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MODEL OF POROUS MATERIALS BY RHEOLOGICAL-DYNAMICAL ANALOGY USING THE PRINCIPLES OF MASS AND ENERGY CONSERVATION

Abstract

It is assumed that the porous material is based on the principle of conservation of mass and the principle of conservation of energy. The validity of both principles relies on experimental observations. Experimental results of different metals were used to compare the Poisson ratio as a function of porosity. A comparison is made between the theory of percolation and the model proposed in this paper based on the rheological-dynamic analogy (RDA). The results show that there is an excellent agreement between percolation theory and the RDA model. Finally, a new relationship between the creep coefficient and porosity was proved for all analyzed metals by defining their damage variable in the range of measurable porosities.

Keywords: Porous materials; Mass and energy conservation; Percolation theory; RDA model, Metals.

МОДЕЛ ПОРОЗНИХ МАТЕРИЈАЛА РЕОЛОШКО-ДИНАМИЧКОМ АНАЛОГИЈОМ ПРИМЕНОМ ПРИНЦИПА ОЧУВАЊА МАСЕ И ЕНЕРГИЈЕ

Сажетак

Претпоставља се да је порозни материјал заснован на принципима очувања масе и енергије. Ваљаност оба принципа се заснива на експерименталним запажањима. Експериментални резултати различитих метала коришћени су за поређење Поасоновог коефицијента као функције од порозности. Упоређена је теорија перколације са моделом предложеним у овом раду који се заснива на реолошко-динамичкој аналогији (РДА). Резултати показују да постоји одлично слагање између теорије перколације и РДА модела. Коначно, доказана је нова веза између коефицијента течења и порозности за све анализиране метале дефинисањем њихове варијабле оштећења у опсегу мерљивих порозности.

Кључне ријечи: Порозни материјали; Очување масе и енергије; Теорија перколације; РДА модел, Метали.

1. INTRODUCTION

So far, significant work has been done to connect the mechanical properties of the material with its microstructure. In particular, relationships have been established between porosity and key mechanical properties such as strength and modulus of elasticity. However, the usefulness and physical significance of many of these relationships are often unclear as most theoretical models are based on some idealized physical microstructure, and the resulting correlations often cannot be applied to actual materials and practical applications, [1].

Spriggs' empirical equation for Young's modulus of porous materials [2], and the similar Ryshkewitch-Duckworth equation for the strength of porous materials [3], have long been accepted from the literature. Also, Phani and Niyogi derived a semi-empirical equation to describe the porosity dependence of Young's modulus of brittle solids [4]. Further, Wang theoretically obtained the relationship between porosity and Young's modulus for porous materials made by powder metallurgy, [5]. The dependence is applicable to the entire range of porosity and is able to treat the transition of the pore structure from interconnected to isolated. However, later, advances in predicting the elastic properties of porous materials over the entire porosity range were closely related to the semi-empirical relationship to Phani and Niyogi, [4].

Porous metals are preferably prepared from powder, the size, and shape of which can vary considerably. During powder consolidation, different porosities can be achieved by varying technological parameters such as temperature, external pressure, or time. Compaction begins only by touching the powder particles and goes to a lower porosity by creating and growing a throat between the particles. Subsequent closure of the pore channels leads to the elimination of the pores. Analyzing these metallic materials, Kovačik concluded that the Phani-Niyogi equation is identical to the equation of percolation theory for Young's behavior and the shear modulus with porosity, [6]. Kovačik determined the percolation threshold for porosity at which the effective Young and the shear modulus become zero. However, the theory of percolation is beyond the content of this paper. This paper only uses the results presented by Kovačik [7] for comparison with the results of the RDA model.

The topic of this paper is related to the RDA model. Taking into account the RDA, the rheological behavior of the sample can be characterized by only one parameter, i.e., the dynamic time of retardation $T^D = 1/\omega$, where ω is the natural angular frequency of the discrete dynamic model. The RDA model has the same phase angle as the discrete dynamic model with damping in the steady state vibration, so from that fact the coefficient of viscous damping was obtained by Milašinović, [8]. Milašinović has already explained the RDA model of material behavior for axially cyclically loaded bars on a macro scale in order to predict their fatigue behavior, [9]. The efficient numerical implementation of RDA and its practical application was also studied by Milašinović for the visco-elasto-plastic behavior of metallic bars where the load function for the Hencky theory is derived, [10]. The main goal of this paper is to predict the relationship between creep coefficient and porosity of materials. It is considered that the principle of conservation of mass and energy is valid during the wave movement between two cross-sections of the sample, regardless of how the bar material behaves in terms of size and arrangement of material particles.

