MULTI-ATTRIBUTE DECISION-MAKING METHODS– INDIFFERENCE ASSESSMENTS

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ABSTRACT

Real situations are based on multiple attributes or criteria. If the assessment problem has multiple dimensions of value, then intuitive judgments can be very difficult. In many situations, the ultimate assessment seems very difficult, especially when the decision maker chooses one of a set of possible options in an unfamiliar environment. This paper analyzes indifference value measurement methods. Indifference value measurement methods rely on studies of indifference (indifference assessments) or comparison of strength of preferences. The values that the decision maker gives to the attributes reflect his preferences, ie. utilities. Utility is only a way to describe preferences. The process of determining (functions) values is only one form of sequential determination of utility.

1. INTRODUCTION

In principle, it is possible to combine the benefits of any single-attribute value measurement method, or any utility measurement method with any weighting procedure and the additive model. All possible combinations of these elements would create a large list of MADM (Multi-Attribute Decision Making Methods) procedures, only a few of which are applied in real situations.

The first combination, SMART (Simple Multi-Attribute Rating Technique), is a procedure of selection of elements that are simple and easy to apply. This combination includes: a single-attribute direct numerical value estimation procedure; weighting procedure (ratio estimation) and the additive model.

The second combination, based on the value measurement theory, emphasizes the use of indifference assessments. This combination includes: difference value measurement and strength of preference assessment for constructing single-attribute value functions; weighting (cross-attribute strength of preference) and the additive model.

The third combination places emphasis on the fact that most of the available options are risky and that procedures should be appropriate for the construction of value functions. A multi-attribute utility theory, ie. a multi-attribute version of the Subjective Equivalent Utility (SEU) includes: variable probabilities or certainty equivalent methods for forming value functions, single-attribute utility; variable probability methods for weight construction and a multiplicative model.

In this paper, we analyze indifference value measurement methods.

2. PREVIOUS RESEARCH

Although the theory and science of decision-making under certainty conditions have existed for a long time, there has been no critical review of some of its parts, in particular, of indifference methods. Few authors have addressed this issue. There are several critical considerations for weighting methods that are integral to the additive, multiplicative, and multilinear forms of the indifference model.

Some authors (Miljković et al., 2017) introduce into the analysis a new weighted sum model that allows for a high degree of interference with the personal preferences of decision makers. The method consists of two main parts, normalization and weighting, which corresponds to our additive model. The advantage of this method is that it is easier to use qualitative and quantitative attributes and it has a wide range of applications.

Multi-criteria methods, which are based on the additive form of the model, allow a kind of “tradeoff” between attributes (criteria). When it comes to the tradeoff between attributes, the small value by the most important attribute can be recovered in the total value with the good values of the less important attributes (Žižović et al., 2016a). New methods of ranking of options have been defined assuming that the first attribute (criterion) is dominant (Žižović, 2018). Attribute value estimates are between 0 and 1. The assumption is that if the value of an option in the dominant attribute (criterion) is 0, then its estimate is equal to 0, regardless of the values of the estimates in other attributes (criteria).

On the other hand, such a situation cannot occur by applying multi-criteria analysis methods based on the multiplicative forms of models. If an option does not satisfy the absolutely important attribute, then its total aggregation value is zero. It follows that ranking of options, from the set of options obtained by the multiplicative method, remains the same relative to the hypothetical option, even if the starting set of options is expanded by a new set of options. Therefore, neither option can be favored.

Some authors (Zardari et al., 2015) provide a broader overview of the weighting methods used in various Multi-Criteria Attribute Decision Making methods (MCDMs). The authors argue that the result of the decision-making process depends mostly on the objectivity of weighting, that is,on determining weights for each attribute (criterion).

3. INDIFFERENCE VALUE ESTIMATE METHODS

We will analyze three classes of indifference measurement methods, MIMV. These methods, somewhat more complex than SMART, differ not only in the way the value (utility) of individual attributes is determined, but also in the procedure of "aggregating" them.

