GLOBAL MATRIX OF REGIONAL GROWTH IN THE FUNCTION OF THE ANALYSIS OF REGIONAL DEVELOPMENT AND INTERREGIONAL RELATIONS

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ARTICLE INFO

Original scientific Article
Received: 20.11.2019
Revised: 3.12.2019
Accepted: 16.12.2019
doi 10.7251/ACE1931009S
UDC 330.34:338.124.4(497)

Keywords: *direct growth rate, indirect growth rate, growth matrix, regional growth matrix, global regional growth matrix, dynamic analysis.*

JEL Classification: C44

ABSTRACT

The subject of research in this paper is the expansion of the growth matrix, which is reflected in the definition of a new matrix - a global regional growth matrix. The assumption for the formation of this matrix is that the synthetic measures of the development of individual regions included in the analysis are first calculated. This matrix provides an opportunity to analyze interregional relations and the dynamics of structural change in the respective regions.

1. INTRODUCTION

Trends in economic quantities (social gross product, production volume, investments, various forms of consumption, etc.) within an economic system are interconnected, directly and indirectly, which results in the emergence of a general interdependence of the observed quantities.

Due to the interdependence of economic quantities, the change in the level of any of them is not the result of a purely qualitative and quantitative increase in the size.

If one assumes that changing one size is a function of only its qualitative and quantitative growth, then direct growth rates would be sufficient to analyze the size. Such starting point in the analysis of the development of an economic size is acceptable only on the assumption that there is complete independence between the observed size on the one hand, and other economic variables on the other. In real life, total independence between sizes within an economic system is impossible.

Similarly, interdependence relations are also observed between economic regions which form the territorial entity of a state and its economic system. There are interdependencies between the regions in the development process, so the level of development of one region changes under the influence of changes within the region itself (direct influences: changes in the volume of investments, changes in the economic structure, etc.) and under the influence of other regions (indirect influence).

2. PREVIOUS RESEARCH

Defining direct and indirect growth rates, as well as growth matrices based on them is originally a work of D. Stojanović (1973). The same author also defined the regional growth matrix. In this regard, there is little previous research. Still, we can single out some authors who have done similar research. We find a similar subject of research in the work of M. Backović (1982).

In later studies, we come across works in which a growth matrix instrument has been applied in various fields,for example: regional management in the maritime transport (Југовић&Лончар,2008), modeling of development potential of logistics operators (Пупавац&Зеленика, 2007), modeling of multimodal transport (Nikolić, 2003).

3. DIRECT AND INDIRECT DEPENDENCE IN THE DEVELOPMENT OF ECONOMIC REGIONS

Based on the previous findings, it follows that in the quantitative analysis of regional development, both direct and indirect rates of development of the observed regions should be considered. In order to show the appropriate model, we will look at m (R_1, R_2,...,R_m),which are from the point of view of economic activities interdependent.

To begin with, we will assume that the level of development of one region is measured by a single indicator, for example, per capita national income.

We will denote income of r and s region in the period t with $P_{r,t}$ and $P_{s,t}$, and income of r and s region in the period $(t-1)$, with $P_{r,t-1}$ and $P_{s,t-1}$, respectively. The change in income of r region in the period $(t-1,t)$,will be

$$\Delta P_{r,t} = P_{r,t} - P_{r,t-1}, \quad \begin{matrix}(t = 1,2,...,T), \\ (r = 1,2,...,m).\end{matrix} \tag{1}$$

Direct income growth rate of r region will show how many units of income of that region come per unit of income of the same region in the period $(t-1)$:

$$p_{rr,t} = \frac{\Delta P_{r,t}}{P_{r,t-1}}, \tag{2}$$

that is

$$p_{rr,t} = \frac{\Delta P_{r,t}}{P_{r,t-1}} - 1, \quad \begin{matrix}(r = 1,2,...,m) \\ (t = 1,2,...,T),\end{matrix}$$

where direct income growth rate of r region in the period t was denoted with $p_{rr,t}$ compared to the period $(t-1)$.[1]

Indirect income growth rate of r region in the period t in relation to income of s region in the period $(t-1)$, denoted with $p_{rs,t}$, shows how many units of increase in income of r region come per unit of income of s region in the period $(t-1)$:

$$p_{rs,t} = \frac{\Delta P_{r,t}}{P_{s,t-1}}, \quad \begin{matrix}(r \neq s = 1,2,...,m), \\ (t = 1,2,...,T).\end{matrix} \tag{3}$$

