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CONFIGURATIONAL FORCE EXERTED ON AN INHOMOGENEITY BY A PILEUP OF SCREW DISLOCATIONS

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Abstract: Configurational forces exerted by a screw dislocation pileup on a circular inhomogeneity or a bimaterial interface are determined for a given number of dislocations, the level of remotely applied stress, and a specified degree of material inhomogeneity. The interface shear stress is also evaluated and discussed.

Keywords: bimaterial interface, configurational force, dislocation pileup, inhomogeneity, screw dislocation.

1. INTRODUCTION

Edge and screw dislocation pileups were originally studied by Eshelby, Frank, and Nabarro [1] under the assumption that the leading dislocation in a pileup is pinned at its position within an infinite homogeneous medium. Chou [2] subsequently considered a screw dislocation pileup against a rigid boundary of a semi-infinite elastic medium. Barnett and Tetelman [3], Thölén [4], and Smith [5] studied the problem of a screw dislocation pileup against a circular inhomogeneity within a matrix material, assuming that dislocations are either in a constant stress field (e.g. lattice friction stress), or the stress field induced by a remotely applied uniform loading. Barnett [6] derived an exact solution for the distribution of a pileup of screw dislocations near a bimaterial interface by using the model of continuously distributed dislocations. Toya [7] analyzed an array of continuously-distributed screw dislocations piled up against a circular rigid inclusion by the complex-variable method, including in the analysis the internal friction stress opposing the movement of dislocations. More recently, Lubarda [8] presented an analysis of edge dislocation pileups against a circular inhomogeneity and a bimaterial interface, with the reference to related other work on the topic. In this paper, we shed additional light to the study of pileups of screw dislocations against a circular inhomogeneity or a plane bimaterial interface by deriving the expressions for the corresponding configurational forces and by evaluating and discussing the interface stresses as a function of material properties.

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2. SCREW DISLOCATION PILEUP AGAINST A CIRCULAR INHOMOGENEITY

The shear stress along the x-axis outside the circular inhomogeneity of radius a due to a single screw dislocation located at $x = x_i$ is (Dundurs, [5])

$$\sigma_{yz}^{\mathrm{disl}}(x,0) = \frac{G_1 b_z}{2\pi}\left[\frac{1}{x-x_i} + C\left(\frac{1}{x-a^2/x_i} - \frac{1}{x}\right)\right],$$

$$C = \frac{G_2 - G_1}{G_2 + G_1}, \tag{1}$$

where b_z is a Burgers vector of the dislocation. The non-singular part of the shear stress at $x = x_i$ is

$$\sigma_{yz}^{\mathrm{disl}}(x_i,0) = \frac{G_1 b_z}{2\pi}\,\frac{Ca^2}{x_i(x_i^2 - a^2)}. \tag{2}$$

The shear stress along the x-axis due to a remotely applied stress $\sigma_{yz}^{\infty} = -\tau$ is

$$\sigma_{yz}^{\mathrm{appl}}(x,0) = -\tau\left(1 - C\frac{a^2}{x^2}\right). \tag{3}$$

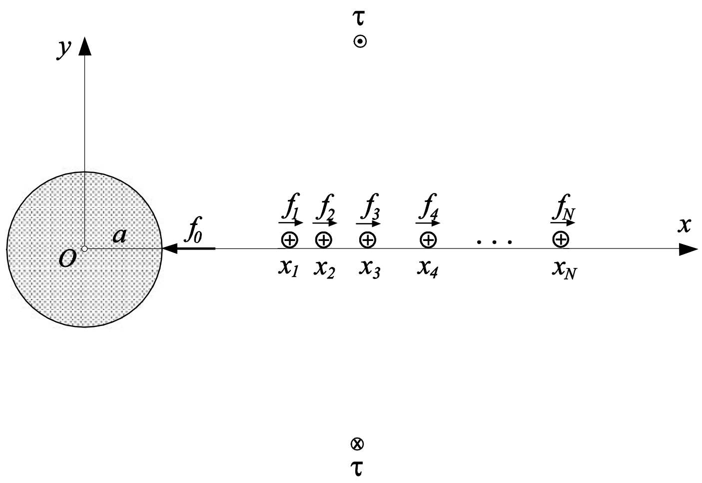

Figure 1. A pileup of N screw dislocations against a circular inhomogeneity of radius a within an infinite medium. The applied remote uniform shear stress is τ. The configurational force exerted on the inhomogeneity by the pileup of dislocations under remote stress τ is f_0. In the equilibrium configuration, the configurational force on each dislocation vanishes ($f_i = 0$), which specifies the corresponding positions of dislocations x_i ($i = 1,2,3,\ldots,N$).

