STOCHASTIC MULTI-ATTRIBUTE UTILITY MODEL

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ABSTRACT

In real situations, the attribute value (mostly variable) can be best represented by introducing the finite number of attribute values level, to which the corresponding probabilities should also be attached. Stochastic Multi-Attribute Utility Model has the ability to analyze such stochastic multi-attribute problems. The choice of one, from the set of available options, is made by choosing the best option based on the maximum expected utility structure. In this paper, we will mention some arguments for the development of the Stochastic Multi-Attribute Utility Model, its advantages (they are closer to reality), disadvantages (analytically difficult technique, subjective assessments of the values of variable attributes), as well as the process of solving the problem.

1. INTRODUCTION

Multi-Attribute Decision Making Methods (MADM) are defined to solve the problem of choosing between complex options in the conditions of certainty, uncertainty and risk. The decision-making problem is defined as the problem of choosing the one from the set of available options that the decision-maker (DM) evaluates on the basis of a number of characteristics (attributes). DM defines each option with its value vector according to the selected attributes, whereby the differences in the significance of individual attributes are expressed through different weight coefficients (ponders) of the attribute. The multi-attribute decision-making theory deals precisely with the procedures for determining the optimal option from the set of available options, which are based on evaluating the options for the selected set of attributes of different significance. Each option is evaluated with a numeric value; then the grades are compared and the best is identified.

Unfortunately, this is not always the case. In some cases, the attribute performance does not have to be determined (”bound”) for just one value. In real situations, the value, usually of variable attribute can be best displayed as the finite number of the attribute value level instead of one value. When considering, for example, the possible results of finding water, the results of drilling can not be presented simply as: ” water found” or ” water not found”. There is a range of possible values that represent a multitude of different, successive levels: from ”0” liters a day (”water not found” really) to 1,000 liters per day (which contains a large number of levels between). In this case, it is more realistic to present the value of each of the attributes as a number of special (separated) levels of value. These problems are solved by applying a *stochastic* multi-attribute utility model (MAUM).

Stochastic MAUM has the ability to analyze such stochastic multi-attribute problems. Stochastic MAUM is a technique that combines the multidimensional properties of MAUM with the ability to make decisions in risky situations (risk-management) from the set of determined options.

2. PREVIOUS RESEARCH

The rules of stochastic dominance and fuzzy approach were used to solve the problem of stochastic multi-attribute decision-making. Fuzzy approach (fuzzy sets) is used to reduce the number of attributes, and the rules of stochastic dominance (defined by Zaras, 2004) are used to determine the dominant relation of options on a smaller set of attributes.

The rules of stochastic dominance can be related to the option pairs by defining strict and/or weak preferences, as well as indifferent estimates between options on individual attributes (Nowak, 2004).

In addition to reducing the number of attributes, it is possible to gradually reduce the number of options according to the DM’s preferences. Thus, some authors (Nowak, 2007) use stochastic dominance rules when choosing a preferred option from the reduced set of options. Obviously, these transformations cause the loss of information.

The concept of stochastic dominance used to measure the dominance power of one option over another is achieved by applying some of the ranking methods, such as PROMETHEE II method (Zhang, 2010).

The values of the options by individual attributes, the intervals of the performance corrected by probabilities, represent a new stochastic degree of dominance. A modified (expanded) TOPSIS method (Yunna et al., 2017) is proposed, including normalization methods, identification of an ideal solution and decision-making process in accordance with the DM's preferences.

In order to solve the problems of multi-attributes (that each attribute value is a stochastic variable instead of the real number), the probability approximation method for solving stochastic attributes is being developed recently. A new method is also suggested (Shenghai et al., 2017) for the calculation of the weight values of stochastic attributes in an uncertain environment.

Non-classical approaches to multi-criteria analysis indicate that it is necessary to differentiate internal uncertainties (relating to values and estimates of decision-makers) and external uncertainties (relating to incomplete knowledge of the consequences of actions). Four broad approaches are considered (Figueira et al., 2016) to address external uncertainties. These are: multi-attribute theory of utility and its extensions; stochastic concept, primarily in the domain of comparing option pairs; the use of surrogate risk measures, such as additional decision criteria; and the integration of multi-attribute methods of decision-making and scenario planning.