Direct and indirect rates can be arranged in a square pattern of numbers so that-direct growth rates are on the main diagonal of that square pattern and indirect

[1] Direct growth rates in relation (2) are expressed relative to the value of income in the period $(t-1)$. They can also be expressed in relation to income in the period t, which is less common in practice. The relations between direct growth rates expressed in relation to the period $(t-1)$ and the period t are given by the following relations:

$$p'_{rr,t} = \frac{p_{rr,t}}{1 + p_{rr,t}}, \quad \begin{matrix}(r = 1,2,...,m) \\ (t = 1,2,...,T),\end{matrix}$$

where

$$p'_{rr,t} = \frac{\Delta P_{r,t}}{P_{r,t}}.$$

A similar observation applies to indirect growth rates, the definition of which follows immediately.

rates are in other places. Such pattern of numbers is called a growth matrix[2] and it has a following appearance:

$$\overline{P}_t = \begin{bmatrix} p_{11,t} & p_{12,t} & \cdots & p_{1m,t} \\ p_{21,t} & p_{22,t} & \cdots & p_{2m,t} \\ \vdots & \vdots & \cdots & \vdots \\ p_{m1,t} & p_{m2,t} & \cdots & p_{mm,t} \end{bmatrix}.$$

In this particular case, the matrix $\overline{P}_t$ is called the regional growth matrix because it contains direct and indirect growth rates of individual regions. It allows monitoring of the growth structure of each region in the period $(t-1,t)$. Direct growth rates show how quantitative and qualitative changes in one region affect the change in per capita income level of the same region in the period $(t-1,t)$. In contrast, the indirect impact of other regions on changes in per capita (regional) income level of the observed region is measured by indirect growth rat

4. FINITION OF THE GROWTH MATRIX BASED ON SYNTHETIC INDICATORS

If we seek to analyze the overall (global) development of the selected regions, for these purposes we cannot use the matrix $\overline{P}_t$, since it is defined on the basis of a single indicator of the level of development. [3]

We defined the economic development of the observed region as the level of development of material production forces, the efficiency or productivity of their use, and the standard of living of the observed region. In short, it follows from the previous definition of the concept of economic development of a region that

[2] A growth matrix for the purposes of analyzing the interdependence of economic quantities was first set by professor Dragiša Stojanović.
See more: Стојановић, Д., Матрица раста, Продуктивност, 1/73, pp. 795-802.

[3] In previous analyses of regional development based on the growth matrix, development has been viewed as a function of only one indicator of the level of development (most often it is the social gross product). The analysis of development using a single indicator, based on the growth matrix, was carried out by professors Dragiša Stojanović and Marko Backović.
See more:
Стојановић, Д., Динамичко програмирање регионалног развоја путем матрице раста, Продуктивност, 7/73, pp. 1126-1129.
Бацковић, М., Анализа и пројекција регионалног развоја методом матрице раста, Економски гласник, 1/82, pp. 128-135.

development is a complex and multidimensional phenomenon and as such cannot be quantified by a single indicator.

In order to analyze the global level of development of *m* regions and interdependencies in the development process, it is necessary to define a growth matrix which is based on indicators that synthetically express the levels of development of the observed regions.[4] This means that instead of per capita national income data, we should have synthetic indicators of the level of development of each of the observed *m* regions and in each time unit *t* (t=1,2,...,T).

Based on synthetic indicators, direct and indirect development rates of the regions would be calculated. Let us assume that $S_{r,t}$ and $S_{s,t}$ are synthetic indicators of the level of development of *r* and *s* region, respectively, in the period *t*, and

$$\Delta S_{r,t} = S_{r,t} - S_{r,t-1}, \quad \begin{matrix} (r = 1,2,...,m) \\ (t = 1,2,...,T), \end{matrix} \tag{4}$$

the change in the level of the synthetic indicator of the development of *r* region in the period (t−1,t). Then the direct growth rate of *r* region in the period (t−1,t) denoted with $s_{rr,t}$, will be:

$$s_{rr,t} = \frac{\Delta S_{r,t}}{S_{r,t-1}}, \quad \begin{matrix} (r = 1,2,...,m) \\ (t = 1,2,...,T), \end{matrix} \tag{5}$$

and will show a relative increase in the level of the synthetic indicator of development of *r* region in the period *t* in relation to the level of the synthetic indicator of the same region in the period (t−1).