2. POISSON RATIO OF POROUS MATERIALS USING RHEOLOGICAL-DYNAMICAL ANALOGY

Mechanical disturbance (deformation) propagates in an elastic sample at phase velocity v_0. Thus, the deformations, which are initiated at the moment t_0 of the wave source, reach an arbitrary point M of the sample at the moment $t \rangle t_0$. The larger the path l of the wave that travels from its source to the point M, the greater the difference $t - t_0$. Accordingly, the vibration at the point M lags behind that at the source of the wave. If l_0 is the distance between the two ends of the sample, follows

$$T^D = t - t_0 = \frac{l_0}{v_0}. \qquad (1)$$

During a small time interval T^D the total strain energy density will move so that the fictitious area A_1 of the cross section 1 will have moved a short distance Δl_1. In the same time the cross section area A_2 will have moved a greater distance Δl_2 such that

$$A_1 \Delta l_1 = A_2 \Delta l_2 = V, \qquad (2)$$

where V is the fictitious volume.

Energy has passed through a fictitious volume without physically transferring material from the source by any cross section in time T^D. Thus, the continuity equation derives from the principle of mass conservation

$$A_1 v_1 = A_2 v_2, \qquad (3)$$

where v_1 and v_2 are the velocities.

Therefore,

$$v_2 = \frac{A_1}{A_2} v_1. \qquad (4)$$

The energy equation is the result of applying the principle of energy conservation to a stable energy transfer. Thus, the reduced area A_2 can now be found by applying Bernoulli's energy theorem

$$\sigma_1 + \frac{1}{2} \rho v_1^2 = \sigma_2 + \frac{1}{2} \rho v_2^2, \qquad (5)$$

where σ_1 and σ_2 are the stresses and ρ is the mass density.

Due to (4), equation (5) becomes

$$\sigma_1 + \frac{1}{2} \rho v_1^2 = \sigma_2 + \frac{1}{2} \rho \left(\frac{A_1}{A_2} \right)^2 v_1^2. \qquad (6)$$

Since the velocity v_1 is the phase velocity

$$v_1 = v_0 = \sqrt{\frac{E_H}{\rho}}, \qquad (7)$$

where E_H is the Young modulus, (6) and (7) imply

$$A_2 = \frac{A_1}{\sqrt{\dfrac{2(\sigma_1 - \sigma_2)}{E_H} + 1}}, \qquad (8)$$

where $\sigma_1 = \sigma_E$ is the elastic stress.

2.1. RELATIONSHIP BETWEEN POISSON RATIO AND CREEP COEFFICIENT

Milašinović, [9], defined the endurance limit $\sigma_e(R)$, where the ratio of minimum and maximum stress is R. In the symmetrical cycle ($R = -1$) follows

$$\sigma_2 = \sigma_e(R) = \frac{\sigma_1}{1 + \varphi^*}, \qquad (9)$$

where φ^* is the structural-material creep coefficient. Taking $\sigma_2 = \sigma_e(R)$, the reduced area A_2 is

$$A_2 = \frac{A_1}{\sqrt{\dfrac{2\sigma_E}{E_H} \left(\dfrac{\varphi^*}{1 + \varphi^*} \right) + 1}}. \qquad (10)$$

Experiments show that under tension, the length of the cylindrical sample increases by Δl, while its diameter Φ_1 decreases. Longitudinal strain can be found by applying Hooke's law

$$\varepsilon_l = \frac{\sigma_E}{E_H} . \qquad (11)$$

If the bar is subjected to tensile or compressive stress in a given direction, not only strain occurs in that direction (longitudinal strain), but also strains in directions perpendicular to it (transverse strains). Within the range of elastic action, the relationship between transverse and longitudinal strain under uniaxial loading conditions is called the Poisson ratio. The transverse strain is

$$\varepsilon_t = \mu_0^* \varepsilon_l , \qquad (12)$$

where μ_0^* is Poisson's ratio of solid material.