Previous studies have significantly enriched the theory and techniques of stochastic multi-attribute problems. However, there are limitations in the existing methods. In the methods used by MAUM, it is difficult to determine the function of utility. Methods use trust indexes and preference indexes. The significance of these indexes is sometimes not easy to interpret.

3. STOCHASTIC MULTI-ATTRIBUTE UTILITY MODEL

Let us imagine that we have a number (set) of options that we estimate. Note "*i*" option of the total "*m*" options. This set of options is presented in the Figure 1.

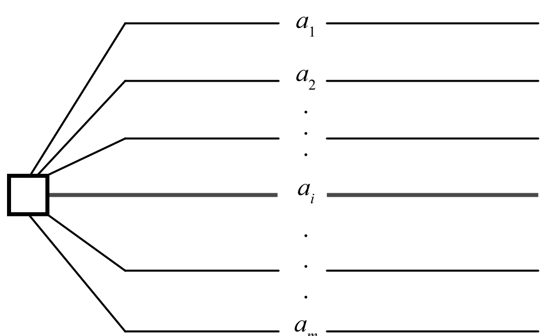

Figure 1: A specific set of options which will be estimated for the stochastic MAUM

In order to compare each of these options, without bias and any external influences, we need to use the same measure, which is a fixed set of attributes. Let us imagine that there is a total of "*n*" of attributes in the set. If we consider *j* attribute, then we have *j* attribute of *i* option, that is, $(x_j)_i$. It is presented in the Figure 2.

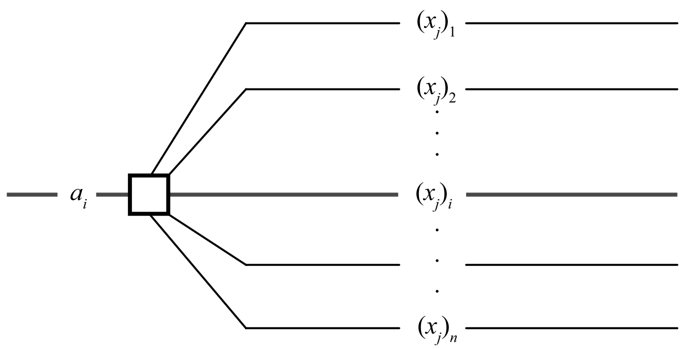

Figure 2: Specific attribute, $(x_j)_i$, from the total set of attributes which will be estimated with the stochastic MAUM

We note that there is a difference in designations compared to the original MAUM process. We will explain this difference in the following way. Each attribute not only has a unique ID, *j*, which is associated with a specific option, i.e. another index, *i*, but also has a *third* index that represents the stochastic property of an attribute. The third index indicates that a specific level of performance, *k*, from the total set of possible values, *q*, can be potentially "sustainable" for each attribute.

Illustration of *k* level of the attribute value for *i* option and *j* combination of attribute, $(x_j)_{ik}$, is shown in the Figure 3.

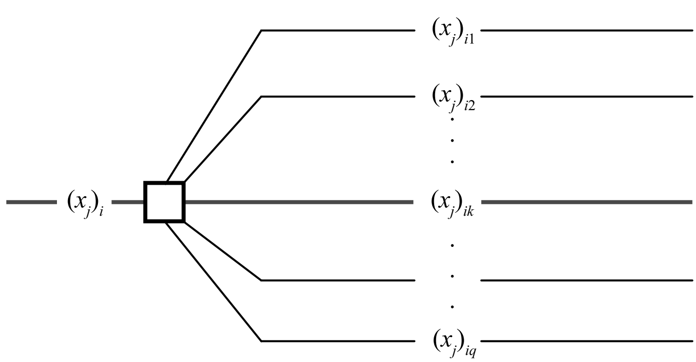

Figure 3: Illustration of *k* level of attribute performance $(x_j)_{ik}$

The process of the stochastic MAUM assessment consists of the following stages:

1. perform stages that are characteristic of the deterministic MAUM process, and refer to options, attributes, weights, limitations of values, and utility curves;
2. determine the value levels (probability distribution) for each attribute. This represents the point of separation of the deterministic and stochastic MAUM processes. The number of different, possible levels of value will depend on the very nature of each problem. It is believed that 3 to 5 levels of attribute values would be sufficient. This information should be displayed in the stochastic attribute value matrix. For example, we estimate the possible range of attribute values; let it be the rate of investment income, $(x_1)_{ik}$, which is between 9% and 18%. We further develop the model, based on the subjective assessments and using expert opinions in these estimates, probability distributions, $P(x_j)_{ik}$, for each combination of options and attribute performance through the range of values previously estimated. We can show this step in the Figure 4.

