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KRATKO ILI PRETHODNO SAOPŠTENJE / SHORT OR PRELIMINARY REPORT

EXTENSION OF THE MINIMUM COST FLOW PROBLEM (MCFP OR CNF) BY CONSIDERATION OF TIME AND QUANTITY OF LOADS WITH MAXIMUM TIME

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Abstract: *This paper extends the general problem of minimizing the total cost of transport on the road network (CNF) by considering the total time, maximum time and total amount of cargo with the longest time. In the literature available to us, models with timing and amount of cargo in the case of a standard transport task were exposed. Optimization is possible by combining 5 criteria, 2 linear and 3 nonlinear ones over the same set of linear constraints. Multicriteria optimization determines Pareto-optimal solutions. Interactive analyst-software algorithms for solving the selected models were defined. The solution of hypothetical problems was illustrated. Closed model with 5 two-way asymmetric communications using software for CNF and it is possible to use software for LP. Four one-criteria problems were solved: total costs, overall transport performance from a time standpoint, transport time (problem of the second type by time) total transport time (problem of the third type by time) and one bi-criteria problem related to the simultaneous minimization of the maximum duration of transport and total costs.*

Keywords: *cost flow problem; total time; Pareto-optimal solutions; algorithms; software.*

JEL classification: *C61*

INTRODUCTORY REMARKS

Network models are well-known examples of problems in practice and are featured in many textbooks and books (for example, (Pašagić, 2003), (Nikolić, 2007), (Pašagić & Kurtanović, 2017) in our country). They have been taught by basic studies of economics and technical colleges in the world as well as in our country.

The **minimum-cost flow problem** (**MCFP**) is an optimization and decision problem to find the cheapest possible way of sending a certain amount of flow through a flow network. A typical application of this problem involves finding the best deliv-

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ery route from a factory to a warehouse where the road network has some capacity and cost associated. The minimum cost flow problem is one of the most fundamental among all flow and circulation problems because most other such problems can be cast as a minimum cost flow problem and also that it can be solved efficiently using the network simplex algorithm (Wikipedia,19.03. 2021).

Minimum Cost flow problem is a way of minimizing the cost required to deliver maximum amount of flow possible in the network. It can be said as an extension of maximum flow problem with an added constraint on cost(per unit flow) of flow for each edge. One other difference in min-cost flow from a normal max flow is that, here, the source and sink have a strict bound on the limits on the flow they can produce or take in respectively, $B(s) > 0$, $B(t) < 0$. Intermediate nodes have no bounds or can be represented as $B(x) > 0$ (Kumar, 2005).

Particularly popular WinQSB software (Long Chang & Desal, 2002) can be downloaded for free from multiple addresses and was included in education at a number of the higher education institutions in the world (especially for Spanish speaking, detailed text instructions such as (Celis) can be found, as well as suitable verbal views on YouTube). It covers the following issues: (1) Network Flow, (2) Transportation Problem, (3) Assignment Problem, (4) Shortest Path Problem, (5) Maximal Flow Problem, (6) Minimal Spanning Tree, (7) Traveling Salesmen Problem. (Nikolić, 2007), (Pašagić & Kurtanović, 2017), (Kurtanović, Destović, & Kulenović, 2019).

This paper shows that optimization is possible by combining 5 minimization type criteria, 2 linear and 3 nonlinear ones, over the same set of linear constraints. For problems by time, we use the two-phase transport classification from (Vukadinović & Cvejić, 1995): (1) total cost, (2) overall transport performance from a time standpoint (problem of type 1 per time), (4) maximum transport time (problem of type 2 per time, (Barsov, 1959)), (3) total transport time (problem of type 3 per time, (Nikolić, 2007)), (5) total cargo with maximum time (HAMER, 1969). Optimizing multiple criteria determines Pareto-optimal solutions (Matthias, 2000)**.**

Defined interactive analyst-software algorithms are displayed on selected hypothetical examples using CNF software from WinQSB.

MATHEMATICAL FORMULATION OF THE PROBLEM

It is appropriate to formulate a general mathematical model in the case of the homogeneous cargo transport closed problem in the following form:

$$\min C(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

$$\min T_{ET}(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{ij} \tag{2}$$

$$\min T(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} (t_{ij} \mid x_{ij} > 0) \tag{3}$$

$$\min t(x) = \min \left\{ \max (t_{ij} \mid x_{ij} > 0) \right\} \tag{4}$$

$$\min q(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_{ij} \mid t_{ij} = t^*) \tag{5}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} x_{ji} = p_i, \; i = 1, \dots, n \tag{6}$$