The first of these is the difference value measurement method, which is based on the assumption of precisely measurable utility of attributes (on an *interval scale*), which are weighted and then the selection criterion is obtained using one of three models (additive, multiplicative or multilinear). The difference measurement method only estimates the strength of preferences (increase in utility). It is no different from a variable-weight SMART, except that it uses multiplicative and multilinear rules if additives are missing. This method does not allow for accurate mutual comparisons of individual values; in this case it is possible to compare only the differences, the intervals between the individual values. Therefore, it is about *interval scales* of preference measurement; equal differences in numbers on the interval scale represent equal differences in the observed property.

Another class of indifference methods for measuring values is called conjoint measurement theory. Unlike difference value measurement, which requires an estimate of the strength of preferences, conjoint measurement requires only *ordinal preferences*, and indifference estimates between multi-attribute objects. This method is based on *ordinarily measurable* utility, and the criterion value is calculated based on the additive and multiplicative model.

The third class of value measurement methods is based on a simple weak order model; it introduces the most rigorous assumption of cardinal measurability not only by individual attributes but also between attributes. The assumption here is that the decision maker is able to make precise comparisons ("tradeoffs") between options, in which the shortages will be compared by some attribute and compensated with the advantages of others.

3.1. Difference value measurement

MADM difference value measurements[1] are constructed on the notion of the relative value of options or the strength of the decision maker's preferences for the options considered. Since it is usually difficult to express such value estimates when it comes to multi-attribute options, difference measurement is based on a number of independence assumptions that allow the task of multi-attribute option estimate to be divided into single-attribute estimates.

Dyer and Sarin (1979) proposed two models of difference value measurement for the MADM case: additive and multiplicative. In addition to these two models, a multilinear difference measurement model is used.

3.1.1. Additive difference value measurement model

The most common form of MADM used to solve real problems is the additive form:

$$v(X) = \sum_{i=1}^{n} w_i v_i(x_i), \quad (1)$$

where:

X - multi-attribute object of estimate, option,

x_i - its realization in *i* attribute,

v_i - single-attribute marginal function of the value, utility[2] of an individual attribute, and

w_i - weight of *i* attribute.

[1] Dyer, J., Sarin, R. (1989). Measurable multi-attribute value functions, Operational Research, No. 27., p. 11.

[2] The value determination process is only one form of sequential value determination. When analyzing indifference methods it should be emphasized that these terms are synonymous.

As with the measurement of single-attribute differences, the assumption is that the additive, that is, the aggregate form "v" expresses the rank of preferences between options and the difference between individual values; these are *interval scales* of measurability. The final rankings (final values) represent the interval scale. The same differences between successive numerical values correspond to identical changes in the intensity of qualitative attributes.

The additive form further indicates that "v_i" can be constructed ignoring other attributes; that is, "v_i" represents a marginal value (utility) function defined for an attribute X_i, with other attributes fixed at some arbitrary level. Formally, the additive form is a strong assumption of independence called *additive difference independence* (ADI).

ADI requires that strength of preferences between two options, X_1 and X_2, which have identical fixed levels in some attributes, do not change when these attributes are fixed at another level. It is about the strength of preferences between two options of two attributes, with other attributes being constant. Some authors, such as Dyer and Sarin, simply call ADI the *difference independence*.

The actual construction of "v", "v_i" and "w_i" can be simple if the ADI assumption holds. Some single-attribute value functions remain unchanged with changes in the levels of other attributes. If, on the other hand, the decision maker considers this task of making such single-attribute estimates difficult or impossible without considering other attributes, ADI may be impaired.

3.1.1.1. Single-attribute value, utility functions

Value, utility functions are constructed with the help of: difference standard sequence method and bisection procedures. Comparisons of several different pairs of attributes were made using these two methods. The decision maker must be aware that the construction of value (utility) functions "v_i", in principle, via i attribute keeps other attribute levels constant.