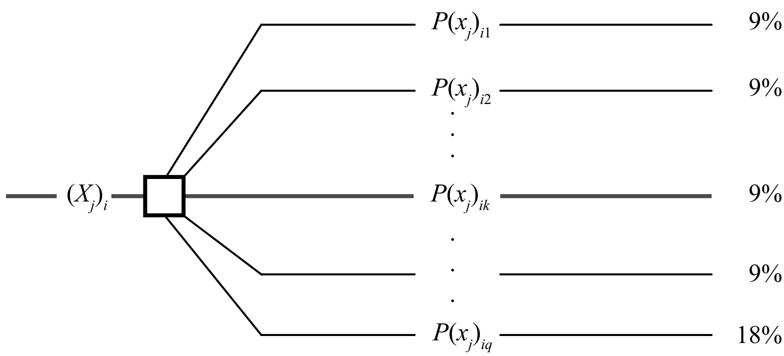

Figure 4: Probability distribution, $P(x_j)_{ik}$, for each combination of option and attribute value through the range of values

3. we proceed from the assumption that the expected value for each attribute, $EV(x_j)_i$, is sufficient to satisfy any limit of the value. This is done by multiplying the value of each attribute, $(y_j)_{ik}$, with the corresponding probability $P(y_j)_{ik}$. This assumption helps us to determine whether the minimum or maximum standard of the expected attribute performance is violated, eliminating from any further consideration any option that has one or more violations of the expected attribute value.

Thus, for the minimum expected attribute value we have:

$$EV(x_j)_i = \sum_{k=1}^{K} P(x_j)_{ik} \times (x_j)_{ik} \geq (x_j)_{\min}, \ \forall i, \ (i = 1,2,...,m), \ \forall j, \ (j = 1,2,...,n).$$

that is, for the maximum expected attribute value we have:

$$EV(x_j)_i = \sum_{k=1}^{K} P(x_j)_{ik} \times (x_j)_{ik} \leq (x_j)_{\max}, \ \forall i, \ (i = 1,2,...,m), \ \forall j, \ (j = 1,2,...,n).$$

4. for those options without limiting restrictions, it is necessary to convert the levels of values (*k* branches) from the second phase to the appropriate utilities. Here we use the utility curves from the first stage, i.e. from the deterministic MAUM process, for each combination of options and attributes, $(x_j)_i$;
5. calculate the expected utility of each attribute, x_j, by multiplying the utility of different levels of attribute values, $u(x_j)_{ik}$, with their corresponding probabilities, $P(x_j)_{ik}$. After that, we sum these products.

This step establishes one size of the expected utility for the given combination with one attribute and one option, $EU(x_j)_i$. So we have:

$$EU(x_j)_i = \sum_{k=1}^{q} P(x_j)_{ik} \times u(x_j)_{ik}, \ \forall i, \ (i = 1,2,...,m), \ \forall j, \ (j = 1,2,...,n).$$

Figure 5 represents it in this way:

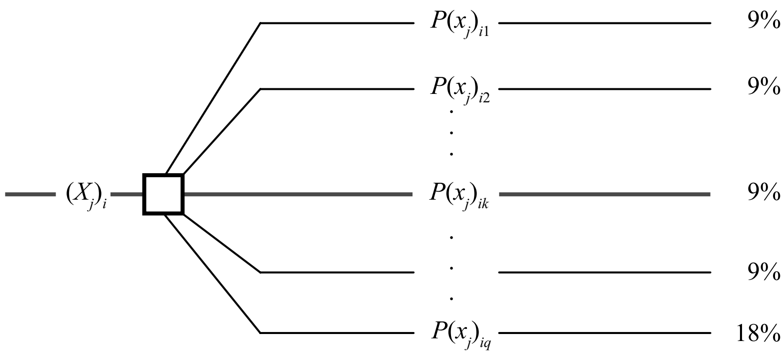

Figure 5: Illustration of the expected utility value for each attribute, $EU(x_j)_i$

6. calculate the complex (composite) expected utility of each attribute, $CEU[a_i]$. This value is obtained by multiplying the weight (relative importance) of the attribute with the expected value of the utility of each attribute, $EU(y_j)_i$, calculated in the previous stage.