The configurational force exerted by the inhomogeneity on the dislocation is

$$f_{i,i}^{\mathrm{image}} = \frac{G_1 b_z^2}{2\pi a}\,\frac{C}{\gamma_i(\gamma_i^2-1)}\,, \quad \gamma_i = x_i/a\,. \qquad (4)$$

The configurational force exerted on the dislocation by the applied stress τ is $\sigma_{yz}^{\mathrm{appl}}(x_i,0)b_z$, i.e., in view of (3),

$$f_i^{\mathrm{appl}} = -\tau b_z\left(1 - C\frac{1}{\gamma_i^2}\right). \qquad (5)$$

If there are two alike screw dislocations near a circular inhomogeneity, one at a distance $x = x_i$ and the other at a distance $x = x_j$ from the center of the inhomogeneity, there is an additional contribution to the force on the dislocation at $x = x_i$ due to its interaction with the dislocation at $x = x_j$. This is $f_{i,j} = f_{i,j}^{\infty} + f_{i,j}^{\mathrm{image}}$, where

$$f_{i,j}^{\infty} = \frac{G_1 b_z^2}{2\pi a}\,\frac{1}{\gamma_i - \gamma_j}\,, \quad f_{i,j}^{\mathrm{image}} = \frac{G_1 b_z^2}{2\pi a}\,\frac{C}{\gamma_i(\gamma_i\gamma_j - 1)}\,. \qquad (6)$$

If there are N dislocations in a pileup (Figure 1), the total force on a dislocation at x_i is specified by

$$f_i = f_i^{\mathrm{appl}} + \sum_{j=1}^{N} f_{i,j}^{\mathrm{image}} + \sum_{j\neq i}^{N} f_{i,j}^{\infty}\,. \qquad (7)$$

In the pileup equilibrium configuration, the dislocation force on each dislocation vanishes $(f_i = 0)$, which defines a system of N nonlinear algebraic equations for the equilibrium dislocation positions,

$$f_i^{\mathrm{appl}} + \sum_{j=1}^{N} f_{i,j}^{\mathrm{image}} + \sum_{j\neq i}^{N} f_{i,j}^{\infty} = 0\,. \qquad (8)$$

2.1. Configurational force on a circular inhomogeneity

The configurational force on a circular inhomogeneity from the dislocation at x_i in the pile of N dislocations is the same in magnitude but opposite in direction to the configurational force on the dislocation at x_i from the image effects of all dislocations in the pileup with respect to the inhomogeneity. Thus,

$$f_i^{\mathrm{inhom}} = \sum_{j=1}^{N} f_{i,j}^{\mathrm{image}}\,. \qquad (9)$$

The total configurational force on the inhomogeneity, directed to the left, is

$$f_0 = \sum_{i=1}^{N} f_i^{\mathrm{inhom}} = \sum_{i=1}^{N}\sum_{j=1}^{N} f_{i,j}^{\mathrm{image}}\,. \qquad (10)$$

However, from the equilibrium conditions of dislocations in the pileup (8), we have

$$\sum_{j=1}^{N} f_{i,j}^{\mathrm{image}} = -\left(f_i^{\mathrm{appl}} + \sum_{j\neq i}^{N} f_{i,j}^{\infty}\right). \qquad (11)$$

Consequently, since

$$\sum_{i=1}^{N}\sum_{j\neq i}^{N} f_{i,j}^{\infty} = 0\,, \tag{12}$$

the substitution of (11) into (10) specifies the total force on the inhomogeneity as

$$f_0 = -\sum_{i=1}^{N} f_i^{appl}\,. \tag{13}$$

In view of (5), this therefore becomes

$$f_0 = \tau b_z \left(N - C\sum_{i=1}^{N} \frac{1}{\gamma_i^2} \right). \tag{14}$$

Physically, f_0 represents the potential energy release rate associated with the increase of the distance between the inhomogeneity and the pileup, keeping all dislocations in the pileup fixed relative to each other. Figure 2a shows the equilibrium positions of $N = 5$ screw dislocations in a pileup against a rigid circular inhomogeneity vs. the applied shear stress τ (solid-line curves). The corresponding configurational force exerted by the loaded pileup on the inhomogeneity is shown in Figure 2b.