Normally, we can construct the order of differences by placing equally spaced x_i, $x_i^{/}$, $x_i^{//}$, and so on, with the trait

$$\begin{aligned}&\left[(x_{1*},x_{2*},\ldots,x_i,\ldots,x_{n*}),(x_{1*},x_{2*},\ldots,x_i^{/},\ldots,x_{n*})\right]\sim\\&\sim\left[(x_{1*},x_{2*},\ldots,x_i^{/},\ldots,x_{n*}),(x_{1*},x_{2*},\ldots,x_i^{//},\ldots,x_{n*})\right]\end{aligned} \tag{2}$$

This complex equation of indifference is verbally translated as follows.

The decision maker prefers x_i over $x_i^{/}$ to the same extent to which he prefers $x_i^{/}$ over $x_i^{//}$, if the values of all other attributes are kept at their worst level.

In practice, the decision maker is unlikely to keep other attributes unchanged in any indifference assessment. This process generates many single-attribute value (utility) functions "v_i". Functional values are arbitrarily assigned to the end values "0" and "1", i.e. $v_i(x_{i*})$=0 and $v_i(x_i^*) = 1$.

3.1.1.2. Weighting

Weighting "w_i" is the next step in measuring the value of differences. This is necessary because we have assigned equal endpoints in values (0 and 1) for each attribute.

If we did not determine the attribute weights by "w_i", we would imply that the increase in preference - utility obtained by moving from the worst to the best level in each attribute - was identical for all attributes. It is quite clear that this is not always true.

Let us compare the relative strength of the preferences of two attributes (from x_i^* to x_{i*}, the best versus the worst attribute level). One way to ask this question is to determine whether a decision maker would prefer to change x_i from x_{i*} to x_i^* or x_j from x_{j*} to x_j^*. Suppose all other attributes are constant (x_k, $k \neq i, j,$ constant); for example, at their worst level x_{i*}.

A decision maker evaluates the strength of preference as follows:

$$\left[(x_i^*, x_{j*}), (x_{i*}, x_{j*})\right] \sim \left[(x_{i*}, x_j^*), (x_{i*}, x_{j*})\right]. \qquad (3)$$

This complex mathematical sentence can be expressed in words as follows.

Let us consider three options identical to each other with respect to all attributes except for *i* and *j*. It is quite clear that the decision maker prefers the option in which *i* attribute has a level x_i^* to the option in which *i* attribute has a level x_{i*}, keeping *j* attribute constant at the level x_{j*}. The same is true for options that differ in the above described way in *j* attribute, with *i* attribute being constant in x_{i*}

Let us consider now the relative strength of the decision maker's preference for the option described as x_i^* and x_{j*} compared to the option described as x_{i*} and x_{j*}. Is that strength of preference greater than, equal to, or less than the decision maker's strength of preference relative to the option described as x_{i*} and x_j^* compared to the option described as x_{i*} and x_{j*}.

If the decision maker thinks that the increase in utility from x_{i*} to x_i^* is greater than that from x_{j*} to x_j^*, then it follows that the strength of preference on the left side of equation (3) is greater; otherwise, it follows that the strength of preference on the right is higher. If the estimate of (value) difference on the left is higher, the conclusion is that, $w_i > w_j$, as can be seen from the following expression:

$$\left[w_i v_i(x_i^*) + w_j v_j(x_{j*})\right] - \left[w_i v_i(x_{i*}) + w_j v_j(x_{j*})\right] > \left[w_i v_i(x_{i*}) + w_j v_j(x_j^*)\right] - \\ - \left[w_i v_i(x_{i*}) + w_j v_j(x_{j*})\right]. \tag{4}$$

This expression can be simplified, so we have

$$w_i v_i(x_i^*) > w_j v_j(x_j^*) \text{ or } w_i > w_j.$$

To determine how much w_i is larger than w_j, we must reduce x_i^* to some central value $x_i^/$, examining the value $x_i^/$ of the indifference

$$\left[(x_i^*, x_{j*}), (x_{i*}, x_{j*})\right] \sim \left[(x_{i*}, x_j^*), (x_{i*}, x_{j*})\right]. \tag{4.1}$$

This difference implies $w_i v_i(x_i^/) = w_j v_j(x_j^*)$ or $\frac{w_j}{w_i} = v_i(x_i^/)$.