On the other hand the transverse strain is

$$\varepsilon_t = \frac{\Phi_1 - \Phi_2}{\Phi_1} . \qquad (13)$$

Φ_1 is the sample diameter while Φ_2 is the reduced diameter.

Hence,

$$\mu_0^* = \frac{\Phi_1 - \Phi_2}{\Phi_1} \frac{E_H}{\sigma_E} . \qquad (14)$$

The reduced sample diameter Φ_2 for all $0 \langle \varphi \langle \varphi^*$ can be obtained using equation(10),

$$\Phi_2 = \frac{\Phi_1}{\sqrt[4]{\dfrac{2\sigma_E}{E_H}\left(\dfrac{\varphi}{1+\varphi}\right)+1}} , \qquad (15)$$

so the relationship between the Poisson's ratio and the creep coefficient is

$$\mu(\varphi) = \left[1 - \frac{1}{\sqrt[4]{\dfrac{2\sigma_E}{E_H}\left(\dfrac{\varphi}{1+\varphi}\right)+1}} \right] \frac{E_H}{\sigma_E} . \qquad (16)$$

The assumption of incompressibility is supported by the fact that the bulk modulus is three orders of magnitude larger than the shear modulus. As a result, compressibility can significantly affect the stress distribution, especially when the Poisson ratio, i.e., μ_0^* approaches 0.5. Generally speaking, it is difficult to experimentally determine the bulk modulus, and only a few special experiments can be found in the literature. For most structural materials, the Poisson ratio has values ranging from one-third to one-fifth; therefore, with ordinary measuring devices, the accuracy of lateral deformation measurements is not as high as with the corresponding axial strain measurements. This discussion of the relationship between variables shows that we can also use E_H and μ_0^* as constants and get the following expression for the creep coefficient φ, which is a new characteristic of the strain state

$$\varphi(\mu) = \left[\left(\frac{1}{1-\dfrac{\mu\sigma_E}{E_H}} \right)^4 - 1 \right] \frac{E_H}{2\sigma_E} \Bigg/ \left\{ 1 - \left[\left(\frac{1}{1-\dfrac{\mu\sigma_E}{E_H}} \right)^4 - 1 \right] \frac{E_H}{2\sigma_E} \right\} . \qquad (17)$$
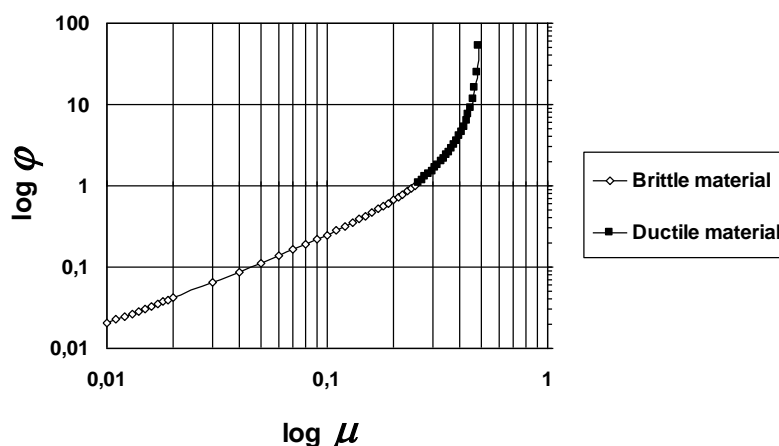
The function $\varphi(\mu)$ is shown in Figure 1.

Figure 1. *Creep coefficient versus Poisson's ratio*

The general approach to the analysis of the problem of the influence of the Poisson's ratio on the development of the creep coefficient must be based on Eq.(17). Assuming that, at the limit of elasticity $\sigma_E/E_H = 0.001$, it follows

$$\varphi(\mu)=\left[\left(\frac{1}{1-0.001\mu}\right)^4-1\right]\frac{1}{2\cdot0.001}\bigg/\left\{1-\left[\left(\frac{1}{1-0.001\mu}\right)^4-1\right]\frac{1}{2\cdot0.001}\right\}. \quad (18)$$

Assuming that μ_0^* is 1/3 for metals, we get $\varphi^* = 2$. The semi-brittle material is determined by the linear dependence of $log\,\varphi$ on $log\,\mu$, as shown in Figure 1. According to this criterion, the upper limit value of the Poisson's ratio for semi-brittle materials is 0.25.

