame" elements.

Since experimentally and numerically obtained shape vectors and frequencies showed differences in values, the initial model was updated based on experimentally obtained results.

Parameters modified in model updating were supports, the properties of rail beams, cable properties, plate properties and the weight of asphalt layer over the concrete deck.

Previously mentioned MAC procedure (1) was used to compare mode shapes that originate from the FE model and OMA. If diagonal values of the MAC matrix are very high (close to 1), and off-diagonal values are very low (close to 0), the FE model is well calibrated.

##### 4.1. Initial FE model

For preliminary dynamic analysis, in order to define measuring locations, initial numerical model was developed (Figure 9).



Figure 9 Initial FEM model

In the initial model, supports were modelled as stiff, and rail and cable properties were defined as in project documentation.

Initial material parameters were chosen based on project documentation and experience, since no laboratory tests for material properties were performed.

As a first step for model updating, quantification of the extent of differences between the initial model frequencies and the ones obtained by OMA is performed, as shown in Table 1.

In the initial model, several modes have close frequency values, but it is emphasized in the case of the first two modes. Compared to the measurement results, the first two mode shapes are reversed.

**Table 1** Frequencies gained in numerical analysis for initial model – “L” stands for lateral, “V” for vertical and “T” for torsional mode shape

Mode number	Measured frequencies		FE initial model		
	Mode shape type	Measured frequencies (Hz)	Mode shape type	Calculated frequencies (Hz)	Difference %
1. mode	V1	1.54	L1	1.79	11.2*
2. mode	L1	1.61	V1	1.81	17.5*
3. mode	V2	2.07	V2	3.30	59.4
4. mode	T1	3.81	T1	3.42	10.2
5. mode	V3	4.25	V3	4.52	6.4
6. mode	V4	6.15	L2	4.76	-

\* Compared to corresponding mode shape

In Table 2, comparison of experimentally and numerically obtained mode shapes is shown.

**Table 2** MAC table for initial FE model

		SAP2000						
		L1	V1	V2	T1	V3	L2	
		1.79	1.81	3.30	3.42	4.52	4.76	
ARTEMIS	V1	1.54	0.010	0.978	0.074	0.000	0.000	0.000
	L1	1.61	0.988	0.003	0.002	0.005	0.000	0.062
	V2	2.07	0.000	0.068	0.985	0.001	0.028	0.000
	T1	3.81	0.005	0.002	0.001	0.283	0.092	0.060
	V3	4.25	0.001	0.035	0.016	0.007	0.964	0.002

## 4.2. FE model updating

Since obtained modal shapes and frequencies did not match measured ones, parameters influencing the dynamic response of the structure were updated to improve the model.

Updating was carried out to minimize the differences between the numerically and experimentally obtained modal frequencies and shapes, so several relevant structure and material parameters were considered.

As for the initial model, differences between obtained frequencies for the updated models were calculated, and mode shapes were compared using MAC procedure.

Some of the model updating procedures are shown in [8]-[10]. In this paper, manual updating results were presented.

### 4.2.1. Selection of updating parameters

In Figure 10, bridge details that affected the uncertainties in the modelling of the structure are shown. Considered parameters for updating were the stiffness of supports, properties of rail beams (which affect the main truss properties and also stiffness of pylons), cable properties, concrete plate properties and the weight of asphalt layer over the concrete deck.





Figure 10. Structure details: pylon, coupled pavement structure, vertical truss beam and bracings

For *rail frame* element in different model variants, torsional moment of inertia, a transverse bending moment of inertia and a vertical bending moment of inertia were modified, in rational amount, as also material parameters of elements in terms of weight and Young's modulus.

*Cable properties* modified were Young's modulus and cross-section in small percentages (maximum 3%).

Parameter that mostly affected mode shapes was the *stiffness of supports*. As footbridge has supports on lower profiles of vertical trusses in 3 points, these supports were modelled as springs, with different lateral and vertical stiffness.

Bending and lateral *stiffness of deck*, and *asphalt cover weight*, were also varied.

#### 4.2.2. Manual model updating

In total, there were 5 updating elements, each having several properties that have been varied in FE modeling. The total number of considered updating parameters was 12.