Repeating this process for all attribute pairs should produce the desired results. This would give redundant information, which would give $\frac{n(n-1)}{2}$ weights, although only (n−1) is needed to solve the entire weight group. However, it would be very difficult to make all $\frac{n(n-1)}{2}$ difference estimates for $n > 4$.

Suppose the analyst wants to construct (n−1)indifference equation as those stated earlier, using the attribute with the smallest possible increase in value, utility as a standard; this attribute is easy to spot by ranking the strengths of preferences of particular values. Let the attribute with the smallest possible increase in value be the utility of n attribute.

Solving (n−1) indifference equation is based on reducing the value x in i attribute from x_i^* to $x_i^/$; if it were necessary to reduce x_n^* to obtain indifference, n attribute would have a greater possible increase in the value than i attribute.

Therefore, we can make ($n-1$) equation of indifference from the following form $\frac{w_i}{w_n} = v_i(x_i^/)$. Together with the restriction $\sum_{i=1}^{n} w_i = 1$, we can solve $w_i = \frac{v_i(x_i^/)}{\sum_{i=1}^{n} v_i(x_i^/)}$.

It is these weights that go into the equation of the additive form of difference measurement.

3.1.2. Multiplicative difference value measurement model

If we cannot apply the additive model due to the *dependency*, while considering other attributes, in solving multi-atribute value (utility) problems, then a *multiplicative difference value measurement model* is suitable. The multiplicative, later multilinear model, requires some kind of, weaker or stronger, dependency. When working with the multiplicative and multilinear model, it is necessary to separate the interaction parameters from the weight questions. In general, what is interdependence? Interdependence is an objective link between attributes. The multiplicative difference value measurement model takes the form:

$$1 + wv(X) = \prod_{i=1}^{n} \left[1 + ww_i v_i\,(x_i) \right], \tag{5}$$

where "w" is a mutual influence parameter.

Its extended form shows more; it explains the presence (strength) of *mutual influences*:

$$v(X) = \sum_{i=1}^{n} w_i v_i(x_i) + \sum_{i<j} ww_i w_j v_i(x_i) v_j(x_j) + \sum_{i<j<m} w^2 w_i w_j w_m v_i(x_i) v_j(x_j) v_m(x_m) + \cdot\ \cdot\ \cdot\ + $$
$$+ w^{n-1} \prod_{i=1}^{n} w_i v_i(x_i). \tag{6}$$

In the previous complex equation, the first term on the right side of the equation is a known additive form of the difference value measurement, the second term refers to the dependence between a pair of attributes, the third term refers to the dependence between three attributes; and so on, down to the last term referring to the interdependence of the entire set of attributes.

Interdependence[3] is obtained when the value of each attribute is multiplied by its weight (weight w_i), this product is multiplied by the mutual influence parameter "w" and then these products are added together.

If the number of terms for mutual influence increases, the value "w", that the product is multiplied by, also increases. Of course, if $w = 0$, this term becomes an additive model. Therefore, we see that "w" can move in the multiplicative difference measurement model, from one to a finite value. If $|w|$ becomes larger, the total mutual influence between all attributes becomes more difficult to calculate. This model makes all the mutual influences dependent on a single parameter and thus the values of the form "A depends a lot on B but much less on C" cannot be properly represented.

The multiplicative form is a kind of dependency we call the *Multiplicative Difference Independency* (MDI). Dyer and Sarin (1979) call them the weak difference dependency. MDI suggests that the effects of changing the value of an attribute pair from one physical measure to another can reduce or extend the value scale of those attributes and move the group elements up or down along the scale. The important feature is that the relative spacing (difference) of elements in the group remains unchanged. It is difficult to provide convincing examples where MDI is maintained and ADI is not maintained.

