Original Scientific paper
UDC 528.482:519.23
DOI: 10.7251/afts.2018.1018.055T
COBISS.RS-ID 7324184

DETERMINATION OF AIR REFRACTION INFLUENCE ON TRIGONOMETRIC HEIGHT DIFFERENCES

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ABSTRACT

Efficiency of trigonometric levelling method for height differences determination is better than those determined by geometric levelling but its accuracy is significantly lower. Development of accuracy of geodetic instruments increases possibilities for improvement accuracy of trigonometric levelling.However, air refraction appears as ultimate limitation factor for accuracy of trigonometric heights determination improvement. This paper aims to research the influence of air refraction on accuracy of height differences determined by trigonometric levelling method. Basic statistical hypothesis is that influence of air refraction on height differences is negligible. The case study is based on the results of regular measurements on “Vlasina” dam by using results of geometric and trigonometric levelling.

Key words: *Air refraction, geometric levelling, trigonometric levelling, accuracy, efficiency*

INTRODUCTION

Efficiency of trigonometric levelling method for height differences determination is better than those determined by geometric levelling but its accuracy in past was significantly lower. Improved accuracy of geodetic instruments in few past decades as well as improved measurement efficiency reached by automated total stations and theodolites implicate further research of possibility of trigonometric levelling method utilization especially for cases with short distances between station and measured points. Utilizing formulae for root mean square error of heights differences determined by trigonometric levelling method results with value of approximately 2 mm for distances up to 250 m and zenith angles in range (90^o±15^o) and with accuracy of horizontal distance$m_D = 1$ mm and zenith angle$m_Z = 1"$. This accuracy is reachable and represents the base for further researches.

Trigonometric levelling and its applications are widely researched in literature [1,2,3,4,5, and 6] which means that it is applicable and useful method for height differences or height of points itself determination. Air refraction and its influences on geodetic measurements also were researched and represented in numerous papers. Very large interval of refraction coefficient variation between -5 and +15 depending on height above ground and micrometeorological condition significantly limit the utilization of trigonometric levelling method [7]. Refractive index of air is also thoroughly researched [8,9, and 10] and it is possible to state that there no exists reliable solution for air refraction influence on geodetic measurements. However if measurements already exists it is possible to determine the value of air refraction coefficient on certain site and under climate condition when the measurements was provided.

Air refraction is ultimate limitation factor for improvement accuracy of trigonometric heights determination. Air refraction in geodetic measurements is usually quantified by refraction coefficient which commonly and its value is commonly taken as $k = 0.13 \pm 0.04$. This value is common and it could vary depending on the condition in field where measurements are provided. Taking the value of air refraction coefficient for a grant is not allowed and it must be determined for each local area where measurements are provided.

For determination of air refraction influence on height differences determined by trigonometric levelling method the model with known horizontal distances is proposed. This method is proposed because the horizontal distances and their accuracy are obtained from horizontal network adjustment. In case of known horizontal distances the efficiency of measurements could be additionally increased because the measurements of slope distances could be avoided.

Determination of air refraction is very complex problem and its variation disables trigonometric levelling method when the maximal accuracy is needed. But in case when the heights differences between benchmarks are determined both with geometric and trigonometric levelling the determination of air refraction influence on height differences determined by trigonometric levelling is possible. In this paper the determination of air refraction coefficient influence is researched when the height differences are determined both by geometric and trigonometric levelling.

The basic hypothesis which will be statistically tested in this paper is that influence of air refraction on height differences determined by trigonometric levelling is negligible. This hypothesis will be tested after air refraction coefficient determination and mean square error of air refraction coefficient. For testing this hypothesis the standard mathematical statistic approach will be provided [11].

Case study was provided on the measurements of height differences obtained both by trigonometric and geometric levelling provided in field on “Vlasina” Dam in Serbia. The network for “Vlasina” dam monitoring is suitable because the lines of sight are shorter than 250 m and benchmarks are measured from different positions of pillars as well as from different heights.