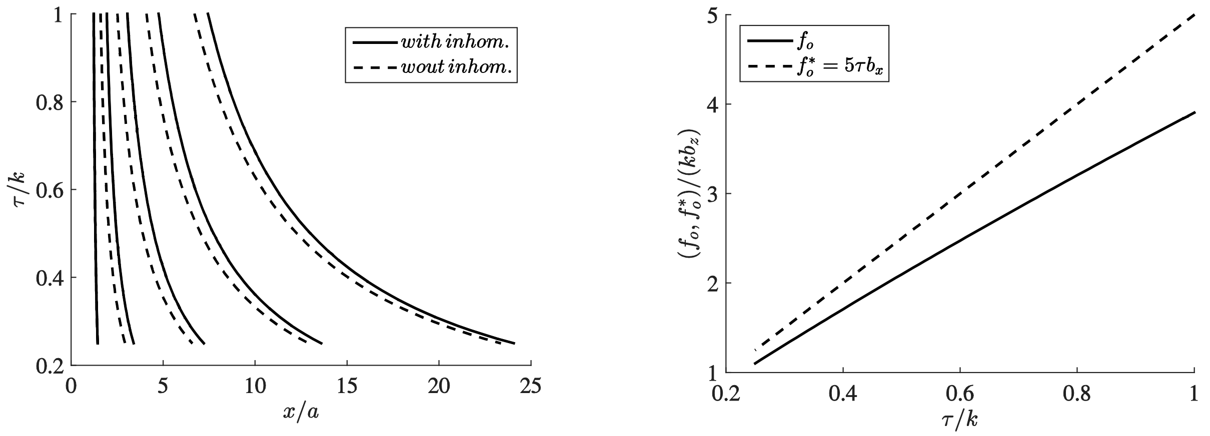

Figure 2. (a) The equilibrium positions of $N = 5$ screw dislocations in a pileup against a rigid circular inhomogeneity vs. the applied shear stress τ (solid-line curves). The dashed curves specify the equilibrium positions of dislocations in a homogeneous infinite medium, assuming that the leading dislocation is pinned at the position of the leading dislocation of the pileup against a rigid inhomogeneity. The scaling factor $k = G_1 b_z/(2\pi a)$. (b) The corresponding configurational force exerted by a pileup on the inhomogeneity.

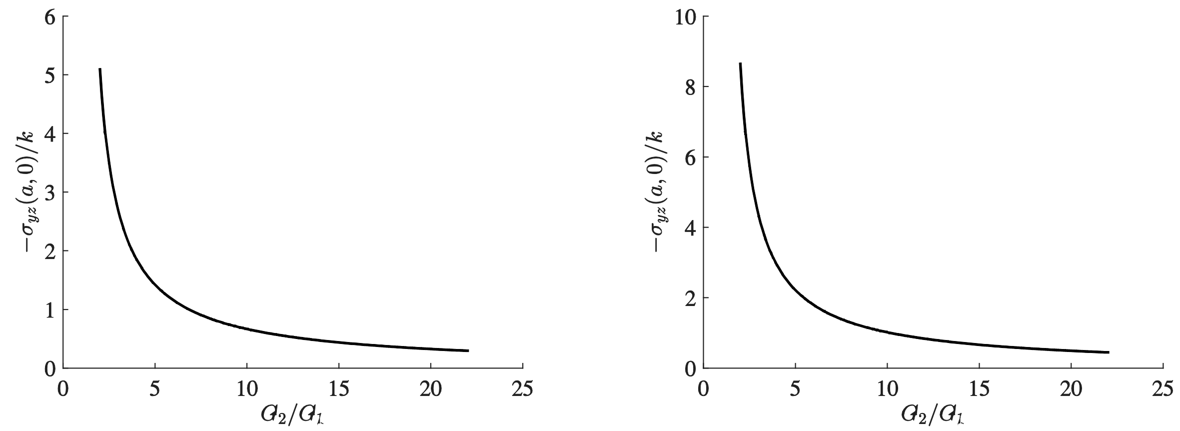

Figure 3. (a) The variation of $\sigma_{yz}(a,0)$ with G_2/G_1 in the case of $N = 5$ dislocations in a pileup under remote stress $\tau = 0.25k$. (b) The same as part (a) in the case of $N = 10$ dislocations in a pileup.