Indirect growth rate, denoted with $s_{rs,t}$:

$$s_{rs,t} = \frac{\Delta S_{r,t}}{S_{s,t-1}}, \quad (r \neq s = 1,2,...,m), \tag{6}$$

will show a relative increase in the level of the synthetic indicator of the development of *r* region in the interval (t−1,t) relative to the level of the synthetic indicator of the development of *s* region in the period (t−1).

[4] Only synthetic indicators that have an essential, cardinal meaning can be used to form this matrix. This will be discussed more after defining the matrix.

If we arrange direct rates $s_{rr,t}$ and indirect rates $s_{rs,t}$ ($r \neq s = 1,2,...,m$; $t=1,2,...,T$) into a square scheme $\overline{S}_t$

$$\overline{S}_t = \begin{bmatrix} s_{11,t} & s_{12,t} & \cdots & s_{1m,t} \\ s_{21,t} & s_{22,t} & \cdots & s_{2m,t} \\ \vdots & \vdots & \cdots & \vdots \\ s_{m1,t} & s_{m2,t} & \cdots & s_{mm,t} \end{bmatrix},$$

we will obtain a growth matrix based on synthetic indicators of the level of development of the regions.

Matrix $\overline{S}_t$ can be called a matrix of global regional growth.

The basic assumption of the application of the global regional growth matrix $\overline{S}_t$ in the dynamic regional analysis is to determine synthetic indicators of development of the observed regions for each time period of observation t ($t=1,2,...,T$), which will approximate the level of the global development of each of m observed regions.

Therefore, synthetic indicators of the level of development of the observed regions, which serve as a basis for the formation of the matrix $\overline{S}_t$, in addition to being used to compile a rank list of the levels of development of the regions, must provide information on approximately real distances in the level of development between the observed regions. Since there is no mathematical-statistical model that would provide synthetic information with cardinal meaning, while fulfilling other conditions (dependence of indicators, elimination of duplicity, condition of asymmetry), we will be satisfied with unweighted index models, because they give results that are expressed in relative numbers, and therefore have the corresponding meaning. Index models provide solutions that, given the phenomenon of duplication of information, we can only accept as approximate and for now usable only for the formation of a global growth matrix. It is possible to mitigate the problem of duplicity by choosing indicators to be included in the analysis. The formation of the matrix $\overline{S}_t$ would be preceded by a qualitative and quantitative analysis of the interdependence of all indicators considered for the derivation of synthetic information, which should result in the creation of a better statistical basis for its establishment.

5. GROWTH SUBMATRICES BASED ON SYNTHETIC INDICATORS

When analyzing interregional relations, it is sometimes necessary to pay attention to the relations between two particular regions, e.g. R_r and R_s. In this case, direct growth rates s_{rr} and s_{ss}, as well as corresponding indirect rates s_{rs} and s_{sr}, should be extracted from the matrix S_t which will be arranged in a special scheme $\overline{S}_{rs,t}$ called the growth submatrix of *r* and *s* region:

$$\overline{S}_{rs,t} = \begin{bmatrix} s_{rr,t} & s_{rs,t} \\ s_{sr,t} & s_{ss,t} \end{bmatrix}.$$

The growth submatrix $\overline{S}_{rs,t}$ is a part of the matrix $\overline{S}_t$, corresponding to *r* and *s* region. It provides information on the reciprocal relations between these regions.

Starting from the growth submatrix $\overline{S}_{rs,t}$, we will emphasize the relations that apply to direct and indirect growth rates.

1) The relation between the indirect growth rate $s_{rs,t}$ and the direct growth rate $s_{rr,t}$ is equal to the ratio of the level of the synthetic indicator of *r* and *s* region in the period (t−1):

$$\frac{s_{rs,t}}{s_{rr,t}} = \frac{S_{r,t-1}}{S_{s,t-1}}. \tag{7}$$

2) The relation between the indirect growth rate $s_{rs,t}$ and the direct growth rate $s_{ss,t}$ is equal to the ration of increment of the level of the synthetic indicator of *r* and *s* region in the period (*t*−1,*t*):

$$\frac{s_{rs,t}}{s_{ss,t}} = \frac{\Delta S_{r,t}}{\Delta S_{s,t}}. \tag{8}$$

3) If we put direct rates $s_{rr,t}$ and $s_{ss,t}$ in the ratio,we get the elasticity of the synthetic indicator of the development level of *r* region relative to *s* region:

$$\frac{s_{rr,t}}{s_{ss,t}} = \frac{S_{s,t-1}}{S_{r,t-1}} \cdot \frac{\Delta S_{r,t}}{\Delta S_{s,t}}. \tag{9}$$