The relationship given by Eq.(18) can be simplified by neglecting the products of second-order exponents, [11]:

$$\left[\left(\frac{1}{1-0.001\mu}\right)^4-1\right]\frac{1}{2\cdot0.001}=$$

$$\left\{\frac{1}{\left[1-0.002\mu+\left(0.001\mu\right)^2\right]\left[1-0.002\mu+\left(0.001\mu\right)^2\right]}-1\right\}\frac{1}{0.002}\approx \quad (19)$$

$$\left[\frac{1}{(1-0.002\mu)(1-0.002\mu)}-1\right]\frac{1}{0.002}=\left[\frac{1}{1-0.004\mu+\left(0.002\mu\right)^2}-1\right]\frac{1}{0.002}\approx$$

$$\left(\frac{1}{1-0.004\mu}-1\right)\frac{1}{0.002}=\left(\frac{0.004\mu}{1-0.004\mu}\right)\frac{1}{0.002}=\frac{2\mu}{1-0.004\mu}\approx2\mu$$

so the relationship between the creep coefficient and the Poisson's ratio is

$$\varphi=\frac{2\mu}{1-2\mu}. \quad (20)$$

If the creep coefficient is a known value, follows

$$\mu=\frac{\varphi}{2(1+\varphi)}. \quad (21)$$

2.2. RELATIONSHIP BETWEEN CREEP COEFFICIENT AND POROSITY

Creep can occur in metals, plastics, rubber, glass, concrete and many other materials. Copper, iron, nickel and their alloys will exhibit creep at high temperatures. When a constant force is applied, some materials gradually deform over time and the result is an increase in length. However, in the

study of material fatigue [9], a very short time T^D was used (T^D represents the time delay that a wave of velocity v_0 takes to traverse the length l_0), due to which the creep coefficient was converted into a quotient of two modules

$$\varphi^* = \frac{E_H}{E_K}, \qquad (22)$$

where E_K is the modulus of viscoelasticity. The creep coefficient is a linear function of both p and E_H, while E_K is independent of porosity and constant of the material, as a consequence of the principle of mass conservation. Figure 2 shows the linear dependence of the creep coefficient on the porosity. φ^* is the creep coefficient at zero porosity while φ_E is the creep coefficient at the end of the porosity range p_E. p_E is a known value as well as p_{max}, which is defined at zero creep coefficient. Considering the points $\left(0, \varphi^*\right)$ and $\left(p_E, \varphi_E\right)$, the linear dependence can be expressed by

$$\varphi(p) = \varphi^* - \frac{p\left(\varphi^* - \varphi_E\right)}{p_E}. \qquad (23)$$

On the other hand, due to points $\left(p_E, \varphi_E\right)$ and $\left(p_{max}, 0\right)$, the linear dependence is

$$\varphi(p) = \varphi^*\left(1 - \frac{p}{p_{max}}\right). \qquad (24)$$
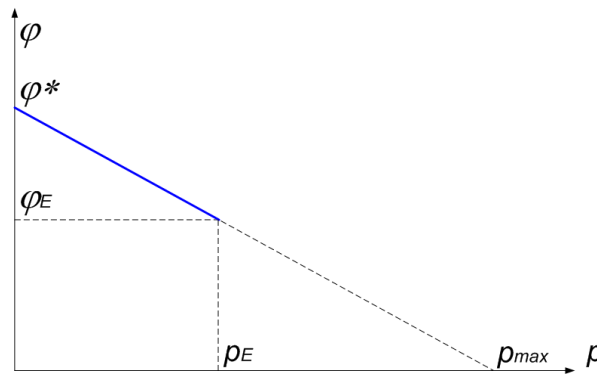

Figure 2. *Creep coefficient versus porosity*

2.3. POISSON RATIO OF POROUS MATERIALS

The relationship between the Poisson ratio and the porosity follows on the basis of Eq.(21)

$$\mu_{RDA}(p) = \left[\varphi^* - \frac{p\left(\varphi^* - \varphi_E\right)}{p_E}\right] \Bigg/ \left\{2\left[1 + \varphi^* - \frac{p\left(\varphi^* - \varphi_E\right)}{p_E}\right]\right\}, \qquad (25)$$

where

$$\varphi^* = \frac{2\mu_0^*}{1 - 2\mu_0^*}. \qquad (26)$$

μ_0^* is the measured value.