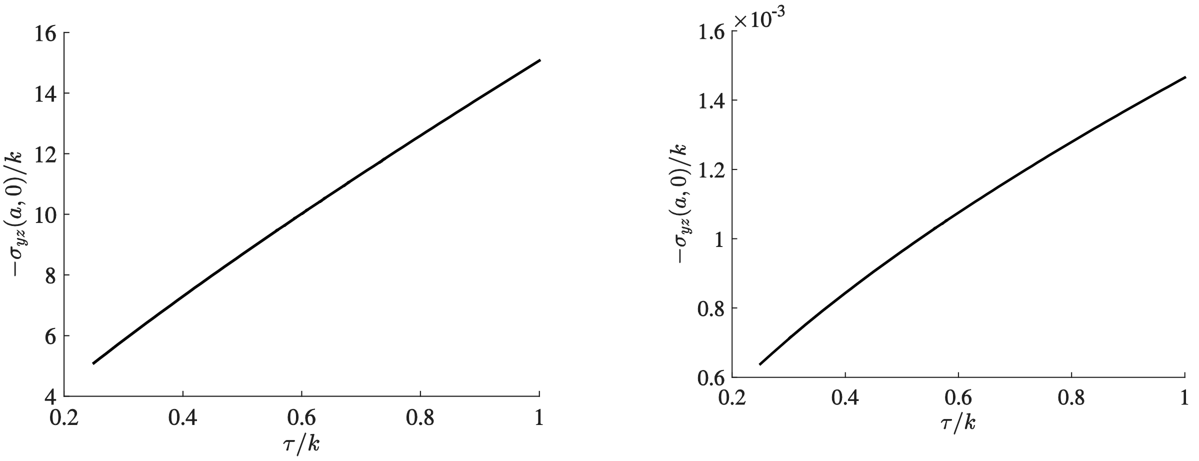

Figure 4. (a) The variation of $\sigma_{yz}(a,0)$ with τ in the case of $N = 5$ dislocations in a pileup against a circular inhomogeneity with $G_2 = 2G_1$. (b) The same as part (a) in the case of rigid inhomogeneity ($G_2 \to \infty$). The scaling factor $k = G_1 b_z/(2\pi a)$.

2.2. Shear stress concentration in front of the inhomogeneity

The shear stress along the plane of the pileup at the interface between the inhomogeneity and the surrounding matrix is

$$\sigma_{yz}(a,0) = -(1-C)\left(\tau - \frac{G_1 b_z}{2\pi a}\sum_{i=1}^{N}\frac{1}{1-\gamma_i}\right). \tag{15}$$

Figure 3 shows the variation of $\sigma_{yz}(a,0)$ with G_2/G_1 in the case of $N=5$ and $N=10$ dislocations in the pileup under remote stress $\tau = 0.25k$. Figure 4 shows the variation of $\sigma_{yz}(a,0)$ with the applied stress τ in the case of $N=5$ dislocations in the pileup against a circular inhomogeneity characterized by $G_2/G_1 = 2$ and $G_2/G_1 \to \infty$.

3. SCREW DISLOCATION PILEUP AGAINST A BONDED BIMATERIAL INTERFACE

If a screw dislocation is near a perfectly bonded bimaterial interface at a distance $x = x_i$ from it, the shear stress along the positive x-axis is (Dundurs, [9])

$$\sigma_{yz}^{\mathrm{disl}}(x,0) = \frac{G_1 b_z}{2\pi}\left(\frac{1}{x-x_i} + \frac{C}{x+x_i}\right). \tag{16}$$

A nonsingular portion of the shear stress at the location of the dislocation is

$$\sigma_{yz}^{\mathrm{disl}}(x_i,0) = \frac{G_1 b_z}{2\pi}\,\frac{C}{2x_i}. \tag{17}$$

The shear stress in the material (1) due to a remotely applied stress $\sigma_{yz}^{\infty} = -\tau$ is $\sigma_{yz}^{appl}(x,0) = -\tau$. It is assumed that a uniform shear strain $\gamma_{\mathrm{yz}} = -\gamma$ is imposed at infinity, so that the corresponding remote shear stress is $G_1\,\gamma_{\mathrm{yz}}$ for $x > 0$, and $G_2\,\gamma_{\mathrm{yz}}$ for $x < 0$.

