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THE STATIC ANALYSIS OF OPTIMUM CLOSURES OF THIN-WALLED CYLINDERS

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Abstract: This paper deals with the problem of optimal vessel end closures subjected to internal pressure. This research aims to analyse the shapes of the middle surfaces in the case of one-arc and two-arc domes. The optimal solutions seek to minimize design complexity while satisfying geometric and strength requirements. For one-arc closures, meridional profiles are defined by a generalized ellipse as well as a Bezier polynomial of the 5th degree, while in the case of two-arc closures, they are represented by Bezier polynomials of the 3rd and 5th degree. The static analysis was conducted using the finite elements method implemented in the Simcenter Femap software package. Appropriate conclusions were derived by assessing equivalent stresses and criterion functions.

Keywords: Geometry optimisation, Pressure vessel head, Internal pressure.

1. INTRODUCTION

The cylindrical shell is one of the most commonly used structural elements when it comes to vessels under the influence of internal pressure. Awareness of the maximum load that the structure can accept without leaking or catastrophic failure is crucial for the designer and the user. It is necessary to guarantee the safety of the contents as well as the integrity of the cylindrical vessel.

A great challenge is certainly represented by different structural solutions when it comes to closed cylinders. There are different types of closures, from flat plates to torispherical, ellipsoidal, and hemispherical cylinder heads. Ellipsoidal and hemispherical cylinder heads are also called one-arc, while torispherical are defined by two meridional curves, and are therefore called two-arc vessel heads. The heads of cylindrical vessels under the influence of internal pressure can be treated as axisymmetric problems.

A significant number of papers have dealt with problems related to cylindrical vessels modelled using shell elements. The authors [1-3] paid special attention to the shell finite elements (FE). The fundamental theories related to thin-walled shells are discussed in cited researches. The burst pressures of thin-walled cylinders constructed of steel with a yield plateau were studied by experimental methods and finite element analysis in research [4]. Paper [5] has established the equations to calculate the burst pressure of thin-wall pipe under capped-end conditions and capped-open conditions. [6, 7] are devoted to shape optimisation of axisymmetric shells. In [8], the optimisation of the flat ends of closed cylinders is shown. The application of flat ends in pressure boilers is inevitably associated with the presence of stress concentration, which is observed in the vicinity of the junction of the cylinder and the closing flat plate. Paper discusses the effectiveness of other designs for flat ends used in pressure vessels and suggests some modifications. [9] presents the experimental and numerical analyses of the pressure vessels with different flat ends. The behaviour under low and high pressures and the influence of the residual welding stresses, material properties, and geometrical tolerances on the level of plastic deformation in the flat end is discussed. [10] considers two optimisation problems regarding the search for optimal shapes and thicknesses of cylindrical closed vessels made of homogeneous and isotropic materials. The region of the junction of ellipsoidal and cylindrical shells is subject to stress concentration, so the subject of the research analysis [11] is a stress concentration problem in a circular cylindrical vessel with ellipsoidal heads. Researches [12-15] deal with a similar topic and proposals for the appropriate optimal types of closure are given through ellipsoidal and torispherical cylinder heads.

The subject of the work [16] is a typical ground-based horizontal cylindrical tank. The optimal ratios of the basic dimensions for an assumed family of tanks of given capacities were determined. The aim of paper [17] is to find the optimum size of a ground based cylindrical liquid storage tank that is supported at both ends and the design considerations are stability and strength constraints. [18] presents a short survey of the optimal design of the pressure vessel and its head using a trigonometric series. [19] discusses shape optimisation of axisymmetric pressure vessels considering an integrated approach in which the entire pressure vessel model is used in conjunction with a multi-objective function that aims to minimize the Von Mises mechanical stress from nozzle to head. [20, 21, 22] are devoted to stresses and their minimization, and [23] analysed ellipsoidal vessels of maximum volume under cyclic pressure. Two design criteria are given for uniform meridian and hoop membrane stresses. Optimal shapes are derived analytically. Paper [24] is a review paper on the strength, stability, and optimisation of horizontal pipes under pressure.

2. VESSEL HEAD OPTIMISATION

Elastic, thin-walled, axisymmetric problems are considered. Pressure vessels for storage and transport usually have a limited length, and at the same time, the aim is to maximize the capacity of the structure. Therefore, from a practical point of view, the cylinder heads should have the smallest depth, to maximize the length of the cylindrical part. On the other hand, it is necessary to choose certain forms of closure so that the cylinder heads with the adopted depth achieve their maximum capacity.

