Professional paper
UDC: 528.9:624.031.573
DOI: 10.7251/afts.2013.0508.061N

ON THE POSSIBILITY OF MEASURED ANGLE CHECKING BY USING MEASURED DISTANCES

Nestorović Žarko[1], Trifković Milan[2], Milutinović Tihomir[3], Pejičić Goran[4], Hamidreza Gholami[4]

[1]*PD „Hidroelektrane Đerdap“ d.o.o. Kladovo, e-mail: zarko.nestorovic@djerdap.rs*
[2]*Građevinski fakultet Subotica, e-mail: tmilutinovic@het.ba*
[3]*HE ,,DABAR,, Trebinje, e-mail: milantri@eunet.rslte*
[4,5]*Tehnički fakultet,Evropski unuverzitet, Brčko , e-mail: gpejicic@gmail.com*

ABSTRACT

Before the electronic distance meters appears the angles measuring dominated in geodetic networks. Development of distance measuring technology and appearance of automated total stations almost equaled the efficiency of measuring angles and distances. This fact allows designer of geodetic networks to combine angles and distances in order to achieve its needed quality. This paper aims to research possibility of checking measured angle by using cosines theorem on the base of measured distances.

Key words: *measured angle, cosines theorem, statistical hypothesis testing*

INTRODUCTION

Before the electronic distance meters appeared the measurement of angles prevailed in the positional (2D) geodetic networks because the distance measurement was resource-demanding, i.e. it was long-lasting and expensive.It was especially in the case ofrequired high accuracy of measured lengths. This fact caused the limitation of quality level of geodetic network design and quality improvement was only possible with considerable enhancement of the number of measured angles. For this reason,analysis and accuracy estimation of angles measuring method were developed in literature [1], [2,3]. With the appearance of electronic distance meters, efficiency and accuracy of measuring distance were considerably increased enabling a reduction in consumption of work of measurement in geodetic networks and an increase of their quality. Accuracy of measuring distances [3,4,5] has also been analyzed and estimated in literature, meaning thatit is possible to obtain their high accuracy when properly performing the distance measurement.

Contemporary development of technologies for measuring angles and distances brought a string of improvements and eliminated considerable number of influences, which limited accuracy of measuring angles and distances by contemporary geodetic total stations [6,7,8]. It is possible to say that development of theodolites reached its maximum as well as that theodolites with standard of measuring direction of $\sigma_p = 0.5''$ represent the border, which will not be overcome in the long-term. If even the accuracy level of measuring angle increases in the domain depending on technology and construction of theodolites, the influence of environment will be an important factor limiting the accuracy of measuring angle. On the other side, the accuracy of measuring distance has been continuously increasing and geodetic instruments (distance meters) with standard of measuring distance of $\sigma_d = 0.6\text{mm} + 1\text{ppm}$ and $\sigma_d = 0.5$ mm for distances up to 120 m have existed on the market for a long time. It means that in future additional improvements could be expected and that high accuracy of longer distances could be reached.

An important issue for geodetic networks design but also for measurement checking, bearing in mind accuracy of measuring angles and distances, could be formulated in the following way: „Under what conditions should measuring distances have advantage in relation to measuring angles in geodetic networks?“ .In other words, what condition allow measuring distances reach more accurate determination of angles value than measuring angle by theodolite. In order to answer this question it is necessary to determine the value and accuracy (root mean square error) of angle determined from measured distances and to compare the values concerned.

In this paper, we will present the comparison of angles obtained on the basis of measured distances and same angles measured by theodolite.

Accuracy analysis of angle obtained by using cosines theorem

Cosines theorem has a form of:

$$a^2 = b^2 + c^2 - 2bccos\beta \tag{1}$$

where a, b, c – sides of triangle and β – angle opposite to side a.
Based on formula (1),it follows that:

$$cos\beta = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\beta = arccos\frac{b^2 + c^2 - a^2}{2bc} \tag{2}$$

Applying the law of error propagation and according to formula (2) the mean square error of angle β could be determined as (considering sides of triangle uncorrelated):

$$\Delta\beta = \frac{\partial\alpha}{\partial\varphi}\frac{\partial\varphi}{\partial a}\Delta a + \frac{\partial\alpha}{\partial\varphi}\frac{\partial\varphi}{\partial b}\Delta b + \frac{\partial\alpha}{\partial\varphi}\frac{\partial\varphi}{\partial c}\Delta c = \frac{\partial\alpha}{\partial\varphi}\left(\frac{\partial\varphi}{\partial a}\Delta a + \frac{\partial\varphi}{\partial b}\Delta b + \frac{\partial\varphi}{\partial c}\Delta c\right)$$