4) The quotient of indirect growth rates $s_{rs,t}$ and $s_{sr,t}$ is equal to the product of the quotient of synthetic indicators $S_{r,t-1}$ and $S_{s,t-1}$, on the one hand, and the quotient of the increment of synthetic indicators $\Delta S_{r,t}$ and $\Delta S_{s,t}$, on the other:

$$\frac{s_{rs,t}}{s_{sr,t}} = \frac{S_{r,t-1}}{S_{s,t-1}} \cdot \frac{\Delta S_{r,t}}{\Delta S_{s,t}}. \tag{10}$$

Relations (7) - (10) are significant because they provide the opportunity to further analyze interregional movements and the dynamics of structural changes.[5]

6. A FORECASTING TERRITORIAL MODEL OF REGIONAL GROWTH

Starting from the assumption that there is an interdependence of the economic activities of the observed regions, a function can be written as

$$\Delta S_r = F(S_{1,t}, S_{2,t}, ..., S_{m,t}), \quad (r = 1, 2, ...m), \tag{11}$$

which shows that the increment of the level of development of *r* region is a function of the level of synthetic indicators of development of all regions. Let functional dependence (11) be specified by the following expression:

$$S_r = \frac{1}{m}\sum_{s=1}^{m} s_{rs} S_{st}, \quad (r = 1, 2, ..., m), \tag{12}$$

where s_{rs} isan indirect rate and *m* is a number of regions.

By supstitution (12) and (4)

$$\begin{array}{l} S_{1,t} - \frac{1}{m}(s_{11}S_{1,t} + s_{12}S_{2,t} + ... + s_{1m}S_{m,t}) = S_{1,t-1} \\ S_{2,t} - \frac{1}{m}(s_{21}S_{1,t} + s_{22}S_{2,t} + ... + s_{2m}S = S_{2,t-1m,t}) \\ \vdots \qquad \vdots \qquad \cdots \qquad \vdots \qquad \cdots \qquad \vdots \\ S_{m,t} - \frac{1}{m}(s_{m1}S_{1,t} + s_{m2}S_{2,t} + ... + s_{mm}S_{m,t}) = S_{m,t-1} \end{array} \quad . \tag{13}$$

[5] The proofs of relations (7) – (10) become apparent if, instead of $s_{rs,t}$, $s_{rr,t}$, $s_{ss,t}$ and $s_{sr,t}$,we put expressions by definition in appropriate places:

$$s_{rs,t} = \frac{\Delta S_{r,t}}{S_{s,(t-1)}}, \; s_{ss,t} = \frac{\Delta S_{s,t}}{S_{s,t-1}}, \; s_{rr,t} = \frac{\Delta S_{r,t}}{S_{r,t-1}}, \; s_{sr,t} = \frac{\Delta S_{s,t}}{S_{r,t-1}}.$$

Written as a matrix, the system (13) has a form:

$$(I - \frac{1}{m}\overline{S_t})S_t = S_{t-1}, \tag{14}$$

where:

I - unit matrix of m order,

$\overline{S_t}$ - growth matrix of m order,

S_t - column vector (dimensions $m\times1$) whose coordinates represent synthetic indicators of the level of development of the observed regions in the period t,

S_{t-1} - column vector (dimensions $m\times1$) whose coordinates represent synthetic indicators of the level of development of the observed regions in the period ($t-1$).

The solution of the system (14) is:

$$\mathrm{S}_t = (I - \frac{1}{m}\overline{S_t})^{-1}\mathrm{S}_{t-1}. \tag{15}$$

Relations (14) and (15) show that the growth matrix $\overline{S_t}$ can be used to establish a relationship between the development levels of selected regions in two successive periods ($t-1,t$).