The creep coefficient of the porous material at the end of the porosity range p_E is

$$\varphi_E = \frac{2\mu_E}{1 - 2\mu_E}, \qquad (27)$$

where μ_E must be defined.

2.4. CRITICAL DAMAGE VARIABLE OF POROUS MATERIALS

Since the development of micro voids leads to a decrease in the stiffness of the material, Milašinović characterized the state of damage of isotropic materials with a critical scalar damage variable, [11], Figure 3.

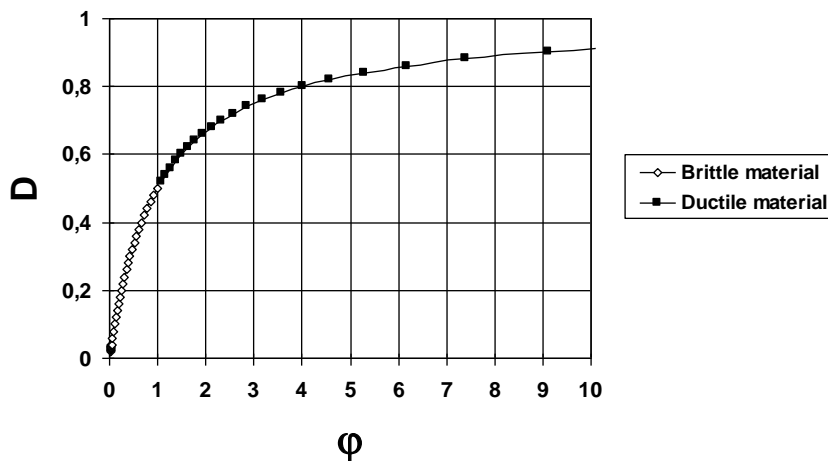$$D = \frac{\varphi}{1+\varphi}, \qquad (28)$$

where $0 \le D \le 1$.

Figure 3. *Critical damage variable versus creep coefficient*

In the case of porous materials, follows

$$D(p) = \left[\varphi^* - \frac{p(\varphi^* - \varphi_E)}{p_E}\right] \Bigg/ \left\{1 + \left[\varphi^* - \frac{p(\varphi^* - \varphi_E)}{p_E}\right]\right\}. \qquad (29)$$

Mot and Rondal, [12], found that all actual materials have a Poisson ratio between 0.2 and 0.5, so for these boundaries are $0.66 \le \varphi \le \infty$ and $0.4 \le D \le 1$. The critical state of elastic parameters of brittle elastic solids containing slits-critical, was also investigated by Krajčinović et al., [13]. According to Lemaitre, [14], the critical value of the damage variable corresponds to the breaking of elements into two parts, where $0.2 \le D \le 0.8$ is for metals. According to the RDA model, the critical damage variable for porous materials in the porosity range $0 \le p \le p_E$ is

$$\frac{\varphi^*}{1+\varphi^*} \ge D \ge \frac{\varphi_E}{1+\varphi_E}. \qquad (30)$$

3. POISSON RATIO OF POROUS MATERIALS USING PERCOLATION THEORY

Phani and Niyogi have developed the power exponent correlation for describing the dependence of Young's modulus and porosity of brittle solids [4]

$$E = E_0(1-ap)^n, \qquad (31)$$

where E and E_0 are the Young's moduli at porosity p and zero, respectively, a and n are material constants. The expression has been derived semi-empirically for describing the porosity dependence of Young's modulus of brittle solids. The equation satisfies quite well the exact theoretical solution for the values of Young's moduli at different porosities for model systems with ideal and non-ideal

packing geometry, where parameters a and n provide information about the packing geometry and pore structure of the material.