Using numerical methods, Kruzelecki and Proszowski [13, 15] gave suggestions for optimal meridian shapes that minimize the design, satisfy the conditions of depth and capacity, and refer to heads of constant thickness that fulfilled appropriate geometric conditions. The authors focused on defining such profiles for which the state of bending in the entire structure is eliminated or minimized. Figure 1 shows a two-arc dome with the following markings:

a – the mean radius of the cylindrical part,
p – internal pressure,
b_0 – the depth achieved by curve C2,
M – joining point of C1 and C2 curvature,
V_a – the capacity (volume) of the hemisphere of the radius a.
L_C – ½ the length of the cylindrical part,
b_M – the depth achieved by curve C1,
b – total depth of the cylinder head,
V – cylinder head volume,

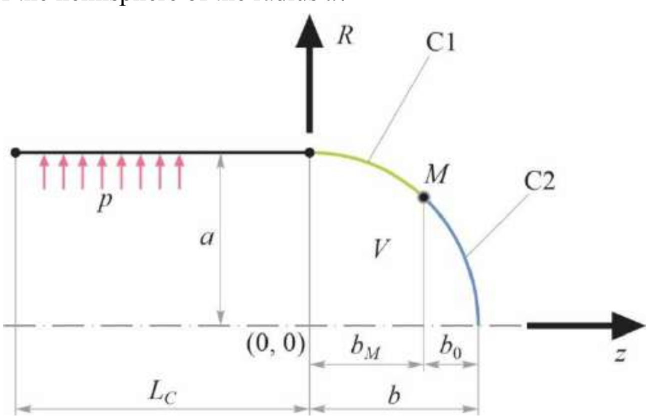

Figure 1. *Pressure vessel loaded by an internal pressure p*

The previously mentioned conditions of depth and capacity will be expressed through the functional *F*, where it is important to find its minimum value:

$$F = \alpha \frac{b}{a} - \delta \frac{\hat{V}}{V_a} \Rightarrow \min \tag{1}$$

The functional *F* contains the depth *b* and the capacity *V* of the vessel end closure, while α and δ represent the weight coefficients satisfying the following condition $\alpha + \delta = 1$.
The following relations will be used in the paper:

$$\begin{gathered} r = R / a,\ \xi = z / b_M,\ \beta = b_M / a, \\ \xi_0 = z_0 / b_M,\ \beta_0 = b_0 / a,\ \bar{r} = R_0 / a \end{gathered} \tag{2}$$

The corresponding geometric conditions are given by expressions (3) and (4):

$z = 0$			
$R = a$		$r = 1$	
$\frac{dR}{dz} = 0$	or in dimensionless form	$r' = 0$	(3)
$\frac{d^2R}{dz^2} = 0$		$r'' = 0$	

$z = z_M$			
$R_{C1} = R_{C2}$		$r_{C1} = r_{C2}$	
$\frac{dR_{C1}}{dz} = \frac{dR_{C2}}{dz}$	or in dimensionless form	$r'_{C1} = r'_{C2}$	(4)
$\frac{d^2R_{C1}}{dz^2} = \frac{d^2R_{C2}}{dz^2}$		$r''_{C1} = r''_{C2}$	

For $z > 0$ it is assumed that the investigation is about convex shells for which the following is valid $\frac{d^2R}{dz^2} < 0$.

In the case of one-arc domes, it is also necessary to fulfil the condition that the tangent to the curve at the point of connection with the axis of the cylinder is perpendicular to the axis of the cylinder:

$$\left. \frac{dr_{C1}}{d\xi} \right|_{\xi=1} = \infty \tag{5}$$

The strength condition indicates that the equivalent stress (Huber-Mises-Hencky) in the entire cylinder head must not exceed the equivalent stress in the cylinder body and is expressed by the following relation:

$$\frac{\sigma_{head}^{eq}}{\sigma_{cyl}^{eq}} \le 1 \tag{6}$$

2.1 Two-arc domes

The curve C1 will be modelled using Bezier polynomials of 3rd and 5th degrees. Curve C2 represents a circular arc of radius R_0, whose centre on the *z*-axis is defined by the z_0 coordinate:

$$R_{C2} = \left[R_0^2 - \left(z - z_0 \right)^2 \right]^{1/2} \tag{7}$$

The smoothness of the curves at the joining point is of great importance.

