Original scientific paper

UDK 004.8.032.26:005.821
DOI 10.7251/COMEN2402179G

CORRELATION ANALYSIS OF FORCE PARAMETERS ASSIGNED TO TIME SERIES DATA

Dragana Malivuk Gak*, Zoran Rajilić

University of Banja Luka, Faculty of Natural Sciences and Mathematics, Banja Luka, Republic of Srpska, B&H

* Corresponding author: dragana.malivuk-gak@pmf.unibl.org

Abstract: In a complex system, determinism without chaos, determinism with chaos, and stochasticity are intertwined, making the behavior of such a system difficult to predict. In this model, time series data are linked to time-dependent force parameters using Newton's second law. The correlations between these force parameters are calculated, and a correlation matrix is created. The values of the determinants of the correlation matrix are then computed. The relationship between the determinant values and the prediction of the behavior of the complex system is considered. This type of rough forecasting model can be valuable for studying earthquakes, movements in stock markets and stock indices, as well as predicting fluctuations in daily and monthly temperatures.

Keywords: Newton's second law, correlation matrix, complex systems.

1. INTRODUCTION

A major earthquake is typically followed by smaller tremors known as "aftershocks." Several methods exist to describe aftershock dynamics. One common approach is the Gutenberg-Richter relation [1]. In this model, it is proposed that the number of earthquakes of magnitude M or greater $N(M)$, is given as:

$$\log N(M) = a - bM, \tag{1}$$

where a and b are two constants. The parameter values are determined experimentally and depend on the regions where earthquakes occur. For instance, the value of the constant b ranges between 0.7 and 1. Another approach known as Omori's law links the time elapsed since the main shock to the number of aftershocks per unit time, $n(t)$ [2-5]:

$$n(t) \approx t^{-p} \tag{2}$$

To avoid divergence at t=0, Omori's law is rewritten as:

$$n(t) = K(t + \tau)^{-p}, \tag{3}$$

where K and t are constants. In order to obtain the number of aftershocks N (t) after the main shock, the previous equation is integrated in the interval from 0 to t [6]. In addition to the previously mentioned models that describe the behavior of the system after a large impact, numerous other studies address the same topic. It is particularly interesting that the physics used to describe the occurrence of earthquakes and the number of aftershocks following the main shock can also be applied to other disciplines, such as economics, specifically in stock market analysis [5-9]. We can apply the model presented in this paper to the prediction of aftershocks in the earthquakes and the stock market with change in the value of stock indices after financial crises.

2. METHOD

We associate parameters of non-conventional force acting on a point in data space with the time series x_k (k = 1,2,3,...) representing daily values of a particular stock index if we investigate stock market in a given year, or values of vertical acceleration in earthquakes.

We define $\nu_k = x_k - x_{k-1}$ for
$k = n + j,\ n = 2,3,4, \ldots 12;\ j = 0,1,2,\ldots$

Then, $$S_j = \frac{1}{12} \sum_{n=1}^{12} x_{n+j} \qquad (4)$$

and by solving ten equations ($3 \le n \le 12$) of the form

$$\nu_k - \nu_{k-1} = a\,(x_k - S_j) + a_2\,(x_k - S_j)^2 + a_3\,(x_k - S_j)^3 + b\nu_k + \omega + \sum_{i=2}^{6} c_i \cos \frac{6.25k}{i} \qquad (5)$$

We obtain ten force parameters:
$a, a_2, a_3, b, \omega, c_2, c_3, c_4, c_5, c_6$.

The relationship (5) represents the Newton Second Law. If x_k is the coordinate of a particle with unit mass, the right side of the relationship (5) represents the conventional force acting on the particle. We calculate the correlations of force parameters and the determinant of the correlation matrix. After considering a large number of examples, we will formulate a rule that links specific values of this determinant with future declines in the stock market index or vertical acceleration.

The correlation matrix of force parameter pairs is a 9 by 9 matrix where correlations are on the diagonal and above the diagonal, while symbols below the diagonal are not relevant. The force parameters are associated with a time series of stock index values over the course of one year (the length of this time series is approximately 250).