To reverse the first two modes in the initial model – vertical and horizontal stiffness of supports and plate parameters were modified, as this had the biggest impact on modal frequencies. Also, to tune frequency values, an asphalt mass, rail frame element properties and cable properties were modified.

The results of two updated FE model solutions are presented herein.

So, in the first modified FE model, the stiffness of supports and plate parameters were modified. In the second modified model, in addition to the modifications made in the first model, the weight of an asphalt layer, rail frame element properties, and cable properties were modified.

Differences in frequencies compared to measurement results are shown in Table 3.

Table 3. Frequencies gained in numerical analysis for two FE models

Mode number / Mode shape type	Experimental results	FE model 1 - updated model -		FE model 2 - updated model -	
	Measured frequencies (Hz)	Calculated frequencies (Hz)	Difference %	Calculated frequencies (Hz)	Difference %
1. mode – V1	1.54	1.62	5.2	1.46	5.2
2. mode – L1	1.61	1.71	6.2	1.50	6.8
3. mode – V2	2.07	2.11	1.9	2.03	1.9
4. mode – T1	3.81	2.91	23.6	3.36	11.8
5. mode – V3	4.25	4.09	3.7	4.12	3.1

As shown in Table 3, for the first updated model, differences in frequencies are somehow lower for the lower modes (2 and 3), compared to the second model. As for higher modes, 4 and 5, differences in frequencies are lower in the case of the second updated model.

In the comparison, only five first modes were shown, as 6th measured mode was 4th vertical, while in numerical model 2nd lateral mode is obtained as 6th.

Mode shape MAC matrix, for the first FE model is shown in Table 4.

Table 4. MAC table for FE model 1

		SAP2000						
		V1	L1	V2	T1	L2	V3	
		1.62	1.71	2.11	2.91	4.09	4.84	
ARTEMIS	V1	1.54	0.986	0.000	0.053	0.000	0.000	0.053
	L1	1.61	0.001	0.938	0.001	0.009	0.063	0.000
	V2	2.07	0.093	0.000	0.975	0.001	0.000	0.005
	T1	3.81	0.000	0.004	0.002	0.294	0.005	0.051
	V3	4.25	0.000	0.001	0.021	0.006	0.005	0.944

The second modified FE model showed the best correlation with the experimental results, regarding mode shape vectors. Max matrix for the second model is shown in Table 5.

Table 5. MAC table for FE model 2

		SAP2000						
		V1	L1	V2	T1	V3	L2	
		1.46	1.50	2.03	3.36	4.12	4.31	
ARTEMIS	V1	1.54	0.993	0.003	0.065	0.000	0.006	0.000
	L1	1.61	0.000	0.994	0.002	0.002	0.000	0.061
	V2	2.07	0.080	0.000	0.990	0.001	0.021	0.000
	T1	3.81	0.001	0.004	0.001	0.289	0.083	0.047
	V3	4.25	0.016	0.001	0.019	0.005	0.984	0.002

Mode shapes for the second modified FE model are shown on Figure 11.