<u>Two-arc domes: Bezier polynomial of the 3rd degree</u>

The shape $r(\xi)$ represents the dimensionless meridian of the dome and curve C1 is defined as follows:

$$r_{C1}(\xi) = \begin{bmatrix} \xi^3 & \xi^2 & \xi & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} \tag{8}$$

Where r_0, r_1, r_2, r_3 are the coordinates of the control points, and ξ is the dimensionless coordinate that includes the dimensionless depth β and is within the limits of $0 \le \xi \le 1$.

By fulfilling the geometric conditions, we get $r_0 = r_1 = r_2 = 1$ and only the coordinate r_3 remains as a free parameter for optimisation.

<u>Two-arc domes: Bezier polynomial of the 5th degree</u>

Dimensionless meridian $r(\xi)$ for curve C1 is defined as follows:

$$r_{C1}(\xi) = \begin{bmatrix} \xi^5 & \xi^4 & \xi^3 & \xi^2 & \xi & 1 \end{bmatrix} \begin{bmatrix} -1 & 5 & -10 & 10 & -5 & 1 \\ 5 & -20 & 30 & -20 & 5 & 0 \\ -10 & 30 & -30 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} \tag{9}$$

Where $r_0, r_1, r_2, r_3, r_4, r_5$ are the coordinates of the control points, and ξ is the dimensionless coordinate that includes the dimensionless depth β and is within the limits of $0 \le \xi \le 1$.

By fulfilling the geometric conditions, we get $r_0 = r_1 = r_2 = 1$ and coordinates r_3, r_4 and r_5 remain as free parameters for optimisation.

2.2 One-arc domes

One-arc domes will be defined in two ways, using the so-called generalized ellipse and using the Bezier polynomial of the 5th degree. Since the curve C2 vanishes, it follows that $b_0 = 0(\beta_0 = 0)$ and $b = b_M$, so the dimensionless depth is obtained as $\beta = b / a$.

One-arc domes: Generalized ellipse

The generalized ellipse takes the form of:

$$r_{C1}(\xi) = \left[1 - \xi^n\right]^{1/m} \tag{10}$$

and it is defined by three parameters, the exponents n and m, and the dimensionless depth which enters the form of the dimensionless coordinate ξ .

One-arc domes: Bezier polynomial of the 5th degree

In this case, the one-arc meridian is defined parametrically:

$$z = b\,f(t),\ R = a\,g(t) \tag{11}$$

where $f(t)$ and $g(t)$ are the Bezier polynomials of the 5th degree:

(12)

$$f(t) = \begin{bmatrix} t^5 & t^4 & t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 5 & -10 & 10 & -5 & 1 \\ 5 & -20 & 30 & -20 & 5 & 0 \\ -10 & 30 & -30 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}$$

$$g(t) = \begin{bmatrix} t^5 & t^4 & t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 5 & -10 & 10 & -5 & 1 \\ 5 & -20 & 30 & -20 & 5 & 0 \\ -10 & 30 & -30 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{bmatrix}$$

(13)

$f_0 ... f_5$ and $g_0 ... g_5$ are the coordinates of the control points, and t is a parameter in the following limits $0 \le t \le 1$. For $t = 0,\ R = a$ and in case $t = 1,\ R = 0$.

By fulfilling the geometric conditions, we get $f_0 = 0,\ f_4 = 1,\ f_5 = 1,\ g_0 = 1,\ g_1 = 1,\ g_2 = 1$ and $g_5 = 0$.

Other coordinates represent free parameters for optimisation.

3. SUGGESTIONS FOR OPTIMISATION

Kruzelecki and Proszowski [13, 15] proposed suitable parameters for the mentioned forms of closure, according to weight coefficients $(\alpha = 1,\ \alpha = 0.5)$. The proposals are presented in Tables 1 and 2.