$$\varphi = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Delta\beta = \frac{-1}{\sqrt{1-\varphi^2}}\left[-\frac{a}{bc}\Delta a + \frac{1}{2c}\left(1 - \frac{c^2 - a^2}{b^2}\right)\Delta b + \frac{1}{2b}\left(1 - \frac{b^2 - a^2}{c^2}\right)\Delta c\right]$$

$$m_{\beta_k}^2 = \left(\frac{-1}{\sqrt{1-\varphi^2}}\right)^2\left\{\left(-\frac{a}{bc}\right)^2 m_a^2 + \left[\frac{1}{2c}\left(1 - \frac{c^2 - a^2}{b^2}\right)\right]^2 m_b^2 + \left[\frac{1}{2b}\left(1 - \frac{b^2 - a^2}{c^2}\right)\right]^2 m_c^2\right\}$$

$$m_{\beta_k} = \sqrt{\left(\frac{-1}{\sqrt{1-\varphi^2}}\right)^2\left\{\left(-\frac{a}{bc}\right)^2 m_a^2 + \left[\frac{1}{2c}\left(1 - \frac{c^2 - a^2}{b^2}\right)\right]^2 m_b^2 + \left[\frac{1}{2b}\left(1 - \frac{b^2 - a^2}{c^2}\right)\right]^2 m_c^2\right\}} \tag{3}$$

where:

m_{β_k} – root mean square (RMS) error of angle obtained by using cosines theorem;
m_a - root mean square error of side a of considered triangle;
m_b - root mean square error of side b of considered triangle and
m_c - root mean square error of side c of considered triangle

Formula (3) calculates the mean square error of angle obtained by using cosines theorem (2). Figure 1 shows the mean square errors of angles obtained by using cosines theorem for equilateral triangle for sides up to 120 m. The curves show the mean square errors for standards of measured distance $\sigma_d = 0.2$ mm and$\sigma_d = 0.5$ mm. These two values are adopted because there are geodetic technologies for distance measurements that could achieve them.

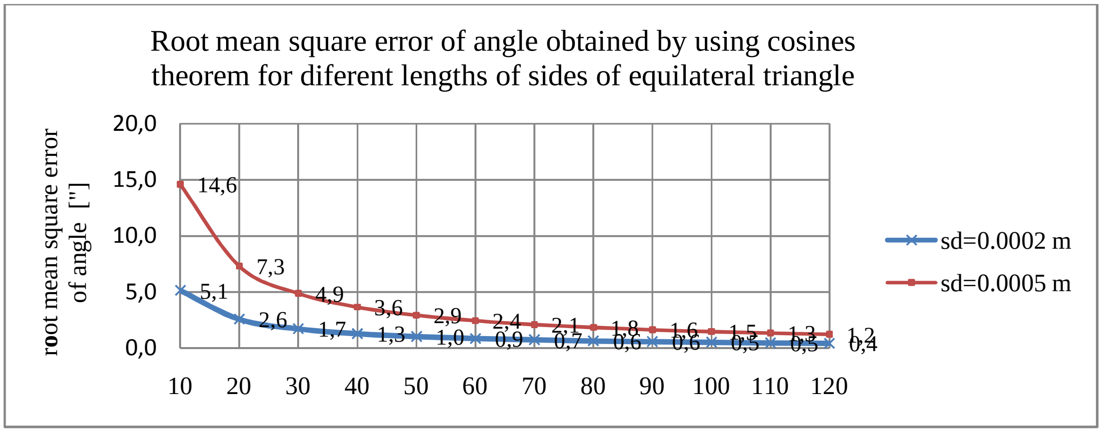

Figure 1 Root mean square error of angle obtained by using cosines theorem for equilateral triangle for sides up to 120 m

According to the diagram in Figure 1, it is possible to conclude that accuracy of angle obtained by using cosines theorem for distances up to 70 m is significantly lower than accuracy of measuring angle, even for extremely high accuracy of measuring distances. It follows from this that in geodetic networks with short length of sides, it is better to measure angles than distances. This conclusion shall be accepted with caution because it is derived under an assumption that declared accuracy of measured direction will be achieved, although it is not that easy in practice. Also, it is necessary to bear in mind that design of geodetic network (point position and observation plan) has certain influence on the quality of geodetic network.

Figure 2 shows root mean square errors of angles obtained by cosines theorem for different standards of measuring distances and for different length of sides of equilateral triangle from 100 m to 1200 m.