According to [6] and [15], Eq.(31) is identical with the percolation theory equation for the behavior of Young's and shear modulus with porosity:

$$E = E_0 \left(1 - \frac{p}{p_{max}}\right)^{f_E} \qquad for \qquad p \le p_{max}, \tag{32}$$

$$G = G_0 \left(1 - \frac{p}{p_{max}}\right)^{f_G} \qquad for \qquad p \le p_{max}, \tag{33}$$

where f_E is the characteristic exponent for the Young's modulus while f_G is the characteristic exponent for the shear modulus of the porous material.

p_{max} is the percolation threshold [16], i.e. the porosity at which the effective Young's E and shear modulus G become zero. From Eq.(22) it follows that $\varphi(p_{max})$ is zero when $E(p_{max})$ is zero, which means that the adopted percolation threshold p_{max} also corresponds to the RDA model.

After substituting Eqs. (32) and (33) in the equation

$$\mu = \frac{E}{2G} - 1, \tag{34}$$

the correlation between Poisson's ratio and porosity from the percolation theory was obtained by Kovačik in [7] as

$$\mu_{PT}(p) = \left(\mu_0^* + 1\right)\left(1 - \frac{p}{p_{max}}\right)^{f_E - f_G} - 1, \tag{35}$$

where

$$\mu_0^* = \frac{E_0}{2G_0} - 1. \tag{36}$$

Poisson's ratio μ_0^* of solid material corresponds to the calculated value using the velocities of P and S waves, [17].

4. MODEL VERIFICATION AND DISCUSSION

A systematic study of the variation of elastic moduli and Poisson's ratio of sintered iron compacts of porosity up to 21.6% has been carried out by measuring longitudinal and shear ultrasonic velocities, [18]. The variation of these parameters with porosity was compared with predictions of elasticity and scattering theories. Using these results, Kovačik [7] presented the results of the correlation of Poisson's ratio and porosity by percolation theory.

According to Table I from [7], for sintered iron are:

$\mu_0^* = 0.303$, $\quad p_{max} = 0.41$, $\quad p_E = 0.22$, $\quad f_E - f_G = 0.09$.

Hence,

$\mu_{PT}(0) = \mu_0^* = 0.303$,

$$\mu_{PT}(p_E) = \left(\mu_0^* + 1\right)\left(1 - \frac{p_E}{p_{max}}\right)^{f_E - f_G} - 1 = \left(0.303 + 1\right)\left(1 - \frac{0.22}{0.41}\right)^{0.09} - 1 = 0.21585.$$

According to the RDA model are:

$$\varphi^* = \frac{2\mu_0^*}{1 - 2\mu_0^*} = \frac{2 \cdot 0.303}{1 - 2 \cdot 0.303} = 1.53807,$$

$$\varphi_E = \frac{2\mu_{PT}(p_E)}{1 - 2\mu_{PT}(p_E)} = \frac{2 \cdot 0.21585}{1 - 2 \cdot 0.21585} = 0.75965.$$

Hence,

$$\mu_{RDA}(0) = \frac{\varphi^*}{2(1+\varphi^*)} = \mu_0^* = 0.303,$$

$$\mu_{RDA}(p_E) = \frac{\varphi_E}{2(1+\varphi_E)} = \frac{0.75965}{2(1+0.75965)} = 0.21585 = \mu_{PT}(p_E).$$