Table 1. *Proposals for parameters of one-arc* domes *[13, 15]*

Weight coefficient	$\alpha = 1$	$\alpha = 0.5$
Generalized ellipse	$n = 2.6000$	$n = 2.6103$
	$m = 1.6721$	$m = 1.8023$
	$\beta = 0.6490$	$\beta = 0.9113$
	$F = 0.6490$	$F = -0.0270$
Bezier polynomial of the 5th degree	$g_3 = 0.8473$	$g_3 = 0.8586$
	$g_4 = 0.5168$	$g_4 = 0.4698$
	$f_1 = 0.0513$	$f_1 = 0.1028$
	$f_2 = 0.4697$	$f_2 = 0.7240$
	$f_3 = 0.6046$	$f_3 = 0.7404$
	$\beta = 0.6066$	$\beta = 0.8992$
	$F = 0.6066$	$F = -0.0504$

Table 2. *Proposals for parameters of two-arc domes [13, 15]*

Weight coefficients	$\alpha = 1$	$\alpha = 0.5$
Bezier polynomial of the 3rd degree	$r_3 = 0.6596$	$r_3 = 0.7007$
	$\beta = 0.5516$	$\beta = 0.6724$
	$\beta_0 = 0.1668$	$\beta_0 = 0.2333$
	$\beta + \beta_0 = 0.7184$	$\beta + \beta_0 = 0.9057$
	$F = 0.7184$	$F = -0.0269$
Bezier polynomial of the 5th degree	$r_3 = 0.8988$	$r_3 = 0.9010$
	$r_4 = 0.8263$	$r_4 = 0.9004$
	$r_5 = 0.5452$	$r_5 = 0.5370$
	$\beta = 0.5466$	$\beta = 0.7707$
	$\beta_0 = 0.1023$	$\beta_0 = 0.1092$
	$\beta + \beta_0 = 0.6489$	$\beta + \beta_0 = 0.8799$
	$F = 0.6489$	$F = -0.0383$

For the displayed values of the corresponding parameters, with the condition of a common point M and a unique tangent to the curves C1 and C2 at the point M, the remaining parameters key to defining curve C2 can be obtained (Table 3).

Table 3. *Parameters of C2*

Weight coefficient	$\alpha = 1$	$\alpha = 0.5$
Bezier polynomial of the 3rd degree	$\bar{r}_0 = 1.3878$ $\xi_0 = -1.2135$	$\bar{r}_0 = 1.1689$ $\xi_0 = -0.3917$
Bezier polynomial of the 5th degree	$\bar{r}_0 = 1.5042$ $\xi_0 = -1.5648$	$\bar{r}_0 = 1.3752$ $\xi_0 = -0.6427$

For the adopted radius of the cylinder $a = 100\,\text{cm}$, the forms of the meridional profiles are obtained as in Figures 2 and 3.

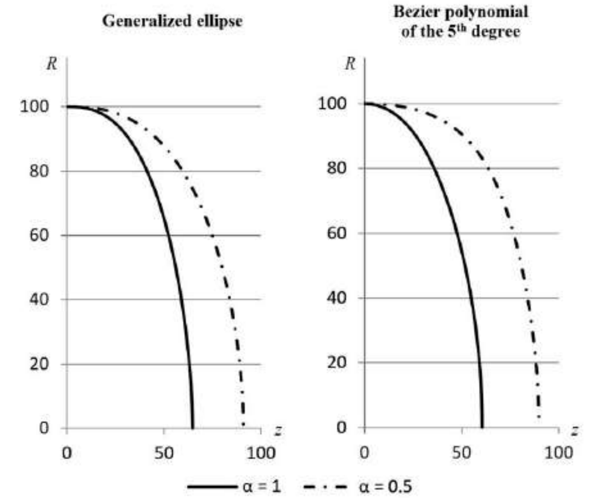

***Figure 2**. One-arc profiles*

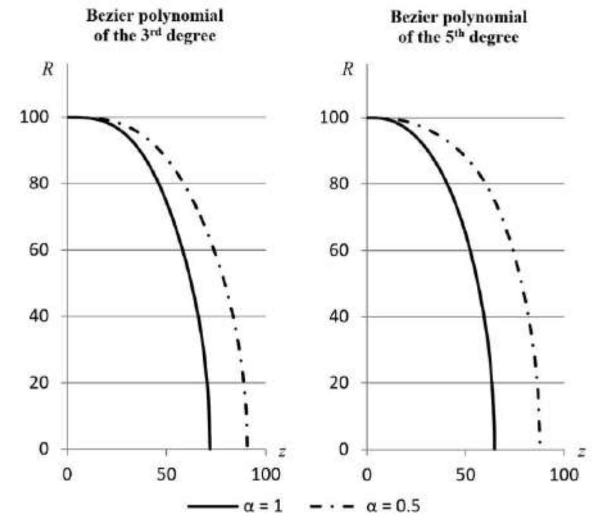

Figure 3. *Two-arc profiles*

Based on Figures 2 and 3, it can be seen that for higher values of the weighting coefficients, smaller dome depths are obtained.