The configurational force exerted by the inhomogeneity on the dislocation is

$$f_{i,i}^{\,image} = \frac{G_1 b_z^2}{2\pi}\,\frac{C}{2x_i}. \tag{18}$$

The configurational force on the dislocation from the externally applied remote stress τ is

$$f_i^{\,appl} = -\tau b_z. \tag{19}$$

If there are two alike dislocations near the interface, one at a distance $x = x_i$ and the other at a distance $x = x_j$ from the interface, there is an additional contribution to the force on the dislocation at $x = x_i$ due to its interaction with the dislocation at $x = x_j$. This is $f_{i,j} = f_{i,j}^{\infty} + f_{i,j}^{\,image}$, where

$$f_{i,j}^{\infty} = \frac{G_1 b_z^2}{2\pi}\,\frac{1}{x_i - x_j}, \quad f_{i,j}^{\,image} = \frac{G_1 b_z^2}{2\pi}\,\frac{C}{x_i + x_j}. \tag{20}$$

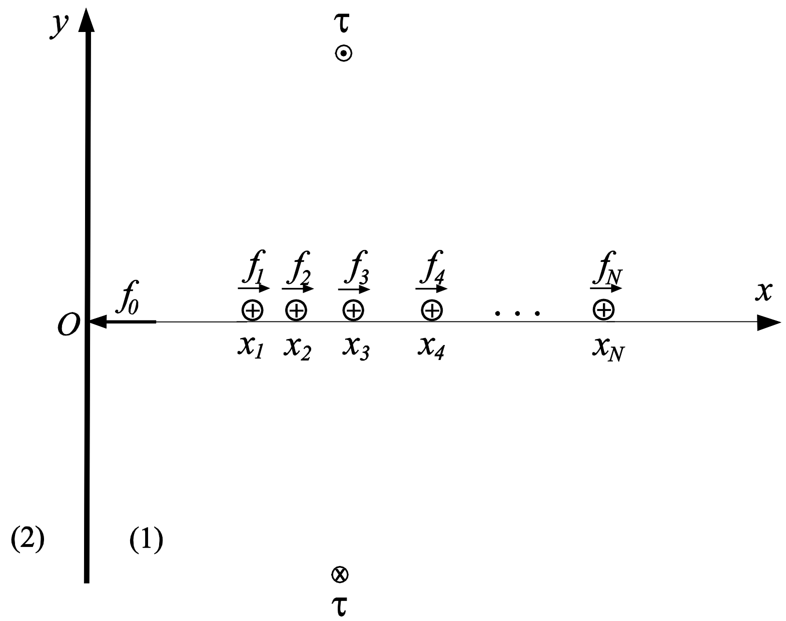

Figure 5. A pileup of N screw dislocations against a bimaterial interface between materials (1) *and* (2) *under applied remote uniform shear stress τ for $x>0$, and $(G_2/G_1)\,\tau$ for $x<0$. The configurational force exerted on the interface by a pileup is f_0. In the equilibrium configuration, the configurational force on each dislocation vanishes ($f_i=0$), which specifies the corresponding positions of dislocations x_i ($i=1,2,3,\ldots,N$).*

If there are N dislocations in a pileup (Figure 5), the total force on a dislocation at x_i is specified by the expression (7), so that the equilibrium positions of dislocations are

$$\sum_{j=1}^{N}\frac{C}{x_i+x_j}+\sum_{j\neq i}^{N}\frac{1}{x_i-x_j}=\frac{\tau}{k_1 b_z}\,,\quad k_1=\frac{G_1}{2\pi}\,. \tag{21}$$

Evidently, if the shear stress is changed from τ to $c_0\tau$ ($c_0>0$), the equilibrium positions of dislocations change from x_i to x_i/c_0. Also, by an analogous analysis to that used in section 2.1, it follows that the configurational force on the interface from the dislocation pileup is