Speaking about multiplicative difference value measurement, let us consider an example for two attributes. In the multiplicative model, v_i and w_i can be estimated as in the additive model, but since the sum w_i will not be equal to one, n equation must be solved. Likewise, the mutual influence parameter must be specified.

Assuming that all $x_i = x_i^*$ and all v_i are such that $v_{i*} = 0 \wedge v_i^* = 1$, we notice that

$$1 + w = \prod_{i=1}^{n} (1 + ww_i). \tag{7}$$

In the case of two attributes, equation (7) becomes

$$1 + w = (1 + ww_1)(1 + ww_2), \tag{8}$$

or

$$w = \frac{1 - w_1 - w_2}{w_1 w_2}. \tag{9}$$

[3] Goodwin, P. (2008). Decision Analysis for Management Judgment 3rd Ed, Utah Valley University.

Since $0 \le w_i \le 1$, the formula (9) also shows limits for *w*:

$$-1 \le w \le \infty.$$

If $w_1 = w_2 = 0$, then $w = \infty$. If $w_1 = w_2 = 1$, then $w = -1$.

Of course, if $w_1 + w_2 = 1$, then $w = 0$, and the condition for applying ADI is satisfied.

3.1.3. Multilinear difference value measurement model

A multilinear difference value measurement model has no limitations in the form of terms of mutual influence and it says:

$$v(X) = \sum_{i=1}^{n} w_i v_i(x_i) + \sum_{i<j} w_{ij} v_i(x_i) v_j(x_j) + \sum_{i<j<m} w_{ijm} v_i(x_i) v_j(x_j) v_m(x_m) + \cdot\cdot\cdot + \\ + \prod_{i=1}^{n} w_{1\ldots n} v_i(x_i). \tag{10}$$

where w_{ij}, w_{ijm} are parameters of mutual influence. It is about the influence of the attribute value "*j*" on the attribute value "*i*".

The multilinear model permits any combination of terms of mutual influence and requires a weaker assumption of independence: *multilinear difference independence* (MLDI), which requires that multilinear difference independence (MLDI) is maintained for only one attribute, while other attributes are fixed. In other words, the strengths of preference in one attribute are unchanged if the values of other attributes are constant. This allows us to construct a single-attribute value function using difference standard sequence. In order to construct a model (10), we need to estimate the parameters of the mutual influence w_{ij}, w_{ijm}, and so on. In fact, when the number of attributes exceeds four, the number of necessary parameters of mutual influence becomes large and consequently the task of estimation becomes impossible. Even so, a multilinear difference measurement model can sometimes be useful, when some mutual influences are pre-identified.

Building multilinear difference measurement models, as opposed to additive and multiplicative building blocks, is sometimes possible or necessary when the decision maker works with a complex value tree.

Often, some options are independent, but weak mutual influence can occur in the attribute subgroups within one option, where the degree of dependency is determined by particular attribute values. The estimate v_i in a multilinear difference measurement model exactly follows the procedures applied in the additive and multiplicative model. The estimate of the parameters of the mutual influence w_{ij}, w_{ijm} follows the procedures used in the multiplicative model as well as those that we can be identified as such.

3.2. Conjoint measurement method

Unlike difference measurement, which estimates the strength of preferences, the conjoint measurement method estimates only ordinal preferences and indifference between multi-attribute objects. This method is based on *ordinarily measurable values* (utilities) and the criterion value is calculated on the basis of the additive or multiplicative model. In the conjoint measurement method, we will construct the value functions without questioning the strength of preference among the attributes. The class of value measurement methods, conjoint measurements, includes:

1. additive conjoint measurement model,
2. multiplicative conjoint measurement model.

3.2.1. Additive conjoint measurement model

The process used requires attributes to be additive and independent of each other. Otherwise, the standard order constructed using this procedure would depend on the level of other attributes. The first additive version of the conjoint measurement method is based on the assumption called *joint independence* (JI). This assumption tells us about the independence between attribute subgroups, ie. independence between groups. In simple terms, JI requires that preferences between options that differ only in the attribute subgroup be independent of the level in the remaining attributes.