4. FEA MODELS

For all calculation forms, pre-processing and post-processing of FEA models were performed in the Simcentar Femap software package. Structural analysis was conducted using NX Nastran software, which operates based on the finite element method. Linear static analysis was performed using the same material data with a Young's modulus (E) of 21000 kN/cm^2 and Poisson ratio (ν) of 0.3. Additionally, an internal pressure of $p = 0.02$ kN/cm^2 was applied.

The length of the modelled cylindrical part is 200 cm and the thickness of the wall of the entire structure is given as $\delta = 0.4$ cm. Due to symmetry 1/8 of the closed cylinder was modelled with defined symmetry conditions as follows:

for nodes with the coordinate $y = 0$, $v = 0, \theta_x = 0, \theta_z = 0$,

for nodes with the coordinate $x = 0$, $u = 0, \theta_y = 0, \theta_z = 0$,

for nodes with the coordinate $z = -200$ cm, $w = 0, \theta_x = 0, \theta_y = 0$.

The 1/8 of the closed cylinder is modelled in detail with 27500 4-node shell finite elements and 27851 nodes. The average length of the sides of elements is about 1 mm. FEA model with corresponding boundary conditions and detailed mesh quality is shown in Figure 4.

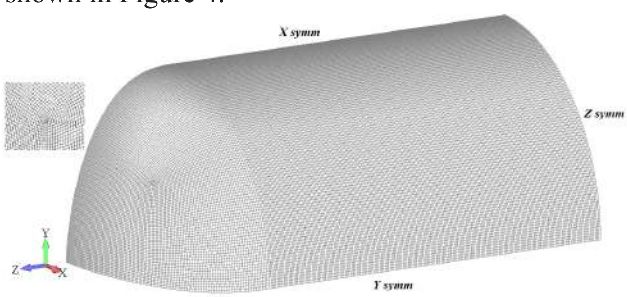

Figure 4. *FEA model with boundary conditions*

5. CALCULATION RESULTS

Figure 5 shows the results for the equivalent stress on the upper surface of the elements, while Figure 6 shows the equivalent stress on the lower surface of the elements in the case of meridian approximation using a generalized ellipse $(\alpha = 1)$. For those surfaces, the equivalent stress reaches its maximum and minimum values per wall thickness. At any point in between, the equivalent stress will have a value between the specified minimum and maximum.