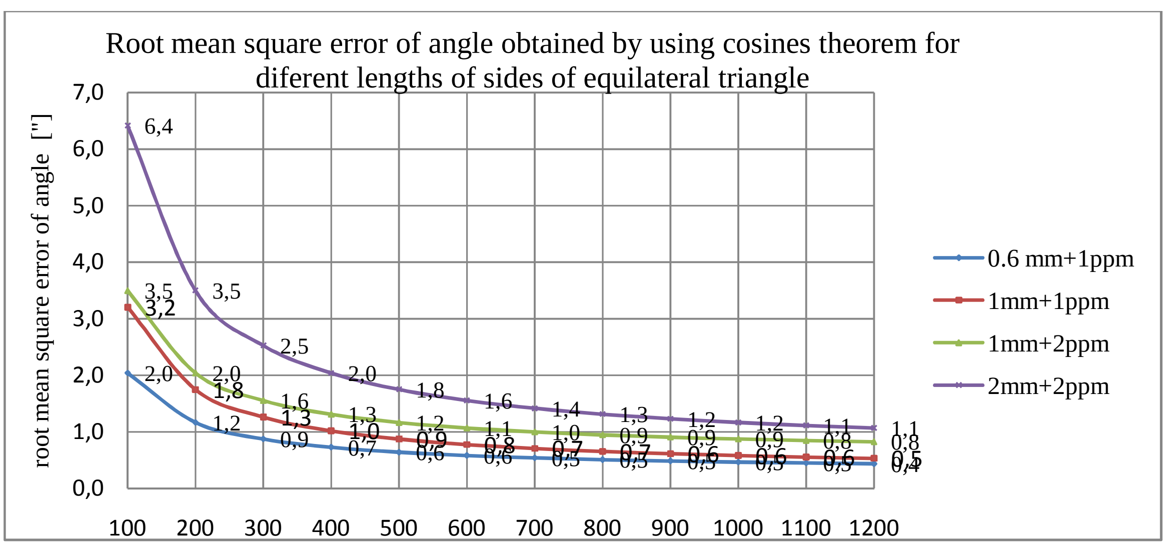

Figure 2 Root mean square errors obtained by cosines theorem for equilateral triangles for length of sides from 100 m to 1200 m

According to the diagram in Figure 2, it is possible to conclude that accuracy of angle obtained by cosines theorem for length of triangle sides up to 400 m is lower than accuracy of measuring angle for distances obtained with lower accuracy ($\sigma_d = 1\text{mm} + 2\text{ppm}$i $\sigma_d = 2\text{mm} + 2\text{ppm}$), while, for distance obtained with higher accuracy($\sigma_d = 0.6\text{mm} + 1\text{ppm}$i $\sigma_d = 1\text{mm} + 1\text{ppm}$) accuracy of measuring angles is almost equal for distances of 300 m. For sides of triangle longer than 800 m even geodetic instruments of standard $\sigma_d = 2\text{mm} + 2\text{ppm}$allow high accuracy of angles obtained by cosines theorem.

Based on the foregoing, it follows that in geodetic networks it is possible to combine instruments of different kind of accuracy. Notes on possible deviations of measuring angle and influence of geodetic network design on quality parameters are also valid in this case.

Statistical hypothesis for measuring angle checking

Formula (2) and (3) allow formulation of two sets of statistical hypothesis for checking measuring angle by theodolite. The first set of statistical hypothesis concerns possible deviations of measuring angle and influence of geodetic network design on quality parameters checking of equality of root mean square errors, while the second set concerns checking of equality of angle values.

Hypothesis for checking equality of root mean squares of angles are:

H_0: root mean square error of angle obtained by cosines theorem based on measuring distances is equal to root mean square error angle measured by theodolite and

H_{0a}: root mean square error of angle obtained by cosines theorem based on measuring distances is not equal to root mean square error angle measured by theodolite.

Since the mean square errors follow F-distribution [8],test statistics are:

$$F = \frac{m_{\beta_M}^2}{m_{\beta_k}^2} \sim F_{f_1,f_2,1-\alpha} \tag{4}$$

where:

F – value of statistics based on values of mean square errors;
$m_{\beta_M}^2$–mean square error of measuring angle;
$m_{\beta_k}^2$ - mean square error based on formula (3);
f_1–degrees of freedom of numerator;
f_2 – degrees of freedom of denominator and
α–levelof significance ($\alpha = 0.05$)

If mean square error of angle is obtained from the declared values (values from producer catalogue) of standard of measuring distance then test statistics (4) have a form of:

$$F = \frac{m_{\beta_M}^2}{m_{\beta_k}^2} \sim F_{f_1,\infty,1-\alpha} \tag{5}$$

Formulae (4) and (5) are valid in case $m_{\beta_M}^2 \geq m_{\beta_k}^2$while in opposite case reciprocal value of F shall be calculated and order of degrees of freedom shall be changed.