In its general form, JI requires that preferences in *any* attribute subgroup be independent of fixed levels in the remaining attributes. For example, when choosing between several job offers, our preferences for pay and benefit combinations are likely independent of, say, the size of the city where we work. The second additive version is based on the assumption called *single independence* (SI). This additive version is based on the fact that preferences between options determined on the basis of only *one* attribute do not change with changes in the value of other attributes; these are linear changes in the value. For example, the size of the apartment, its location and its rent can be represented by attributes that are SI. If the size of the apartment is important to us, then individually it does not matter where it is and how much it is priced. The most interesting cases of violation of the JI condition are those in which the SI condition is satisfied. These violations are usually well-concealed and difficult to identify.

The process used to construct an additive function of the value "v" within the conjoint measurement method is called a *dual standard sequence* (Krantz, 1971), or *saw tooth* (Fishburn, 1967), that is *lock step* (China&Raifa, 1976). Here, we will briefly outline the basic idea behind this procedure. The process can be best represented in two dimensions: these are, say, two-attribute options given in a form (x_1, x_2).

We first arbitrarily choose a *zero point* on a scale, usually the worst available (possible) point, or the actual combination of attribute values, label them with (x_{1*}, x_{2*}), and suppose that the value function for the worst combination of attribute values equals zero, that is, we have that $v(x_{1*}, x_{2*}) = 0$.

Also, let us arbitrarily select the unit of value of the first attribute, mark it with x_1^1. Then we construct the effect of the second attribute, which is the effect x_2^1, which has the same difference in value. In order to guarantee equal differences in value, we assume that any increase in the second attribute equals the standard increase in the first attribute from, for example, x_{1*} to x_1^*; therefore, the standard effect is determined by the following estimates:

$$(x_1^1, x_{2*}) \sim (x_{1*}, x_2^1), \tag{11}$$

$$(x_1^1, x_2^1) \sim (x_{1*}, x_2^2), \tag{12}$$

$$(x_1^1, x_2^2) \sim (x_{1*}, x_2^3), \tag{13}$$

or, in general

$$(x_1^1, x_2^{i-1}) \sim (x_{1*}, x_2^i). \tag{14}$$

Graphically, this procedure can be represented by Figure 1, which shows a derived value function with two attributes.

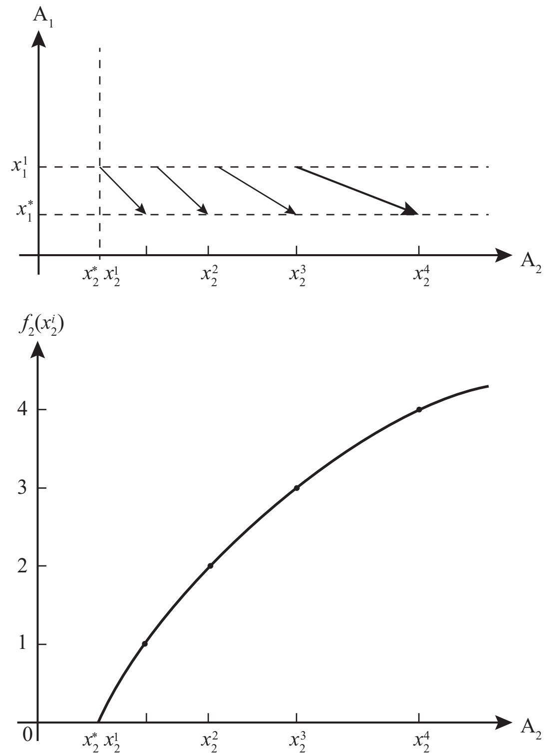

Figure 1. Illustration of a double standard series

3.2.2. Multiplicative conjoint measurement model

The conjoint measurement method is also developed to solve multiplicative model forms. Unlike the multiplicative and multilinear difference measurement model, the multiplicative conjoint measurement models are indeed multiplicative, having zero multipliers and cancellations. In fact, cancellations are necessary in these models. Such cancellation signs can be expressed by a multiplicative conjoint measurement model which has a form

$$f(x) = \prod_{i=1}^{n} f_i(x_i), \tag{15}$$

where levels of some “f_i” can produce neutral amounts (figures).