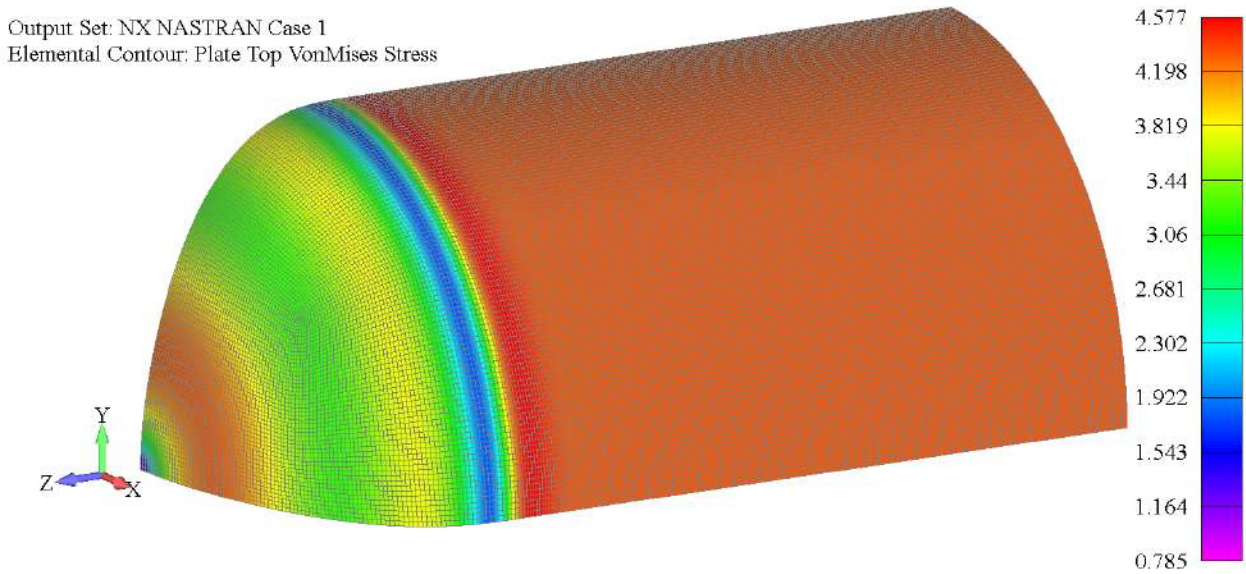

Figure 5. *Top Von Mises Stresses (generalized ellipse, α = 1)*

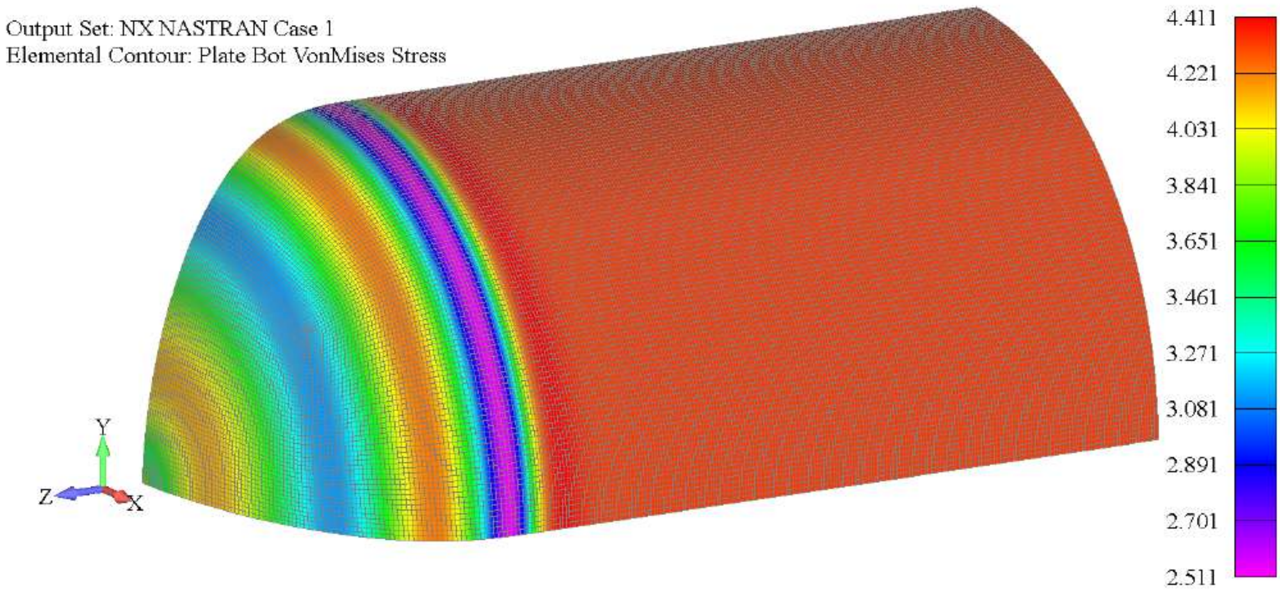

Figure 6. *Bottom Von Mises Stresses (generalized ellipse, α = 1)*

The graphs in Figures 7 and 8 show the dimensionless equivalent stress $\frac{\sigma_{head}^{eq}}{\sigma_{cyl}^{eq}}$ of the cylinder head at the outer and inner surface as a function of the dimensionless coordinate $\xi_1 = z / a$.