Hypothesis for checking equality of angles are:

H_0: angle obtained by cosines theorem on the base of measured distance and measuring angle are equal and

H_a: angle obtained by cosines theorem on the base of measured distance and measuring angle are not equal

Since the equality of values follows t-distribution[8], test statistics are:

$$t = \frac{d_\beta}{m_\beta} = \frac{\beta_M - \beta_k}{m_{\beta_M - \beta_k}} \sim t_{f,1-\alpha} \tag{6}$$

$$m_{\beta_M - \beta_k} = \sqrt{{m_{\beta_M}}^2 + {m_{\beta_k}}^2} \tag{7}$$

where:

t – value obtained from measurement;
β_M – value of measuring angle;
β_k–value of angle obtained by cosines theorem based on measuring distances;
m_{β_M} – root mean square error of measuring angle;
m_{β_k} - root mean square error of angle obtained by cosines theorem based on measured distances;
$m_{\beta_M - \beta_k}$ – root mean square error of angle obtained from measurement and by cosines theorem based on measured distances;
f–degree of freedom (number of redundant measurement)
α - level of significance ($\alpha = 0.05$)

Degree of freedom (number of redundant measurement) shall be calculated in the following way [8]:

$$\frac{1}{f} = \frac{c^2}{f_{\beta_M}} + \frac{(1-c)^2}{f_{\beta_k}} \tag{8}$$

where

$$c = \frac{\frac{m_{\beta_M}^2}{n_{\beta_M}}}{\frac{m_{\beta_M}^2}{n_{\beta_M}} + \frac{m_{\beta_k}^2}{n_{\beta_k}}} \tag{9}$$

f_{β_M} – degree of freedom (number of redundant measurements) of measuring angle;
f_{β_k}– degree of freedom (number of redundant measurements) of angle obtained by cosines theorem based on measured distances;
n_{β_M}–number of angle measurements; and
n_{β_k} – number of distance measurements for angle determination using cosines theorem.
Accuracy estimation of measuring distance could be done using the following formula:

$$m_d = \sqrt{\frac{[\Delta\Delta]}{2n}} \tag{10}$$

where:

m_d–root mean square error of measured distance;
$[\Delta\Delta]$ - sum of square values of distance between forward-backward values;
$\Delta = d_{backward} - d_{forward}$ – difference between measured forward–backward; and
n–number of measured distances.

Comparison of values of measuring angle and angle obtained by using cosines theorem

Check of the model for checking measuring angle is performed by means of an experiment in the field conditions.

Measurements were carried out in the following way:

- Instrument with standards $\sigma_p = 0.5''$for measuring directions and$\sigma_d = 1\text{mm} + 2\text{ppm}$for measuring distances;
- Instrument is checked in a laboratory;
- Forward – backward measurement of distances;
- Measurement of directions, distances and vertical angles was performed at the same time;
- Measurements were performed by the means of forced centring of prisms and instrument;
- Angle was measured in twelve (12) sets;
- During the measurement the temperature, pressure and humidity of air were recorded in order to calculate their influence on measured distances;
- Instrument was protected by parasol during the measurement; and
- Duration of measurement was 80 minutes

The results of distance measurements are given in Table 1.

Table 1 Results of distance measurements including accuracy estimation

	Forward [m]	Backward [m]	Average [m]	Δ [mm]	[ΔΔ]
a=	309.6328	309.6330	309.6329	0.2	0.04
b=	323.8402	323.8406	323.8404	0.4	0.16
c=	22.6032	22.6034	22.6033	0.2	0.04
				[ΔΔ]=	0.24
				m_d=	0.2 mm

The results of measurements including accuracy estimation with degree of freedom for hypothesis testing are given in Table 2.

Table 2 Results of measurements, number of measurements and degrees of freedom

	Measuring values	RMS from measured results	RMS based on declared values
a=	22.6032 m	0.2 mm	1.6 mm
b=	309.6353 m	0.2 mm	1.6 mm
c=	323.8424 m	0.2 mm	1.0 mm
β_M=	3°10'52.2"	1.2"	0.7"
β_k=	3°10'52.9"	0.2"	1.7"
n_{β_M}=	12		
n_{β_k}=	6		
f_1=	11		
f_2=	3		
f=	13		

Hypothesis testing about equality of root mean square errors leads to following results:

$$F = \frac{m_{\beta_M}^2}{m_{\beta_k}^2} = \left(\frac{1.2"}{0.2"}\right)^2 = 37.1567 > 8.7633 = F_{11,3,0.95};$$