3.3. Weak order model

All of the methods described above take explicit or implicit assumptions about attribute independence. The model we are considering does not require the assumption of independence, but it does require very complicated modification procedures. This model is called the weak order model.

The weak order model assumes that only the decision maker is able to make consistent tradeoffs between multiple attributes. Tradeoff involves comparing options with at least two attributes; in other words, how much will we give up the value of the attribute "*i*", to increase the value of the attribute "*j*".In practice, the decision maker will not be able to consistently make multi-attribute comparisons for all attributes. The weak order model tries to systematize tradeoffs, that is, to make precise comparisons in which the disadvantages will be compared, compensated with advantages over others; it is ready to "trade", to make concessions to others because of the advantages of certain attributes.

The basic procedure for constructing value functions using the weak order model consists of the following substitution questions. We can make for each resultant multi-attribute value an equal result with certain constant values in all attributes except for *i* attribute. In other words, we find for each option *x* and *y*, having the same attributes, the following:

$$x = (x_1, x_2, ..., \hat{x}_i, ..., x_n) \sim (a_1, a_2, ..., \hat{x}_i^1, ..., a_n), \tag{16}$$

$$y = (y_1, y_2, ..., \hat{y}_i, ..., y_n) \sim (a_1, a_2, ..., \hat{y}_i^1, ..., a_n), \tag{17}$$

for some constant values a_{ij}, where $j \neq i$.

Here, and in what follows in the superscript, for tradeoff values of *i* attribute it means that in *r tradeoff* step this attribute value is changed to, say, $\hat{x}_i^r$.

After *tradeoff*, we determine the order of preference for *i* attribute; so, we construct the order of functions of value (utility) f_i.

The decision maker will prefer "*x*" over "*y*" if and only if

$$f_i(\hat{x}_i^1) \geq f_i(\hat{y}_i^1). \tag{18}$$

We can improve this *tradeoff* process by breaking it down into ($n-1$) step, as shown in the following example:

$$
\begin{aligned}
x = (x_1,\ x_2,\ ...,\ \hat{x}_i,\ ...,\ x_n) &\sim (a_1,\ a_2,\ ...,\ \ \hat{x}_i^1,\ ...,\ a_n) \\
&\sim (a_1,\ a_2,\ ...,\ \ \hat{x}_i^2,\ ...,\ a_n) \\
&\ \vdots \qquad\qquad \vdots \qquad\quad \vdots \\
&\sim (a_1,\ a_2,\ ...,\ \hat{x}_i^{n-1},\ ...,\ a_n).
\end{aligned} \tag{19}
$$

Later we can consistently show the same tradeoff for the option “*y*”; “*x*” would be preferred over “*y*” if and only if

$$
f_i(\hat{x}_i^{n-1}) \geq f_i(\hat{y}_i^{n-1}). \tag{20}
$$

4. CONCLUSION

The modern way of doing business requires from decision makers to increasingly make important business decisions in the face of constant changes in the environment and situations when exact data cannot be obtained for all the parameters that influence a business decision. On the other hand, wrong decisions can be catastrophic and irreversible, so decision makers need to be able to make low-risk decisions. This is achieved by modern methods using multi-attribute decision-making methods.

Indifference value measurement methods rely on studies of indifference (indifference estimates) or comparison of the strength of preferences. The values that the decision maker gives to the attributes reflect his preferences, that is utilities. Value functions have been calculated based on attributes expressed in units, but are expressed in some units of utility. The process of determining (function) values is only one form of sequential determination of utility. The paper analyzes three classes of indifference value measurement methods. These methods, somewhat more complex than SMART, differ from each other not only in the way the value (utility) of individual attributes is determined, but also in the procedure of “aggregating” them.”