In the first row of the matrix, the correlations are between a and a_2 , a and a_3,… a and c_6 . In the second row, the correlations are between a_2 and a_3, a_2 and b, …, a_2 and c_6 .

In the ninth row, the correlation are between c_5 *and* c_6. When we want calculate determinat of the matrix we take condition
$x = a_2x = a_3x = bx = \omega x = c_2x = c_3x = c_4x = c_5x = 1$

3. RESULTS

3.1. Artificial matrices

For a more thorough understanding of the depicted model, artificial matrices were generated, and the determinants' values were subsequently scrutinized and commented upon. The level of stochasticity is determined by the amount of correlation and the regularity of matrix elements. In the case of moderate stochasticity, it is moderate the amount of determinant, which announces a decline. Let us look at three examples (Tables 1-3).

Table 1. *An example of a matrix with a high level of stochasticity*

0.48	0.12	0.14	0.06	0.17	0.15	-0.49	-0.12	-0.34
a_2x	-0.19	-0.18	0.28	-0.14	-0.17	0.12	0.39	0.43
a_3x	x	-0.38	-0.11	-0.45	0.17	0.15	0.01	-0.06
bx	x	x	-0.21	0.37	-0.32	-0.08	-0.19	0.32
wx	x	x	x	0.14	0.09	0.12	0.17	-0.48
c_2x	x	x	x	x	-0.11	0.14	-0.16	0.18
c_3x	x	x	x	x	x	-0.13	0.18	-0.36
c_4x	x	x	x	x	x	x	-0.50	-0.39
c_5x	x	x	x	x	x	x	x	-0.49

In this case value of determinant is -3.23 and correlations are in interval from 0 to – 0.5.

Table 2. *An example of a matrix with moderate level of stochasticity*

0.58	0.62	0.64	0.66	0.77	0.55	-0.69	-0.52	-0.78
a_2x	-0.69	-0.53	-0.73	0.51	-0.57	0.72	0.69	0.63
a_3x	x	0.71	-0.79	-0.51	0.70	0.69	0.61	-0.76
bx	x	x	-0.71	0.72	-0.62	-0.58	-0.59	-0.72
wx	x	x	x	0.58	0.55	0.62	0.67	-0.48
c_2x	x	x	x	x	-0.71	0.54	-0.76	0.18
c_3x	x	x	x	x	x	-0.53	-0.64	-0.56
c_4x	x	x	x	x	x	x	-0.50	0.52
c_5x	x	x	x	x	x	x	x	-0.74

In this case value of determinant is -0.16 and correlations are in interval from 0.5 to – 0.8. The eighth and ninth columns in the determinant are roughly proportional.

Table 3. *An example of a matrix with a low level of stochasticity*

0.87	0.87	-0.89	0.86	0.87	0.85	-0.99	0.82	-0.81
a_2x	0.99	-0.85	-0.83	-0.84	-0.97	0.82	0.89	-0.88
a_3x	x	0.95	-0.89	-0.91	0.90	0.89	0.83	-0.82
bx	x	x	0.98	0.97	-0.87	-0.94	-0.88	-0.87
wx	x	x	x	0.96	-0.95	0.92	0.86	-0.85
c_2x	x	x	x	x	-0.94	0.86	0.94	-0.93
c_3x	x	x	x	x	x	-0.83	0.94	-0.93
c_4x	x	x	x	x	x	x	0.83	0.18
c_5x	x	x	x	x	x	x	x	-0.99

In this case value of determinant is -10^{-10} and correlations are in interval from 0.8-1. The first and second and eighth and ninth columns in the determinant are roughly proportional.

We can now formulate the rule: If the determinant of the correlation matrix is between 0.10 and 0.72, a downturn is likely to occur. A moderate determinant value of the correlation matrix (0.10-0.72) corresponds to a moderate level of stochasticity. In the case of a high level of stochasticity, there is no bubble inflation nor strong government intervention. In the case of a moderate level of stochasticity, a bubble inflates but there is no strong government intervention.