$$f_0=N\tau b_z\,. \tag{22}$$

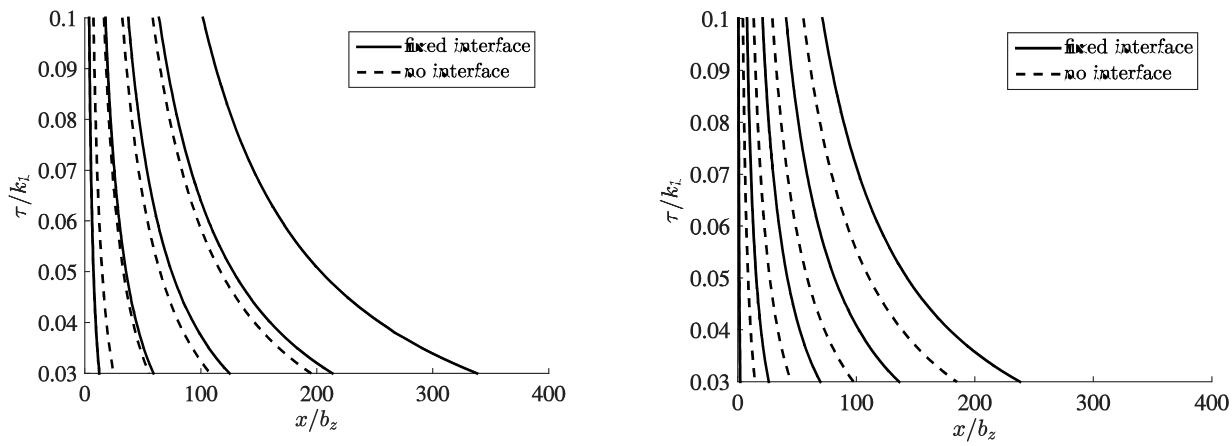

Figure 6. The equilibrium positions of $N=5$ dislocations vs. the applied shear stress τ in a screw-dislocation pileup against a rigid interface ($G_2\to\infty$). The scaling factor $k_1=G_1/(2\pi)$. (b) The equilibrium positions of $N=5$ screw dislocations vs. the applied shear stress τ in a pileup against a bimaterial interface with $G_2=2G_1$.

Figure 6 illustrates the effect of the inhomogeneity parameter Γ on the equilibrium configuration of the pileup of $N = 5$ screw dislocations. In Figure 6a, the interface is assumed to be rigid $(\Gamma = \infty)$, while in Figure 6b the material (2) has the shear modulus $G_2 = 2G_1$. In the latter case, the repulsion from the interface is milder, so that the dislocations are closer to the interface, while the extent of the pileup $(x_N - x_1)$ is smaller.

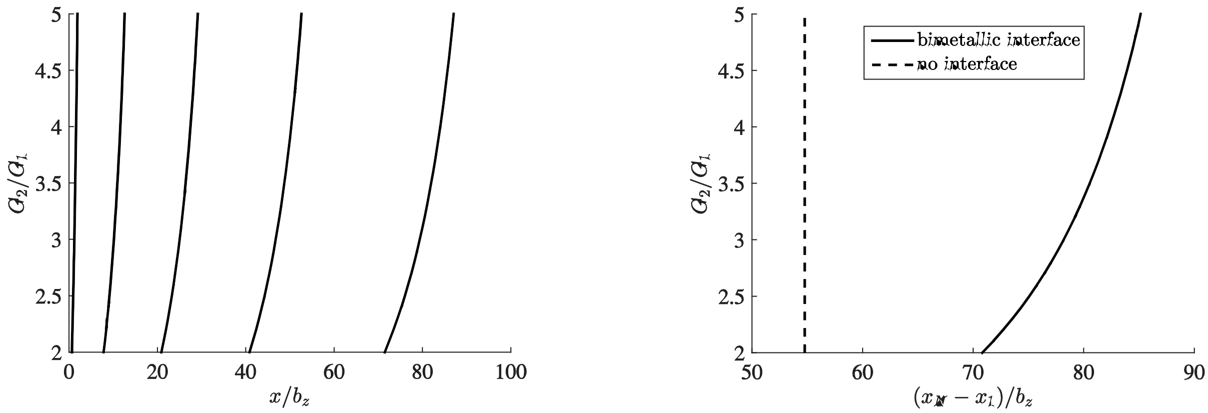

Figure 7. The equilibrium positions of $N = 5$ dislocations vs. the applied shear stress τ in a screw-dislocation pileup against a rigid interface ($G_2 \to \infty$). The scaling factor $k_1 = G_1/(2\pi)$. (b) The equilibrium positions of $N = 5$ screw dislocations vs. the applied shear stress τ in a pileup against a bimaterial interface with $G_2 = 2G_1$.