$$\sum_{i=1}^{n} p_i = 0 \qquad (7)$$

$$0 \le x_{ij} \le d_{ij}, \; i = 1,...,n; \; j = 1,...,n \qquad (8)$$

where: n = number of network nodes, (i-j) and/or (j-i) = network or communication ports (i≠j), x_{ij} = unknown load volume [tonnes, t] on (i-j), d_{ij} = upper capacity limit for (i-j),c_{ij} = unit costs [monetary units per 1t, m.u./t] on (i-j), t_{ij} = transport time [minutes] for x_{ij}[t], p_i = characteristic of i-type node [t]: (p_i> 0 for the supply, p_i< 0 for demand, p_i = 0 for transit), (6) the condition that the own supply with the eventual receipt of cargo from other nodes be forwarded to other nodes (if p_i> 0), or that its own demand can be satisfied and possible over load forwarded to the other nodes (if p_i< 0), that is, the transits received load forwarded further (if p_i = 0), (7) equilibrium of total supply and total demand, and (8) conditions that non-negative variables x_{ij} do not exceed d_{ij}. Communications can be: (a) one-way, only (i-j) or (j-i), two-way symmetrical, c_{ij} = c_{ji} and t_{ij} = t_{ji}, two-way asymmetric c_{ij}≠ c_{ji} or t_{ij}≠ t_{ji}

ALGORITHMS AND ILLUSTRATIVE EXAMPLES

Example 1. Closed model, 5 two-way asymmetric communications

$n = 7, \; [p] = [50, 70, 0, 0, 0, -30, -90]$

(1) $\min C(x) = \sum_{(i,j)} c_{ij} x_{ij}$

Matrix Form, The Unit Costs c_{ij}[m.u.]

From \ To	S1	S2	T1	T2	T3	D1	D2	Supply
S1		1	3	2				50
S2	2			4	7			70
T1				2	6	5		0
T2			3		1	7	9	0
T3			8	2		8	6	0
D1							1	-30
D2						2		-90
Demand	0	0	0	0	0	0	0	

Matrix Form, The Variables x_{ij}[t]

$$
\begin{array}{c|ccccccc}
x_{ij} & S_1 & S_2 & T_1 & T_2 & T_3 & D_1 & D_2 \\
\hline
S_1 & \backslash & x_{12} & x_{13} & x_{14} & - & - & - \\
S_2 & x_{21} & \backslash & - & x_{24} & x_{25} & - & - \\
T_1 & - & - & \backslash & x_{34} & x_{35} & x_{36} & - \\
T_2 & - & - & x_{43} & \backslash & x_{45} & x_{46} & x_{47} \\
T_3 & - & - & x_{53} & x_{54} & \backslash & x_{56} & x_{57} \\
D_1 & - & - & - & - & - & \backslash & x_{57} \\
D_2 & - & - & - & - & - & x_{76} & \backslash
\end{array}
$$

min $C^{(1)} = C^{(1)*}$ = 1,190 (m.u.)

Graphic Solutions

Alternative solutions

$x_{ij}^{(1.1)}*,\ x_{ij}^{(1.2)}*$

Consequences, values of other criteria

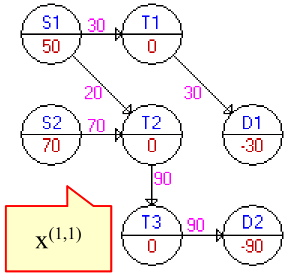
$T_{TE}^{(1.1)} = 43.380$ [tonnes-min]

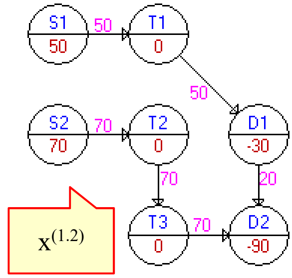
$T_{TE}^{(1.2)} = 48.180$ [tonnes min]

$T^{(1.1)} = 894$ [min.]

$T^{(1.2)} = 912$[min.], $t^{(1.1)} = t_{36} = 300$ [min.] $= t^{(1.2)}$

$q^{(1.1)} = x_{36} = 30$ [t] $= q^{(1.2)} = 30$ [t]





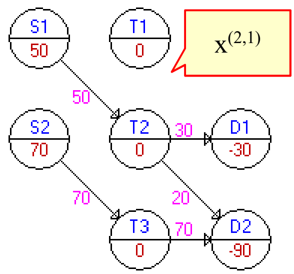
Marginal alternatives. All solutions are determined by their linear combination with with $\alpha \in [0,1]$

$x^{(1)} = \alpha x^{(1.1)} + (1-\alpha) x^{(1.2)}$

(2) $\min T_{ET}(x) = \sum_{(i,j)} t_{ij} x_{ij}$

The Times t_{ij} (minutes)