Figure 4 shows the comparison between the percolation theory and the RDA model in the analyzed porosity range for sintered iron.
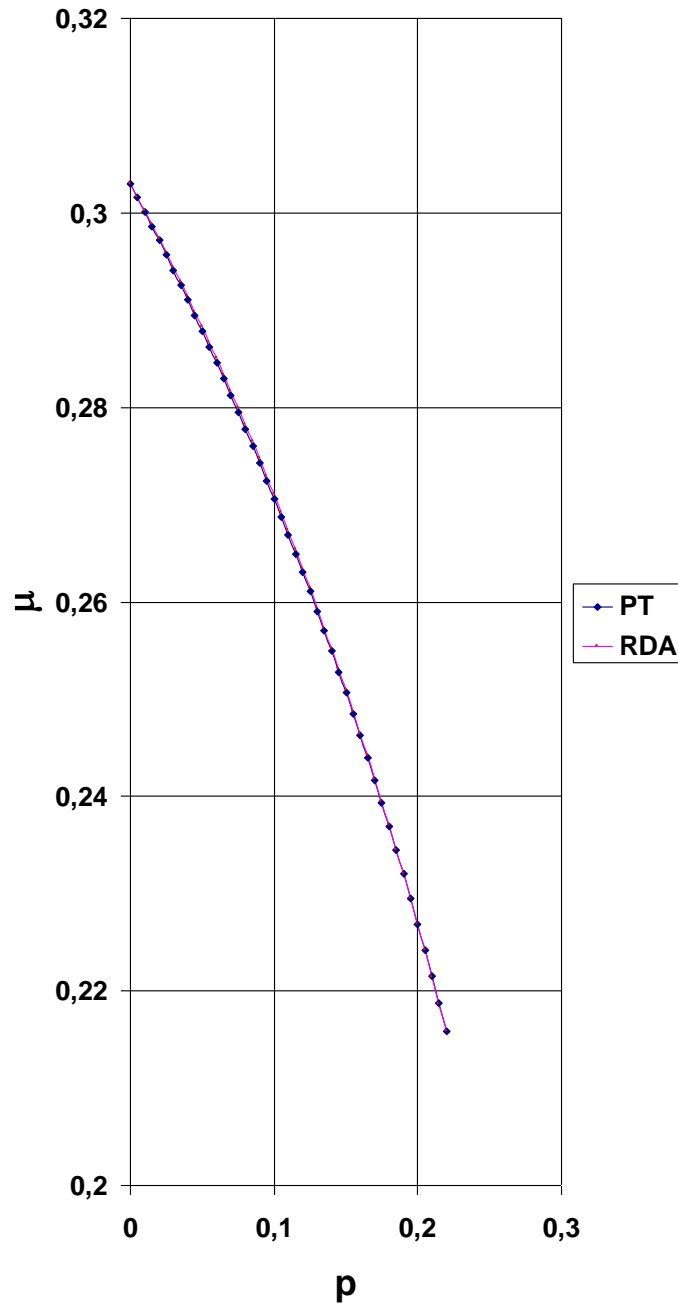

Figure 4. *Graphical comparison of Poisson's ratio versus porosity according to the percolation theory and the RDA model for sintered iron from Figure 1 in [7]*

The curve of the function of difference $\mu_{PT}(p) - \mu_{RDA}(p)$ for the interval $[0, \ p_E]$ is shown in Figure 5. It can be concluded that the differences are very small, i.e., insignificant in this example.
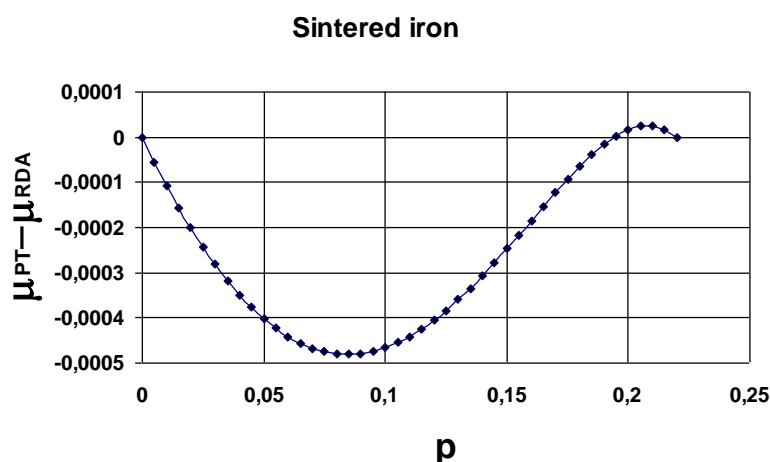
Figure 5. *Differences between the percolation theory and the RDA model for sintered iron from Table I in [7]*

At the limits of the porosity range, both models always give the same results. However, the theory of percolation gives the limit value of the Poisson's ratio -1 at the percolation threshold, while the damage variable of -2 is not a real value.

On the other hand, the RDA model gives the Poisson ratio close to zero at the percolation threshold

$$\mu_{RDA}\left(p_{max}\right)=\frac{\varphi^{*}-\dfrac{p_{max}\left(\varphi^{*}-\varphi_{E}\right)}{p_{E}}}{2\left[1+\varphi^{*}-\dfrac{p_{max}\left(\varphi^{*}-\varphi_{E}\right)}{p_{E}}\right]}=\frac{1.53807-\dfrac{0.41\left(1.53807-0.75965\right)}{0.22}}{2\left[1+1.53807-\dfrac{0.41\left(1.53807-0.75965\right)}{0.22}\right]}=0.04 .$$

Hence,

$$\varphi\left(p_{max}\right)=\frac{2\cdot0.04}{1-2\cdot0.04}=0.087 ,$$

$$D\left(p_{max}\right)=\frac{0.087}{1+0.087}=0.08 .$$

The damage variable of 0.08 is within the prescribed range of $0\le D\le1$, but is not in the range of critical values according to the RDA model, where $0.606\ge D\ge0.43$.