Figure 7a shows the equilibrium positions of $N = 5$ dislocations vs. the applied shear stress τ in a screw-dislocation pileup against a rigid interface. Figure 7b shows the corresponding length (extent) of the pileup. Clearly, the stiffer the interface, the stronger the repulsion exerted on the pileup and thus the larger the extent of the pileup.

3.1. Shear stress concentration at the interface

The shear stress at the intersection of the interface and the plane of the pileup is

$$\sigma_{yz}(0,0) = -\tau - (1-C)\frac{G_1 b_z}{2\pi}\sum_{i=1}^{N}\frac{1}{x_i}. \tag{23}$$

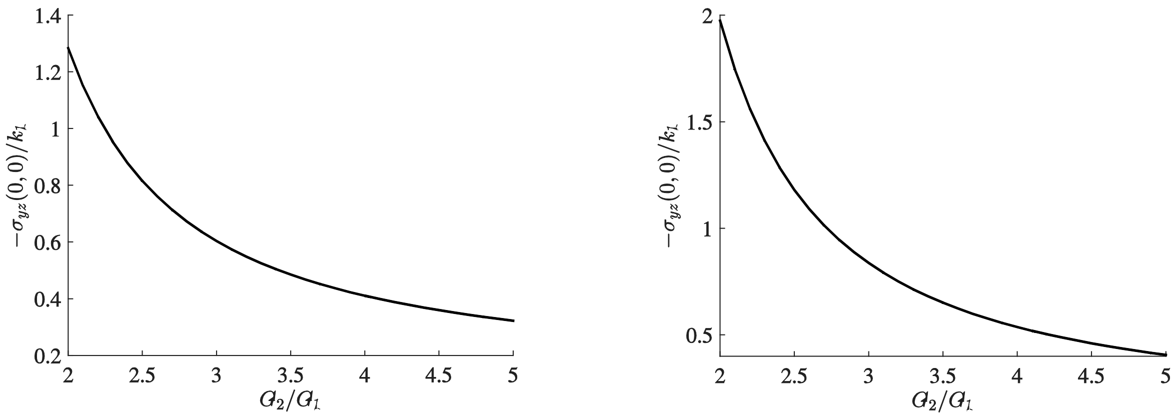

Figure 8. (a) The variation of $\sigma_{yz}(0,0)$ with G_2/G_1 in the case of $N = 5$ dislocations in a pileup under remote stress $\tau = 0.1k_1$. (b) Same as part (a) in the case of $N = 10$ dislocations in a pileup.

The variation of $\sigma_{yz}(0,0)$ with G_2/G_1 in the case of $N = 5$ and $N = 10$ dislocations is under the remote stress $\tau = 0.1k_1$ is shown in Figure 8. The stiffer the interface the smaller $\sigma_{yz}(0,0)$, because a stiff interface repels the pileup further away from it. For an analysis of interface stresses caused by dislocation walls parallel to bimaterial interface, see Lubarda [10], and Asaro and Lubarda [11].

4. CONCLUSIONS

Screw dislocation pileups against a circular inhomogeneity within an isotropic infinitely extended material, or against a perfectly bonded interface between two isotropic half-spaces of different shear moduli are considered. The equilibrium configurations of pileups under remote antiplane shear loading are determined numerically for a specified number N of dislocations, the applied level of stress, and the specified material properties. The configurational force exerted on the inhomogeneity by the pileup is equal to $N\tau b_z$, plus a term dependent on the dislocation positions and the shear moduli ratio G_2/G_1. The stiffer the inhomogeneity, the smaller the configurational force, for any given level of applied shear stress, because a stiff inhomogeneity repels the pileup further away from its center. The configurational force exerted by a pileup on a plane bimaterial interface is equal to $N\tau b_z$, independently of dislocation positions and material properties.

5. ACKNOWLEDGMENTS

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КОНФИГУРАЦИОНА СИЛА НА МАТЕРИЈАЛНУ НЕХОМОГЕНОСТ УСЉЕД НАГОМИЛАВАЊА ЗАВОЈНИХ ДИСЛОКАЦИЈА

Сажетак: У раду је одређена конфигурациона сила која дјелује на цилиндричну материјалну нехомогеност или на међуграничну површ биматеријала усљед нагомилавања завојних дислокација под дејством спољашњег оптерећења. Концентрација напона између нехомогености и околног материјала је израчуната и анализирана.

Кључне ријечи: биматеријал, конфигурациона сила, нагомилавање дислокација, нехомогеност, завојне дислокације.