From \ To	S1	S2	T1	T2	T3	D1	D2	Supply
S1		60	162	108				50
S2	66			90	60			70
T1				120	210	300		0
T2			150		114	66	90	0
T3			240	144		132	120	0
D1							126	-30
D2						150		-90
Demand	0	0	0	0	0	0	0	



$\min T_{ET}^{(2)} = T_{ET}^{(2)}* == 21,780$ [t-min]

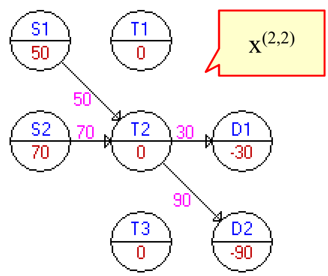
Alternative solutions

$C^{(2.1)} = 1,400$ [m.u.], $C^{(2.2)} = 1,610$ [m.u.]

$T^{(2.1)} = 444$ [min], $T^{(2.2)} = 324$ [min]

$t^{(2.1)} = t_{37} = 120$ [min], $t^{(2.2)} = t_{14} = 108$ [min]

$q^{(2.1)} = x_{37} = 70$ [t], $q^{(2.2)} = x_{14} = 50$ [t]



(3) $\min T(x) = \sum_{(i,j)} (t_{ij} \mid x_{ij} > 0)$, [12]

Table 1. Iteration 1 Selection of the starting solution $x^{(3.1)}$ by the diagonal method

			1	2	3	4	5	6	7		$\sum t_{ij}$	max t_{ij}	$\sum c_{ij}x_{ij}$
			S_1	S_2	T_1	T_2	T_3	D_1	D_2	p	$x_{ij} > 0$	$x_{ij} > 0$	
1	S_1	t_{1j}	M	60	162	108	M	M	M		60	60	50
		x_{1j}		[50]						50			
2	S_2	t_{2j}	66	M	M	90	60	M	M		150	90	690
		x_{2j}				[50]	[70]			70			
3	T_1	t_{3j}	M	M	M	120	210	300	M		0	0	0
		x_{3j}											
4	T_2	t_{4j}	M	M	150	M	114	66	90		114	114	50
		x_{4j}					[50]–δ	+δ					
5	T_3	t_{5j}	M	M	240	144	M	132	120		252	132	780
		x_{5j}						[30]–δ	[90]				
6	D_1	t_{5j}	M	M	M	M	M	M	126		0	0	0
		x_{5j}											
7	D_2	t_{5j}	M	M	M	M	M	150	M		0	0	0
		x_{5j}											
		-p						30	90		576 $T^{(3.1)}$	132 $t^{(3.1)}$	1.570 $C^{(3.1)}$

Source: authors

<u>*Iteration 2*</u> Optimization testing and improvement of non-optimal solution.

- *Definition 1.* The solution is optimal if there are no non-base variables with a negative increment in the total time.
- *Definition 2.* The total time increment is equal to the difference between the time of the variable that becomes base and the time of the variable that becomes non-base.

$$
\begin{array}{l|l|l}
x_{46}^{(3.1)} = 0, & x_{46}^{(3.2)} = \delta > 0 & \Rightarrow x_{46}^{(3.2)} = \delta = 30 \\
x_{45}^{(3.1)} = 50, & x_{45}^{(3.2)} = x_{45}^{(3.1)} - \delta = 50 - \delta > 0 \Rightarrow \delta \le 50 & x_{45}^{(3.2)} = 20 \\
x_{56}^{(3.1)} = 30 & x_{56}^{(3.2)} = x_{56}^{(3.1)} - \delta = 30 - \delta > 0 \Rightarrow \delta \le 30 & x_{56}^{(3.2)} = 0
\end{array}
$$

$\Rightarrow \Delta T^{(3.2)} = t_{45} - t_{56} = 66 - 132 = -66$
$T^{(3.2)} = T^{(3.1)} + \Delta T^{(3.2)} = 576 + (-66) = 510$

The solution $x^{(3.1)}$ is not optimal due to $\Delta T^{(3.2)} = -66 < 0$

- New base $x_{46}^{(3.2)} = 30$ determines non-base $x_{56}^{(3.2)} = 0$ and better $x^{(3.2)}$ with $T^{(3.2)} < T^{(3.1)}$.

The unit cost increment is determined by c_{ij} for all x_{ij} as modified.