This relation is given as:

$$CEU\left[a_i\right]=\sum_{j=1}^{n}EU(y_j)_i=\sum_{j=1}^{n}\sum_{k=1}^{q}w_j\times P(y_j)_{ik}\times u(y_j)_{ik},\ \forall i,\ (i=1,2,...,m),\ \forall j,\ (j=1,2,...,n),$$

where:

w_j – is a nominal weight of j attribute,

$P(y_j)_{ik}$ – is a probability of realization of k level of utility for j attribute and i option, $(y_j)_{ik}$,

$u(y_j)_{ik}$ – is the appropriate utility of the attribute, $(y_j)_{ik}$,

$EU(y_j)_i$ – is the expected utility of j attribute and i option.

From the expression,

$$EU(y_j)_i=\sum_{k=1}^{q}P(y_j)_{ik}\times u(y_j)_{ik},$$

we have:

m – is a total number of options,

n – is a total number of attributes and

q – is a total number of value levels for a specific attribute,

and finally, we get $CEU[a_i]$, the complex (composite, total) utility of i option.

7. the choice of the optimal strategy, i.e. the option with the highest complex (composite) utility. We can present this as a choice that is defined in the following way:

$$\max CEU[a_i]$$

Deterministic and stochastic MAUM are not analytically difficult techniques, but the aspects of the organization of these techniques are complicated. Therefore, it would be good to use appropriate computer software to calculate deterministic and stochastic models and methods whenever possible. There is easy and simple software to use when implementing the standard MAUM estimates. These software packages are: Microcomputer software for MAUM, Lotus 1-2-3, Quatro (if possible, Excel can be used). Specifically to solve the problems presented in this paper, we will mention the Confidence factor software package, which solves problems step by step as it is stated in the theoretical part. Unfortunately until now, the products of stochastic MAUM software are not available for us.

4. THE APPLICATION OF THE STOCHASTIC MULTI-ATTRIBUTE UTILITY MODEL

The decision-maker examines three options for investing:

1. the place of residence for one family in New Belgrade, (a_1),
2. the property under lease at Avala, with 33% ownership, (a_2),
3. the apartment in the center of Belgrade with three bedrooms in Knez Mihajlova, (a_3).

When choosing, he defines the most important factors that determine his investment decision as:

1. level of investment income, $(y_1)_i$,
2. share, $(y_2)_i$,
3. interest rate, $(y_3)_i$.

DM eliminates any option whose value does not meet a certain minimum (or maximum) value. Each option will be eliminated if one or more of the following three minimum standards are not present:

1. provide a minimum income of 12%,
2. do not require more than € 200,000 for his part in the share (including all financial obligations related to it),
3. do not demand more than the expected 12% interest rate on the loan.

In addition, DM determines the relative importance of attributes in ratio: 50%: 35%: 15%; for the level of investment income, stake and monetary rate, respectively. Let us start with the process of applying the MAUM assessment. The options, the importance of attributes, constraints, and utilities are given in the table 1.

Table 1: Attributes, weights and restrictions on real estate investment

Attributes for real estate investment			
Attributes	Name	Relative importance	Limit values
y_1	Level of investment income	50%	≥ 12%
y_2	Share	35%	≤€ 200,000
y_3	Interest rate	15%	≤ 12%

Let us define a range of values (probability distribution) for each attribute. DM estimates that this information can be presented in the table 2:

Table 2: Value levels for individual attributes

Value levels for individual attributes	
Attributes	Level of value
Level of income, $(y_1)_{ik}$	between 9% and 18%
Share, $(y_2)_{ik}$	between € 50,000 and € 25,.000
Interest rate, $(y_3)_{ik}$	between 9% and 13%

The expected values for the *level of income* are:

$EV(y_1)_1$=13.5%

$EV(y_1)_2$=14.7%

$EV(y_1)_3$=12.9%.

The expected value for all options and attributes clearly shows that there is no violation of the limit values (between 9% and 18%, ≥ 12%).

The expected values for the *share* are:

$EV(y_2)_1$=120,000,

$EV(y_2)_2$=155,000,

$EV(y_2)_3$=105,000.

The expected value for all options and attributes clearly shows that there is no violation of the limit values (between € 50,000 and € 250,000, ≤ € 200,000).