Although the negative Poisson's ratio for solid materials is not a measurable quantity, it is theoretically provable. According to the Cauchy–Hooke law for isotropic materials and as a consequence of the second law of thermodynamics, the following inequality must hold for isotropic materials, $-1\langle\mu\langle0.5$. However, in some older literature, the prevailing opinion was that the Poisson's ratio should always be positive for isotropic materials, which means that the results of the RDA are closer to reality.

On the other hand, the negative Poisson's ratio implies a negative damage variable, which is a very little researched theoretical case. Furthermore, the negative damage variable implies a negative crack density in the part of the range, [19]. In the appropriate regions, the actual cracks are replaced by the stiffening-rigid lamellae. These solid elements are referred to in the literature as negative cracks or anti-cracks, [20].

The differences in the results between the percolation theory and the RDA model depend primarily on the choice of the percolation threshold p_{max}, but also on the characteristic exponents f_E and f_G that are in the percolation formulas.

It is generally accepted that the value of the percolation threshold is a function of powder size, shape, size and distribution of shapes, and methods of preparation, [21]. In this paper, the RDA model is related to percolation theory in order to compare the results. The connecting parameter is the Poisson's ratio μ_E calculated according to the percolation theory at the limit p_E of the predicted porosity range. It is then included in the formulas according to the RDA model to obtain the creep

coefficient φ_E. The obtained differences for all analyzed metals are small and different, but they are a consequence of the choice of percolation threshold and exponents f_E and f_G.

The curves of the critical damage variables as a function of porosity for all analyzed metals in [7], but based on the RDA model, are shown in Figure 6. In the case of porous ZnO, the results show that the critical damage variable is probably independent of porosity.
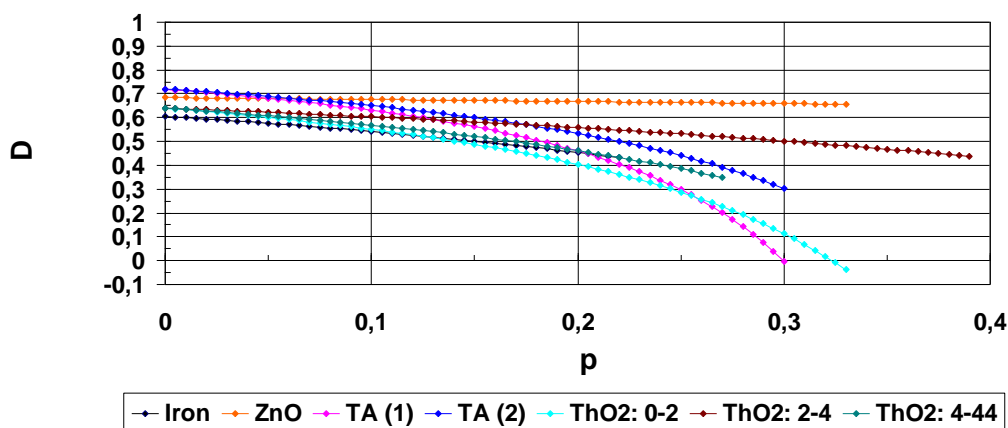

Figure 6. *Graphical comparison of the critical damage variable as a function of porosity for all analyzed metals [7] according to the RDA model*

5. CONCLUDING REMARKS

Percolation theory and the RDA model are based on completely different physical principles, so comparing their results with experimental ones is extremely important. Based on the principles of conservation of mass and energy during the wave movement between two cross sections of the sample, a new linear relationship between the creep coefficient and porosity is defined. The validity of this relationship is confirmed by comparing the dependence of the Poisson ratio on porosity according to the RDA model with the correlation results obtained according to the percolation theory.

It is obvious that the RDA model can be applied independently of the percolation theory, but it is first necessary to determine the limit of the porosity range p_E and Poisson ratio at that limit.

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