METHODOLOGY

Starting from well-known formula for trigonometric heights determination by using known horizontal distance and measured zenith angle:

$$h = DctgZ + \frac{D^2}{2R} - k\frac{D^2}{2R} \qquad \dots (1)$$

where:

- h - height difference between station (tilting axis of theodolite) and measured point determined by trigonometric levelling method;
- D – horizontal distance between station and measured point;
- Z – zenith angle;
- k – coefficient of air refraction (usually adopted k=0.13±0.04) and
- R – radius of Earth (in this paper is adopted R=6377000 m).

it is possible to determinate height difference between theodolites tilting axis and measured point if the horizontal distance is known and zenith angle is measured.

The Figure 1 illustrates model of trigonometric heights described by formula (1).Horizontal distance could be obtained in different ways but it is presumed that it is not obtained on the base of measured slope distance because in that case the formula for trigonometric height could be obtained by formula for slope distances.

In this case we will assume that horizontal distance is obtained as a result of horizontal network adjustment. Zenith angle is assumed to be a result of measurement. The radius of Earth will be adopted as a mean value for certain area what in this case is R=6,377,000 m. The only unknown parameter is air refraction which is represented by coefficient of air refraction.

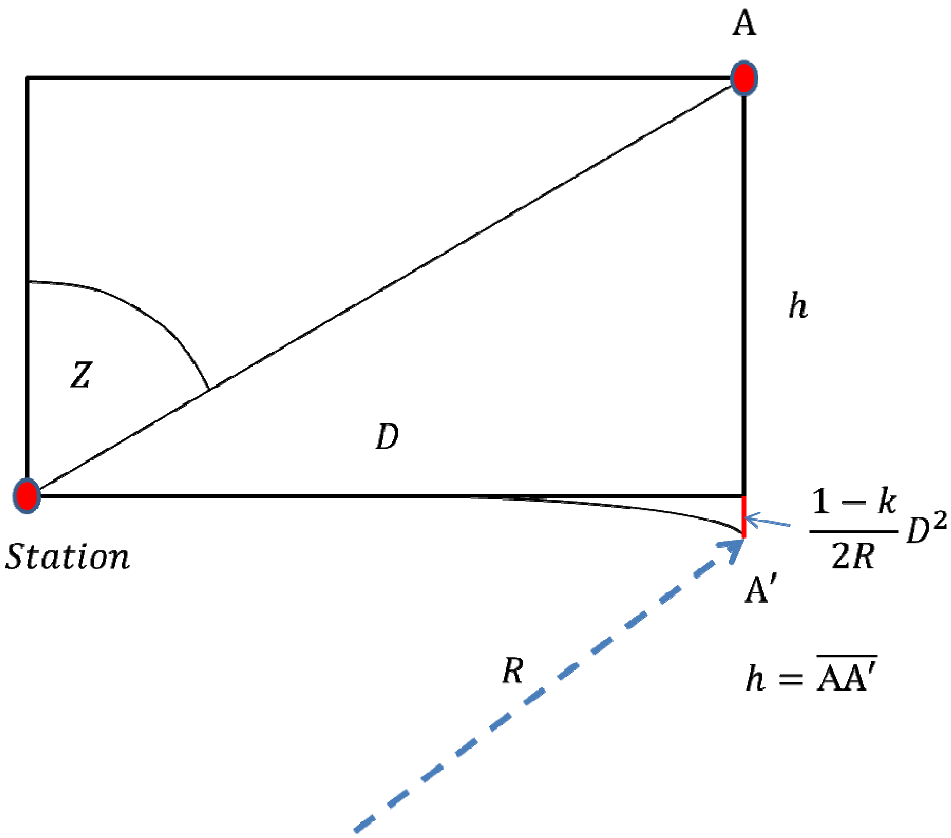

Figure 1 Model of trigonometric heights determination when horizontal distance is known and zenith angle is measured

Mean square error of height differences obtained by trigonometric levelling method is given as follows:

$$m_h^2 = m_D^2 ctg^2 Z + D^2 \frac{1}{sin^2 Z} m_Z^2 \qquad ...(2)$$

$$m_h = \sqrt{m_D^2 ctg^2 Z + D^2 \frac{1}{sin^2 Z} m_Z^2} \qquad ...(3)$$

RMSE for height difference between points A and B, if the horizontal distances are the same and zenith angles are symmetrical to the horizon (or zenith angle are equal but azimuths are different) is then:

$$m_{h_{AB}} = m_h \sqrt{2} \qquad ...(4)$$

The root mean square errors (RMSE) of height differences obtained by formula (3) and adopted values$m_D = 1$ mm; $m_Z = 1''$ for different horizontal distances and zenith angles respectively areshown on figure 2.