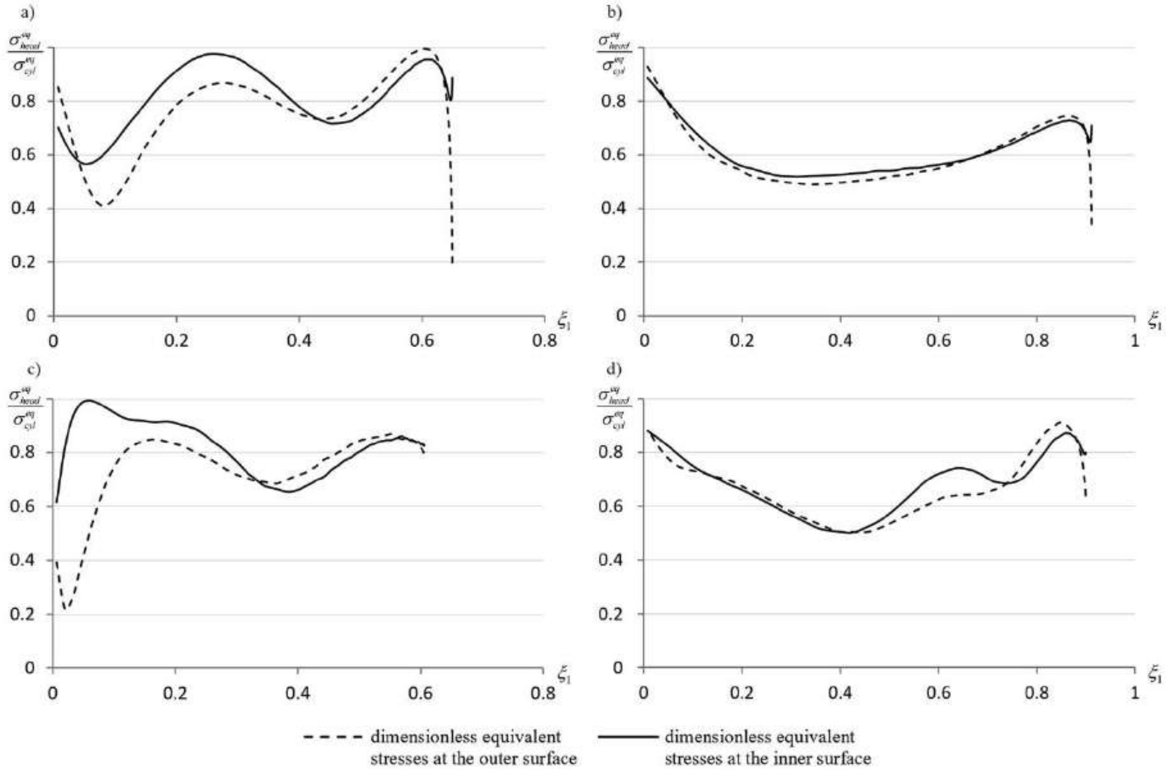

Figure 7. *Dimensionless equivalent stresses in case of one-arc domes defined via generalized ellipse: a) α = 1, b) α = 0.5 and Bezier polynomial of the 5th degree: c) α = 1, d) α = 0.5*