The expected values for the *interest rate* are:

$EV(y_3)_1$=10.6%

$EV(y_3)_2$=9.5%

$EV(y_3)_3$=9.6%.

The expected value for all options and attributes clearly shows that there is no violation of the limit values (between 9% and 13%, ≤ 12%).

The expected utility for the *level of income* is:

$EU(y_1)_1$=0.606 utility,

$EU(y_1)_2$=0.728 utility,

$EU(y_1)_3$=0.568 utility.

The expected utility for the *share* is:

$EU(y_2)_1$=0.396 utility,

$EU(y_1)_2$=0.331 utility,

$EU(y_1)_3$=0.499 utility.

The expected utility for the *interest rate* is:

$EU(y_3)_1$=0.517 utility,

$EU(y_3)_2$=0.875 utility,

$EU(y_3)_3$=0.830 utility.

Let us calculate the expected utility ($EU(y_1)_1$) for the first option, a family home, a_1. First, we determine the contribution of expected utility from the income rate attribute, $(y_1)_1$, by multiplying each level of distribution of probability, $P(y_1)_{1k}$, with its utility, $u(y_1)_{1k}$. We get
$EU(y_1)_1$=0,606 utility.

Calculating the remaining two expected utility for attributes $(y_1)_2$ - share and $(y_1)_3$ - interest rate, is a simple operation:

$EU(y_1)_2$=0.396 utility,

$EU(y_1)_3$=0.517 utility.

Now we can calculate the complex (composite) expected utility of each attribute, CEU[a_i]. This value is obtained by multiplying the weight (relative significance) of the attribute, w_j, with the expected value-utility of each attribute, $EU(y_j)_i$.

Therefore, for a_1, we have $CEU\left[a_1\right]=\sum_{j=1}^{n} w_j u_{1j}=0{,}519$ utility.

for a_2 we have $CEU\left[a_2\right]=\sum_{j=1}^{n} w_j u_{2j}=0{,}601$ utility.

for a_3 we have $CEU\left[a_3\right]=\sum_{j=1}^{n} w_j u_{3j}=0{,}573$ utility.

The choice of an optimal strategy, i.e. the option with the largest complex (composite) value-utility, can be presented as the choice which is defined in the following way:

$$\max CEU[a_i].$$

DM is making a choice in the following way:

$$CEU\left[a_2\right]>CEU\left[a_3\right]>CEU\left[a_1\right],$$

that is, he will invest in the property under lease at Avala, whereby he will have 33% of the ownership.

5. CONCLUSION

A modern way of doing business requires more often from decision-makers to make important business decisions in the conditions of constant changes in the environment and situations when it is not possible to obtain exact data for all parameters that influence making business decision. On the other hand, wrong decisions can be catastrophic and irreversible, so decision-makers must be able to make decisions in conditions of uncertainty and risk. This is achieved with modern methods where stochastic multi-attribute utility models are used.

The stochasticity of a multi-attribute problem can be best analyzed by introducing the finite number of levels for the attributes values, together with the corresponding probabilities. The choice is made by selecting the best option based on the maximum expected utility composition.

Deterministic and stochastic MAUM are not analytically difficult techniques, but the aspects of the organization of these techniques are complicated. Therefore, it would be good to use the appropriate computer software to calculate deterministic and stochastic models and methods whenever possible.

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СТОХАСТИЧКИ ВИШЕАТРИБУТИВНИ МОДЕЛ КОРИСНОСТИ

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САЖЕТАК

У реалним ситуацијама вриједност, у већој мјери промјенљивог, атрибута може се најбоље приказати увођењем коначног броја нивоа вриједности атрибута, којима треба придружити и одговарајуће вјероватноће. Стохастички вишеатрибутивни модел корисности има могућност да анализира такве стохастичке, вишеатрибутивне проблеме. Избор једне, из скупа расположивих опција, врши се тако да се бира најбоља опција на бази максимално очекиване композиције корисности. У овом раду, навешћемо само неке од аргумената за развој стохастичког вишеатрибутивног модела корисности, предности (ближи су реалности), недостатке (аналитички тешка техника, субјективне процјене нивоа вриједности промјенљивих атрибута), као и поступак рјешавања.

Кључне ријечи
Стохастички вишеатрибутивни модел корисности, вриједност, корисност, вјероватноћа, тежине.