The idea for air refraction influence determination on trigonometric height differences is based on measuring zenith angles from one station points to the sets of points. The difference of trigonometric heights between two measured points from one station shall contain the influence of air refraction. Determination of air refraction coefficients is described by the following mathematical model:

$$h_A = D_A ctgZ_A + \frac{D_A^2}{2R} - k\frac{D_A^2}{2R} \quad \ldots (5)$$

$$h_B = D_B ctgZ_B + \frac{D_B^2}{2R} - k\frac{D_B^2}{2R} \quad \ldots (6)$$

where A and B are the measured points station from which the measurement of zenith angle are provided.

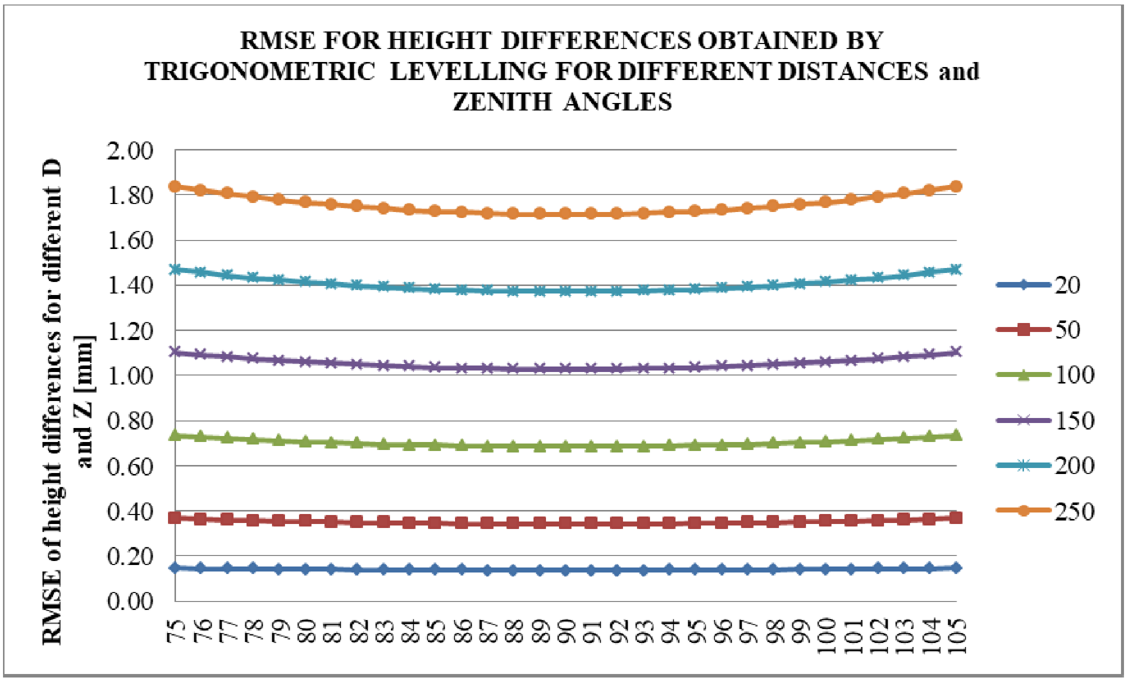

Figure 2 RMSE of height differences determined by trigonometric levelling method for different horizontal distances and zenith angles

The height difference between points A and B then reads:

$$h_{AB} = h_B - h_A = D_B ctgZ_B - D_A ctgZ_A + \frac{D_B^2}{2R} - k\frac{D_B^2}{2R} - \frac{D_A^2}{2R} + k\frac{D_A^2}{2R} \quad \ldots (7)$$

$$h_{AB} = h'_{AB} + k\left(\frac{D_A^2}{2R} - \frac{D_B^2}{2R}\right) \quad \ldots (8)$$

At the same time the height difference between points A and B determined by geometric levelling method, which is considered as a conditionally absolutely accurate and denoted by: h_{AB}^G.