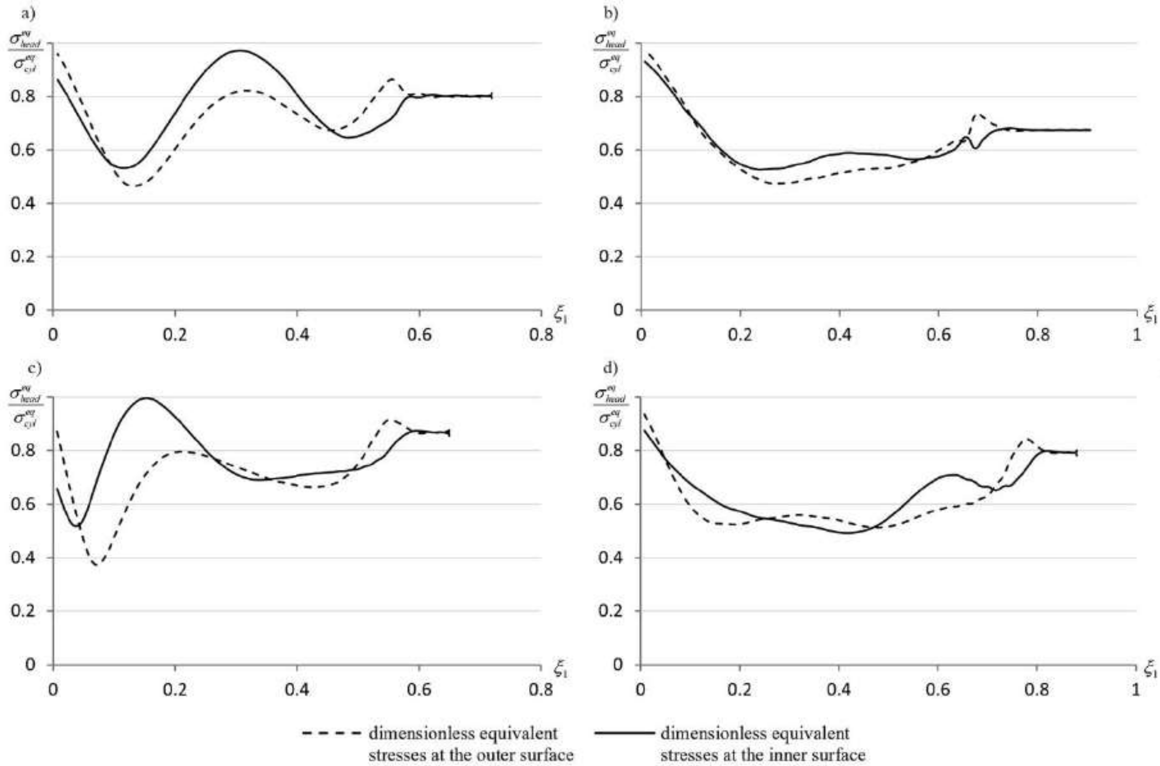

Figure 8. *Dimensionless equivalent stresses in case of two-arc domes defined via Bezier polynomial of the 3rd degree: a) α = 1, b) α = 0.5, and Bezier polynomial of the 5th degree: c) α = 1, d) α = 0.5*

6. CONCLUSIONS

Based on the presented graphs (Figures 7 and 8), it can be concluded that the proposed meridian forms meet the strength requirements in all cases. Differences in the equivalent stresses on the inner and outer surfaces indicate a change in the value of the equivalent stress per wall thickness. At any other point in between, the value of the equivalent stress will be between those previously mentioned, one of which is minimum and the other one maximum. Namely, the difference in these values causes the bending of the dome.

Domes with greater depths were obtained for lower values of the weight coefficients for both one-arc and two-arc meridional profiles. However, for those domes, better results were obtained regarding the functional (1). Also, Figures 7 and 8 indicate that the state of bending is minimized precisely for domes with a weight coefficient $\alpha = 0.5$.

When comparing one-arc and two-arc domes, lower values of criterion function (1) are obtained in the case of one-arc domes, so in terms of optimisation, can serve as a better solution.

If a comparison is made between the meridians defined by the generalized ellipse and the meridians defined by the Bezier polynomial of the 5th degree, we can see that for the same values of the weighting coefficients, smaller depths and lower values of the criterion function (1) are obtained for the meridians defined by the Bezier polynomial of the 5th degree.

In terms of eliminating bending, the best solution turned out to be the generalized ellipse for the value of the weight coefficient $\alpha = 0.5$.

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STATIČKA ANALIZA OPTIMALNIH VIDOVA ZATVARANJA TANKOZIDNIH CILINDARA

Sažetak: Ovaj rad razmatra problem optimalnih vidova zatvaranja tankozidnih cilindara pod dejstvom unutrašnjeg pritiska. Cilj je da se analiziraju oblici srednje površi u slučaju jednolučnih i dvolučnih kupola. Optimalna rješenja minimiziraju dizajn, te zadovoljavaju geometrijske zahtjeve i uslove čvrstoće. U slučaju jednolučnih kupola, meridijalni profili su definisani generalizovanom elipsom i Bezijerovim polinomom 5. stepena, dok su u slučaju dvolučnih kupola definisani pomoću Bezijerovih polinoma 3. i 5. stepena. Statička analiza je sprovedena primjenom konačnih elemenata kroz Simcentar Femap softverski paket. Uz osvrt na ekvivalentne napone i kriterijumske funkcije doneseni su odgovarajući zaključci.

Ključne riječi: Geometrijska optimizacija, posude pod pritiskom, unutrašnji pritisak.

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