If the problem of interdependence of indicators of development of selected regions is viewed through T of time units ($t=1,2,...,T$), then the level of development of all regions in T period can be expressed through the growth matrix from all previous periods. To establish such a connection, we will use relation (14) in which we will replace t with: $t=1$, $t=2$,..., $t=T$:

$$\begin{array}{l}
(I - \frac{1}{m}\overline{S_1})S_1 = S_0 \\
(I - \frac{1}{m}\overline{S_2})S_2 = S_2 \\
\vdots \quad \vdots \qquad \vdots \\
(I - \frac{1}{m}\overline{S_T})S_T = S_{T-1}.
\end{array} \tag{15.1}$$

It follows from system (15.1) that:

$$(I-\frac{1}{m}\overline{S_1})\cdot(I-\frac{1}{m}\overline{S_2})\cdot \quad \cdots \quad \cdot(I-\frac{1}{m}\overline{S_T})=S_0,$$

with the solution by S_T that follows:

$$S_T=\left[(I-\frac{1}{m}\overline{S_1})\cdot(I-\frac{1}{m}\overline{S_2})\cdot \quad \cdots \quad \cdot(I-\frac{1}{m}\overline{S_T})\right]^{-1}S_0. \tag{16}$$

With solution (16) the level of development of all regions in T period was put into function of the level of development of the same regions in all previous periods.[6]

The regional growth matrix can be used to predict the level of development indicators in the future. This problem can be solved in two ways.

First, it is assumed that the same relations between regions will be maintained in the future, which means that the growth matrix does not change. In this case, the growth matrix from the previous period will serve as a basis for estimating the level of development in the future period.

Second, it is assumed that interdependence relations between regions will change. This assumption results in a change in the growth matrix for the future period (e.g. $t=T+1$). As the actual interdependence relations, expressed by direct and indirect growth rates, are unknown, their estimation must be made. The estimation of direct and indirect growth rates is most often done using the trend method (linear, exponential or logarithmic).

If we denote the vector of the level of development indicators of regions in the period (T+1) with S_{T+1}, it is predicted that the vector of these quantities will be:

$$S_{(T+1)}=(I-\frac{1}{m}\overline{S_c})^{-1}S_T,$$

where we denoted the estimated growth matrix with $\overline{S_c}$.

The main problem in the application of this model is the estimation of direct and indirect growth rates in the period for which the level of synthetic indicators of

6 Стојановић, Д., Динамичко програмирање регионалног развоја путем матрице раста, op. cit., p. 1127.

the observed regions is projected, that is, how to obtain a changed growth matrix $\overline{S_c}$.

7. CONCLUSION

In modern conditions of economy, classical methods of analysis are not sufficient to make the choice of the appropriate variant of development, so the need to introduce new models and methods is imposed.

Economic development is a complex and multidimensional phenomenon and as such cannot be quantified by a single indicator. In order to analyze the global level of development of regions and interdependencies in the development process, it is necessary to define a growth matrix that is based on indicators that synthetically express the levels of development of the observed regions. Direct and indirect development rates of regions are calculated based on synthetic indicators.

By applying this matrix, it is possible to predict the level of development of the observed regions, as well as the development rankings.

Since there is no mathematical-statistical model that would provide synthetic information with cardinal meaning, while fulfilling other conditions (dependence of indicators, elimination of duplicity, condition of asymmetry), we will be satisfied with unweighted index models, because they give results that are expressed in relative numbers, and therefore have the corresponding meaning. Index models provide solutions that, given the phenomenon of duplication of information, can only be accepted as approximate and for now usedonly for the formation of the global growth matrix.

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ГЛОБАЛНА МАТРИЦА РЕГИОНАЛНОГ РАСТА У ФУНКЦИЈИ АНАЛИЗЕ РАЗВОЈА РЕГИЈА И МЕЂУРЕГИОНАЛНИХ ОДНОСА

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САЖЕТАК

Предмет истраживања у овом раду јесте проширење матрице раста, које се огледа у дефинисању нове матрице - матрице глобалног регионалног раста. Претпоставка за формирање те матрице јесте да се претходно израчунају синтетичке мјере развијености појединих регија које су укључене у анализу. Та матрица пружа могућност анализе међурегионалних односа и динамике структурних промјена у одговарајућим регијама.

Кључне ријечи:

Директна стопа раста, индиректна стопа раста, матрица раста, матрица регионалног раста, матрица глобалног регионалног раста, динамичка анализа.