Differences between height differences obtained from two different stations shall be the same i.e.:

$$\delta h_{AB} = h_{AB}^G - h_{AB} = 0 \quad \ldots (9)$$

This is case if all parameters are known and if the errors of measurements are excluded. But the errors in measurements are unavoidable and coefficients of air refraction are also unknown. This leads to following formula:

$$0 = h'_{AB} + k\left(\frac{D_A^2}{2R} - \frac{D_B^2}{2R}\right) - h_{AB}^G \quad \ldots (10)$$

where:

$$h'_{AB} = D_B ctgZ_B - D_A ctgZ_A + \frac{D_B^2}{2R} - \frac{D_A^2}{2R}$$

If we denote

$$\delta h'_{AB} = h^G_{AB} - h'_{AB} \quad ...(11)$$

formula (10) will then read:

$$\delta h'_{AB} = k\left(\frac{D_A^2}{2R} - \frac{D_B^2}{2R}\right) \quad ...(12)$$

The final form of formula (12) then reads:

$$\delta h'_{AB} = ak \quad ...(13)$$

where

$$a = \left(\frac{D_A^2}{2R} - \frac{D_B^2}{2R}\right) \quad ...(14)$$

When we have n points between which the height differences are determined both by trigonometric and geometric levelling method it is possible to make combinations of height differences for each two points. Formula (9) also points out that height difference is determined by trigonometric levelling is influenced only by difference of air refraction coefficient which characterizes each measured point. The total number of combinations is calculated as follows:

$$C_n^2 = m = \frac{(n-1)*n}{2}$$

And finally formula (13) may be written as follows:

$$\delta h'_{AB_i} = a_i k; i = 1 \sim m \quad ...(15)$$

Bearing in mind that results of measurements are erroneous formula (14) must contain also the term which represents the measurement error. This could be described as follows:

$$\delta h'_{AB_i} = a_i k + \varepsilon_i \quad ...(16)$$

We will assume that measurement errors are normally distributed i.e. $\varepsilon \sim N(0, \sigma^2)$. This presumption allows the adjustment of data by the least square method where an air refraction coefficient is unknown.

Finally formula (15) will read:

$$\delta h'_{AB_i} + v_i = a_i k \quad ...(16)$$

$$v_i = a_i k - \delta h'_{AB_i} \quad ...(17)$$

Having m equations and only one unknown (k) the system of equations is predetermined and solution should be obtained by least squares method utilization.

The model of least squares is [11]:

$$v = Ax + l$$

$$l = -\delta h'_{AB_i}$$

$$x = -(A^T P A) A^T P l$$

where:

$$A = \begin{bmatrix} a_1 \\ \cdots \\ a_m \end{bmatrix}$$

$$x = [k]$$

$$P = diag\left[\frac{1}{\left(m_{\delta h'_{AB_1}}\right)^2} \quad \frac{1}{\left(m_{\delta h'_{AB_2}}\right)^2} \quad \cdots \frac{1}{\left(m_{\delta h'_{AB_i}}\right)^2} \quad \cdots \quad \frac{1}{\left(m_{\delta h'_{AB_m}}\right)^2}\right]$$

$$m_{\delta h'_{AB}} = \sqrt{m_{h_A}^2 + m_{h_B}^2}$$

Solution of system equations is the coefficient of air refraction. The residualsv_i shall be the random errors of height differences determined from the two different station points.Meaning of air refraction coefficient in this model is the average of its influence on one station in given range of azimuth, zenith angles, distances and time of measurement duration on one station.

The conclusion about air refraction influence is based on statistical hypothesis testing. Statistical hypotheses are:

H_0: $k = 0$ (i.e. the influence of air refraction on height differences determined by trigonometric levelling method is negligible) against

H_a: $k \neq 0$ (i.e. the influence of air refraction on height differences determined by trigonometric levelling method is not negligible).

Test statistics is [11]:

$$t = \frac{k}{m_k} \sim t_{1-\alpha;f}$$

where:

- t – statistics;
- $t_{1-\alpha;f}$ – students' distribution for given level of significance α (α =0.05) and degrees of freedom f;
- k – air refraction coefficient obtained from adjustment after outliers elimination;
- $m_k = m_0\sqrt{Q_k}$ – root mean square error of air refraction coefficient obtained from adjustment after outliers elimination;
- α– significance level ($\alpha = 0.05$) and
- f – degrees of freedom.