Of course, the use of these methods is not sufficient in itself, since the decision maker will always play a decisive role in defining the problem, determining weighting coefficients and evaluating the qualitative attributes.

The paper could not provide answers to all questions in the field of multi-attribute decision-making, but its content and structure will contribute to adequate assessment of the importance and application of indifference estimates in measuring attribute values.

REFERENCES

Atadil, H. A., Sirakaya-Turk, E., Meng, F., & Decrop, A. (2018). Exploring travelers' decision-making styles. *International Journal of Contemporary Hospitality Management*, *30*(1), 618-636. doi.org/10.1108/IJCHM-11-2016-0613.

Brauers, W.K.M. (2004). *Optimization methods for a stakeholder society. A revolution in economic thinking by multiobjective optimization: Nonconvex Optimization and its Applications*, Kluwer Academic Publishers, Boston, USA.

Dyer, J. S., & Sarin, R. K. (1979). Measurable multiattribute value functions. *Operations research*, *27*(4), 810-822. doi.org/10.1287/opre.27.4.810

Goodwin, P. (2008). *Decision Analysis for Management Judgment 3rd Ed*, Utah Valley University.

Zardari, N. H., Ahmed, K., Shirazi, S. M., & Yusop, Z. B. (2015). Weighting methods and their effects on multi-criteria decision making model outcomes in water resources management. *Springer*. doi.org/10.1007/978-3-319-12586-2

Žižović, M. (2018). NEW MULTI-CRITERIA VALORIZATION OF HUNTING TOURISM DESTINATION. *Quaestus*, (12), 77-83.

Žižović, M.R., Damljanović, N., Pavlović, K., Janjić, M. (2016). *Modification of Fuller's triangle method for calculation of weighted coefficients in MCDM models*. Proc. UNITECH Gabrovo, p. IV166-IV170.

Zizovic, M. M., Damljanovic, N., & Zizovic, M. R. (2016). Multiplicative multi-criteria analysis method for decision-making. *Maejo International Journal of Science and Technology*, 10(2), 233-241. doi.org/10.2298/fil1710981z

Miljković, B., Žižović, M. R., Petojević, A., & Damljanović, N. (2017). New weighted sum model. *Filomat*, 31(10), 2991-2998. doi.org/10.2298/fil1710991m

Santocanale, L., & Wehrung, F. (2018). The equational theory of the weak order on finite symmetric groups. doi.org/10.4171/jems/804

Станић, С., Рачић, Ж. (2019). *Економско-математички модели и методе*, Универзитет у Бањој Луци, Економски факултет.

Triantaphyllou, E. (2000). *Multi-criteria decision making methods. In Multi-criteria decision making methods: A comparative study* (pp. 5-21). Springer, Boston, MA. doi.org/10.1007/978-1-4757-3157-6_2

МЕТОДЕ ВИШЕАТРИБУТИВНОГ ОДЛУЧИВАЊА - ПРОЦЈЕНЕ ИНДИФЕРЕНТНОСТИ

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САЖЕТАК

Реалне ситуације базирају се на вишеструким атрибутима или критеријумима. Ако проблем процјене има вишеструке димензије вриједности, тада интуитивна просуђивања могу бити веома тешка. У многим ситуацијама крајња процјена изгледа веома тешка, нарочито када доносилац одлуке врши избор једне из скупа могућих опција и то у условима непознатог окружења. У овом раду анализирају се методе индиферентности за мјерење вриједности. Методе индиферентности за мјерење вриједности ослањају се на истраживања индиферентности (индиферентних процјена) или поређења јачине преференција. Вриједности које доносилац одлука даје атрибутима одражавају његове преференције, тј. корисности. Корисност се сматра само начином за описивање преференција. Поступак утврђивања (функција) вриједности је само један вид секвенцијалног утврђивања корисности.

Кључне ријечи:

Методе вишеатрибутивног одлучивања, методе индиферентности, адитивни модел, мултипликативни модел, мултилинеарни модел, пондерисање.