$F = \frac{m_{\beta_M}^2}{m_{\beta_M}^2(D)} = \left(\frac{1.2"}{0.7"}\right)^2 = 2.9388 > 1.7886 = F_{11,\infty,0.95}$; where$m_{\beta_M}^2(D)$is mean square error obtained by using declared standard of direction $\sigma_p = 0.5"$for used instrument; and

$F = \frac{m_{\beta_k}^2(D)}{m_{\beta_M}^2} = \left(\frac{1.7"}{1.2"}\right)^2 = 2.0326 < 3.3567 = F_{3,11,0.95}$; where$m_{\beta_k}^2(D)$mean square error of angle obtained by cosines theorem based on measured distances with declared standard of $\sigma_d = 1\text{mm} + 2\text{ppm}$ for used instrument.

Hypothesis testing about equality of angle obtained by measurement and by cosines theorem based on measuring distances leads to:

$$t = \frac{d_\beta}{m_\beta} = \frac{\beta_M - \beta_k}{m_{\beta_M - \beta_k}} = \frac{|-0.7"|}{1.2"} = 0.5756 < 1.7709 = t_{13,0.95}$$

According to the obtained results it may be concluded:

- There is no reason to accept the null hypothesis about equality of root mean squares of measuring angle and the same angle obtained by cosines theorem based on the measuring distances and by using root mean square errors of distances obtained by formula (10). This means that accuracy of measuring angle is significantly lower than accuracy of angle obtained by cosines theorem when the root mean square error is calculated on the basis of difference of forward – backward measured distance;

- There is no reason to accept null hypothesis about equality of root mean square error of measuring angle and declared standard for measuring directions for used instrument $\sigma_u = \sigma_p\sqrt{2} = 0.7"$, where σ_u is standard of measured angle;

- There is no reason to reject null hypothesis about equality of root mean square errors of measuring angle and angle obtained by cosines theorem based on the measuring distances and declared accuracy of measuring distances for used instrument; and

- There is no reason to reject null hypothesis about equality of measuring angle and angle obtained by cosines theorem based on the measured distances.

It is here possible to ask the following question: „Is it possible to check other angles in a triangle?“ The answer is that even in the best case (when the root mean squares obtained from forward – backward results),the root mean square error of measuring angle is significantly lower than the root mean square error obtained by formula (3) because$m_{\beta_k} = 3.1"$and$F = \frac{m_{\beta_k}^2}{m_{\beta_M}^2} = \left(\frac{3.1"}{1.2"}\right)^2 = 6.6736 > 3.5874 = F_{3,11,0.95}$. It means that, for checking measuring angle by cosines theorem, it is necessary that a certain triangle has satisfactory geometry.

CONCLUSION

In respect of the results of the foregoing analysis, it may be concluded:

- It is possible to check the measuring angle by cosines theorem based on the measuring distance;
- It is necessary that triangle has satisfactory geometry for reliable checking of measuring angle;

- Accuracy of measuring distances could be better than the declared one for certain instrument; and
- During the measurement it is possible that some influences could appear and cause the significant deviations of measuring angle from the declared values.

The obtained results make it possible to carry out statistical checking of measuring angles during geodetic network design with the purpose of gross errors detection.

REFERENCES

[1] Činklović, N. (1983): Metode preciznih geodetskih merenja. Beograd. Građevinski fakultet, Univerzitet u Beogradu.
[2] Činklović, N. (1978): Analiza i prethodna ocena tačnosti metoda preciznih geodetskih merenja. Beograd. Građevinski fakultet, Univerzitet u Beogradu.
[3] Mrkić, R. (1991): Geodetska metrologija, Beograd. Naučna knjiga.
[4] Kontić, S., Mrkić, R. (1987): Elektronski teodoliti i njihova primena u geodeziji. Struga, Makedonija. Savetovanje o osnovnim geodetskim radovima, Zbornik radova.
[5] Kontić, S., Mrkić, R. (1987): Elektronsko merenje dužina. Beograd. Građevinski fakultet Beograd, Naučna knjiga.
[6] Solarić, N., Benčić, D., Barković, Đ., Zrinjski, M. (2007): Reading and Registering of Directions by Electronic Theodolites and Electronic Tacheometers Leica, KiG special issue. Zagreb. University of Zagreb, Faculty of Geodesy.
[7] Solarić, N., Solarić, M., Barković, Đ., Zrinjski, M. (2011): New Technological Achievement in the Construction of Integrated Measuring Stations, Geodetski list, Vol. 65 (88) No.4, 2011.
[8] Perović, G. (1989): Račun izravnanja, teorija grešaka merenja. Beogard. Naučna knjiga.