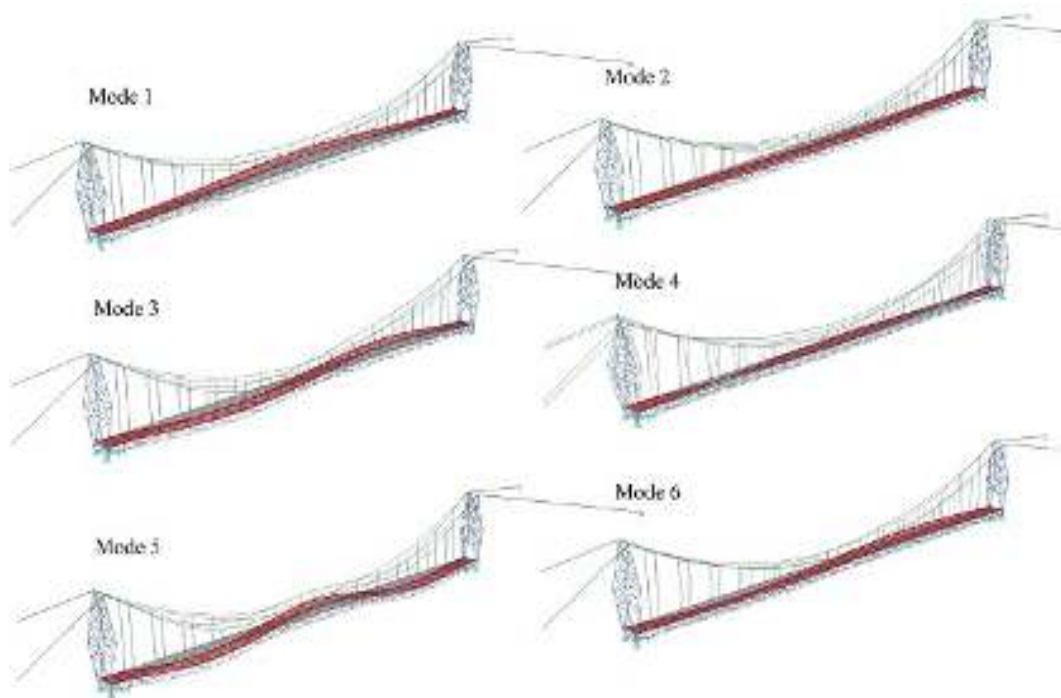


Figure 11. Mode shapes for updated FM model

An obvious difference between the numerically and experimentally obtained results is that the second horizontal mode, shown in the FE model, was not found in the measurement results.

#### 4.2.3. Updated model analysis and further updating recommendations

As can be noted, the first torsional mode showed a low correlation with measurements, and the reason can be insimetricity of suspenders, several slack suspenders and also some deterioration damages, that were not considered in this stage of analysis.

Also, in the analysis of measuring results, the second horizontal mode was not detected in the first nine modes of vibration, unlike in the FE model. The possible reason could be that it was not „awakened“ enough, or/and its frequency was closely spaced with another mode and was not detected during the measurement result analysis.

In the process of finding an optimal solution for FE model, many data are unknown, and many parameters affect the mode shapes. Regarding that, it is difficult, and not practical to choose the best fit manually. Numerous combinations can be made considering possible structural and material properties. Also, subject footbridge has several deterioration damages, that also affect dynamic behavior, and should not be omitted from model updating.

Manual analysis is a good starting point for the further analysis of optimum parameters defining dynamic behavior, as it can indicate parameters that have the biggest impact on this behavior.

All parameters that affect dynamic behavior should be considered with a certain distribution of possible values, and the best modal shape and frequency match should be found using a probabilistic approach, and software with solver tools.

## 5. ANALYSIS AND CONCLUSIONS

An ambient vibration modal identification of so-called lively footbridge was performed, based on the Frequency Domain Decomposition method and Multiple Test Setups Measurement Procedure.

Results obtained by presented measuring layout and using described parameters in the process of measurement and data processing, resulted in clear modal frequency values and mode shapes, for five vertical, three torsional and one lateral mode.

In ARTeMIS software package, candidate modes were estimated by the “Peak picking” method and the obtained results were verified using MAC procedure.

For the subject bridge, the numerical model was developed using main design documentation, after visual verification of design parameters.

After comparing to the measurement results, numerical model was manually updated, choosing relevant updating parameters. The updated model’s frequencies were compared to the experimental results in terms of a difference in percentages. Numerical comparison of mode shapes obtained in FE models and by OMA was performed using, again, MAC formulation.

Considering that the bridge has several deterioration damages, geometrical irregularities (mostly regarding cable sag and suspenders), and also some other unknown parameters (as the behavior of the supports, and properties of rail beams), manually modified parameters provided a good correlation for the updated model’s values and measurement results, as shown in MAC tabular results.

Although the numerical model, manually updated, showed a relatively good fit with the measurement results, considering the nature of the bridge, further update is recommended using probabilistic analysis. Manual FE model update is a good starting point, as it helps in understanding the dynamic behavior of the subject structure, in terms of the effects of the main structure parameters on modal properties.

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