3.2. Earthquakes

Japanese seismologist Omori discovered the first law of earthquake physics [5-6]. It is seems to be an additional correlation between stock market crashes and earthquakes, alongside Omori's Law. Now let us examine the results obtained using the method presented here for three earthquakes: Kobe 1995 (Table 4.), Van 2011 (Table 5.) and Encino 2014 (Table 6).

Table 4. *Earthquake Kobe 1995*

time	interval	det. value	trend
1-250	from -10 045 to 11 252	0.17	
251-500	from -5415 to 12036	-0.29	up-up
501-750	from -8116 to 11194	0.04	down-down
751-1000	from -7733 to 14597	-0.02	up-down
1001-1250	from -7673 to 10151	0.27	up-down
1251-1500	from -10258 to 13567	10^{-5}	down-up
1501-1750	from -11289 to 27920	0.004	down-up
1751-2000	from -28860 to 42428	-0.02	down-up
2001-2250	from -34522 to 38691	-0.0005	down-down
2251-2550	from -19894 to 25791	-0.0005	down-up
2501-2750	from -12369 to 18402	-0.76	down-up
2751-3000	from -9148 to 11794	-0.02	down-up

Table 5. *Earthquake Van 2011*

time	interval	det. value	trend
1-200	from 0.009 to 0.01	0.0003	
201-400	from 0.012 to 0.015	0	
401-600	from 0.015to 0.018	0.04	
601-800	from 0.018 to 0.020	-0.003	
801-1000	from 0.020 to 0.021	2.9	
1001-1200	from 0.021 to 0.022	0.13	prediction
1465-1664	from 0.020 to 0.021	0.002	
1665-1864	from 0.018 to 0.022	-0.16	prediction
1865-2064	from -0.03 to 0.04	-0.01	
2065-2264	from -0.01 to -0.03	-0.006	
2265-2464	from -0.2 to -0.004	0.55	prediction
2465-2664	from -0.4 to -0.1	0.01	
2665-2864	from -0.3 to 0.1	0.02	
2865-3064	from -0.2 to 0.2	-0.01	
3065-3264	from -0.1 to 0.2	-0.18	prediction
3265-3464	from 0.05 to 0.4	-0.0002	
3465-3664	from -0.07 to 0.2	-0.008	
3665-3864	from -0.1 to 0.1	-0.0002	
3865-4064	from -0.4 to 0.1	0.17	prediction
4065-4264	from -0.2 to 0.1	0.30	prediction
4265-4464	from -0.2 to 0.1	-0.06	
4465-4664	from 0.06 to 1.2	0.0005	

Table 6. *Encino 2014.*

time	interval	det. value	trend
201–900	from -0.0003 to 0.0007	-10^{-6}	
901–1600	from 0.0008 to 0.0008	0.05	
1601–2300	from -0.0008 to 0.001	10^{-8}	
2301–3000	from -0.03 to 0.03	-0.19	prediction
3001–3700	from -0.2 to 0.09	0.83	
3701–4400	from -0.2 to 0.02	-10^{-5}	

In the second column of Table 5, the interval represents the vertical acceleration measured by seismographs at the University of Tasmania during the Kobe earthquake in 1995. The unit of time is 0.01 seconds. The note 'up-down' in the fourth column of the table indicates that the lower limit of acceleration increased while the upper limit decreased compared to the previous period. The increase in amplitude (down-up) in the sixth, seventh, and eighth rows in Table 5 is predicted by moderate determinant values (0.17, -0.29, and 0.27; first, second, and fifth rows). From the first to the eighth row in Table 6 an amplitude order of magnitude is 0.001. In the ninth row, there is a sudden increase in amplitude predicted in the sixth row. The eleventh row shows the increase predicted in the eighth row. A new increase occurs in the thirteenth row, predicted in the eleventh row. In the nineteenth row, the amplitude is 2.5 times larger than in the previous one (predicted in the fifteenth row). The largest amplitude (last row of Table 6) is predicted in the nineteenth and twentieth rows. We have the same case in Table 7, where the amplitude growth is predicted in the fourth row.