RESULTS AND DISCUSSION

Vlasinadam is an earth filled dam built in the middle of twentieth century on the river Vlasina in south-eastern Serbia. The geodetic networks for dam monitoring were developed around the dam and measurements are provided regularly over the decades. On the dam 30 benchmarks are stabilized and measured for dam monitoring. Benchmarks are stabilized by concrete base and are near the surface of the dam. Surface of the dam is covered by grass. Around the dam the five pillars are stabilized and the measurements of zenith directions and zenith angles were provided. Zenith angles were measured on the small prism centred on each benchmark. The height of the prism over benchmark was 0.100 mm in order to minimize the centring error. Pillars from which the measurements of zenith angles were provided are regularly distributed around the dam and named: C, F, G and H. During the measurement atmospheric conditions were stable and favourable for measurement. The position of "Vlasina" dam with pillars positions from Google Earth view is shown on Figure 3.

Figure 3 "Vlasina" dam and positions of pillars for dam monitoring

The practical value of air refraction influence on height differences determined by trigonometric levelling determination is in improvement of efficiency of dam monitoring. Actually trigonometric levelling method is more efficient than geometric levelling method and its utilization could increase efficiency i.e. to shorten the time of dam monitoring duration. At the same time the accuracy shall not be lost because the importance of reliable conclusion about state and behaviour of the dam.

The reason for research was found in the fact that lines of sights from station points to the benchmarks on the dam are relatively short and that fact could lead to high accuracy of height difference determined by trigonometric levelling method. According to presented model and results of measurements the influence of air refraction was calculated and results are shown in table 1.

Table 1 the results of data processing about air refraction influence on height differences determined by trigonometric levelling method

	k	m_k	f	t	$t_{0.95;f}$	Accepted
C	-0.002	0.244	148	0.0082	1.9761	H_0
F	0.353	0.153	252	2.3136	1.9694	H_a
G	0.203	0.353	130	0.5718	1.9784	H_0
H	0.776	0.268	123	2.8964	1.9794	H_a

According to initial hypothesis testing it is possible to conclude that at pillars C and G null hypothesis shall be adopted and at pillars F and G shall be adopted alternative hypotheses.

Analysing results given in table 1 it is obvious that in two cases (measurements from pillar C and from pillar G) influence of air refraction could be negligible and in two cases (measurements from pillar F and H) it is not negligible. However the RMSEs of air refraction coefficient are relatively high for all pillars in spite of the high number of measurements. The range of distances and zenith angles as well as the number of measured points and influence of RMSE of air refraction coefficient for each pillar are given in table 2.

The influence of RMS of air refraction coefficient is calculated by following formula:

$$I(m_k) = 2 * m_k \left(\frac{D_{Amax}^2}{2R} - \frac{D_{min}^2}{2R} \right)$$

where $I(m_k)$ denotes the influence of RMSE of air refraction coefficient.

Table 2 the range of distances and zenith angles from pillars and influence of RMSE of air refraction coefficient on height difference

	D_{min}[m]	D_{max}[m]	Z_{min}[° ′]	Z_{max}[° ′]	n	$I(m_k)$[mm]
C	28.61	250.15	91.20	104.54	18	2.36
F	112.80	220.50	86.22	95.50	24	0.99
G	119.70	200.66	82.07	89.14	19	1.83
H	79.70	232.67	80.18	90.00	18	1.99

Comparing the influence of air refraction coefficient with influence of errors of horizontal distance and zenith angle it follows that those two influences are equal. This fact implies that influence of air refraction around the Vlasina dam is rather high in spite of the relatively short distances and high accuracy of their determination. Also this result implies that measurements provided for dam monitoring contain capacity for research and improvement of accuracy of trigonometric levelling method.

CONCLUSION

Air refraction as a limiting factor of accuracy of height differences determined by trigonometric levelling method was researched based on results of regular measurements for Vlasina Dam monitoring. Results of geometric levelling was treated as absolutely accurate and model was developed on the base of common formula for trigonometric levelling when horizontal distance are known and zenith angle are measured.

The results of analysis shown that influence of air refraction on height differences determined by trigonometric levelling is significant and that it is still limitation factor for improving their accuracy. Also further researching and improvement of measurement plan is justified because every chance of improving efficiency of measurements for dam monitoring is very important for dam management.

(Received September 2017, accepted October 2017)

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