4. CONCLUSION

This part of the research highlights an additional correlation between stock market crashes and earthquakes, building on Omori's Law, the first law of earthquake physics. The results of the analysis of three earthquakes—Kobe 1995, Van 2011, and Encino 2014—demonstrate that changes in the amplitude of vertical acceleration can be predicted based on moderate determinant values. This analysis suggests that certain patterns in amplitude, such as sudden increases, may be associated with specific determinants, opening up possibilities for further research on predicting natural and market events using mathematical models.

5. REFERENCES

[1] B. Gutenberg, C.F. Richter, *Earthquake magnitude, intensity, energy and accelaration*, Seismol. Soc. Am. Bull. (46) (1956) 105 –145.

[2] F. Omori, On the after-shocks of earthquakes, J. Coll. Sci. Imp. Univ. Tokyo (7) (1894) 111–200

[3] F. Lillo, R.N. Mantegna, Power law relaxation in a complex system: Omori law after a .nancial market crash, Phys. Rev. E 68 (2003) 016119

[4] A. Helmstetter, D. Sornette. Diffusion of Earthquake Aftershock Epicenters, Omori's Law and Generalized Continuous-Time Random Walk Models. Physical Review E : Statistical, Nonlinear, and Soft Matter Physics, 2002, 66, pp.061104. ff10.1103/PhysRevE.66.061104ff. ffhal-00023353f

[5] Utsu, T., Ogata, Y., Matsu'ura, R.S.: The centenary of the Omori formula for a decay law of aftershock activity. J. Phys. Earth 43, 1–33 (1995)

[6] F. Lillo, R.N. Mantegna, Dynamics of financial market index after a crash, Manuscript, (2002) http://arXiv.org/abs/cond-mat/0209685

[6] F. Lillo, R.N. Mantegna, Power law relaxation in a complex system: Omori law after a financial market crash, Phys. Rev. E 68 (2003) 016119.

[7] Gao, J., Hu, J. (2013). *Financial Crisis, Omori's Law, and Negative Entropy Flow*. In: Greenberg, A.M., Kennedy, W.G., Bos, N.D. (eds) Social Computing, Behavioral-Cultural Modeling and Prediction. SBP 2013. Lecture Notes in Computer Science, vol 7812. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-37210-0_49

[8] ao, J.B., Cao, Y.H., Tung, W.W., Hu, J.: Multiscale Analysis of Complex Time Series — Integration of Chaos and Random Fractal Theory, and Beyond, pp. 38–40. Wiley, Hoboken (2007)

[9] Weber, P., Wang, F., Vodenska-Chitkushev, I., Havlin, S., Stanley, H.E.: Relation between volatility correlations in financial markets and Omori processes occurring on all scales. Phys. Rev. E 76, 016109 (2007)

АНАЛИЗА КОРЕЛАЦИЈА ПАРАМЕТАРА СИЛЕ ПРИДРУЖЕНИХ ВРЕМЕНСКИМ НИЗОВИМА

Сажетак: У комплексном систему, детерминизам без хаоса, детерминизам са хаосом и стохастика су испреплетени, што отежава предвиђање понашања таквог система. У овом моделу, временске серије података су повезане са временски зависним параметрима силе користећи Њутнов други закон. Корелације између ових параметара силе се рачунају, а затим се креира матрица корелација, те се рачунају вриједности детерминанти матрице корелација. Разматра се зависност између вриједности детерминанти и предвиђања понашања комплексног система. Ова врста грубог модела за предвиђање може бити корисна за проучавање земљотреса, кретања на берзама и берзанским индексима, као и за предвиђање осцилација дневних и мјесечних температура.

Кључне ријечи: Други Њутнов закон, матрица корелација, комплексни системи.

Paper received: 3 September 2024 **Paper accepted: 11 December 2024**