$\Delta C^{3.2)} = -c_{45} + c_{46} - c_{56} = -1 + 7 - 8 = -2$
$C^{(3.2)} = C^{3.1)} + \Delta C^{(3.2)} \cdot \delta = 1.570 + (-2) \cdot 30 = 1.510$

Table 2. $x^{(3.2)}$ Lower total costs incurred $C^{(3.2)} < C^{(3.1)}$

			1	2	3	4	5	6	7	p	$\sum t_{ij}$ $x_{ij} > 0$	max t_{ij}	$\sum c_{ij}x_{ij}$
			S_1	S_2	T_1	T_2	T_3	D_1	D_2				
1	S_1	t_{1j}	M	60	162	108	M	M	M		60	60	50
		x_{1j}		[50]						50			
2	S_2	t_{2j}	66	M	M	90	60	M	M		150	0	690
		x_{2j}				[50]	[70]			70			
3	T_1	t_{3j}	M	M	M	120	210	300	M		0	0	0
		x_{3j}											
4	T_2	t_{4j}	M	M	150	M	114	66	90		180	115	230
		x_{4j}					[20]	[30]					
5	T_3	t_{5j}	M	M	240	144	M	132	120		120	129	540
		x_{5j}							[90]				
6	D_1	t_{5j}	M	M	M	M	M	M	126		0	0	0
		x_{5j}											
7	D_2	t_{5j}	M	M	M	M	M	150	M		0	0	0
		x_{5j}											
		p						30	90		510	132	1.510

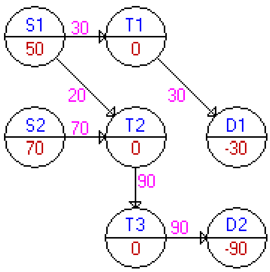
Source: authors

This is followed by repeating the process of testing the optimality of the solution and improving it if it is not optimal. The process is interrupted when there are no negative increments in total time.

(1) (4) $\min\left\{C=\sum_{(i,j)} c_{ij}x_{ij},\ \ t=\max\ (t_{ij}\,\middle|\,x_{ij}>0)\right\}$

Iteration 1. Step 1. min

C



$x^{(4.1)} = x^{C*}$, $C^{(4.1)} = C*$

Table 3. Times t_{ij} (min). Marked fields (i,j) with $x_{ij} > 0$

		1	2	3	4	5	6	7	max t_{ij}	$\sum t_{ij}$
	t_{ij}	S_1	S_2	T_1	T_2	T_3	D_1	D_2	$x_{ij} > 0$	$x_{ij} > 0$
1	S_1		60	162	108				162	270
2	S_2	66			90	60			90	90
3	T_1				120	210	300		300	300
4	T_2			150		114	66	90	114	114
5	T_3			240	144		132	120	120	120
6	D_1							126	-	-
7	D_2						150		-	-
									300	894

Source: authors

Step 2 Estimate of the longest time

$t^{(4,1)} = \max\{162,108,90,\underline{300},114,120\} = 300$[min]$= t_{36}$

Consequences on other criteria: $T^{(4,1)}$= 894 [min], $q^{(4,1)}= x_{36}$= 30[t]

The first Pareto solution was obtained $\{x^{(4,1)}, C^{(4.1)}, t^{(4.1)}, q^{(4.1)}\}$

Step 3 Interdiction of communication (i,j) with $t_{ij} \geq t^{(4.1)} = 300$ and repeating the process of k=1.

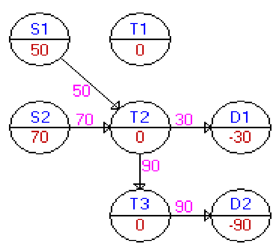
Unit costs adjustment:

$c_{ij}^{(4.2)} = \propto$ za $t_{ij} \geq t^{(4.1)} = 300$, only communication (3,6) with $t^{(4.1)} = 300$

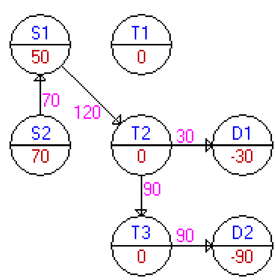
$c_{ij}^{(4.2)} = c_{ij}$ for $t_{ij} < t^{(4.1)} = 50$

Iteration 2 Step 1 min $C^{(4.2)}$

From \ To	S1	S2	T1	T2	T3	D1	D2	Supply
S1		1	3	2				50
S2	2			4	7			70
T1				2	6			0
T2			3		1	7	9	0
T3			8	2		8	6	0
D1							1	-30
D2						2		-90
Demand	0	0	0	0	0	0	0	



Step 2 Alternative solutions $x^{(4.2.1)}$ and $x^{(4.2.2)}$



- The continuation of the solution process indicates that the problem has 6 Pareto-optimal solutions.

- Basic Pareto solution property: Better value for one criterion requires worse value for at least one of the remaining criteria.

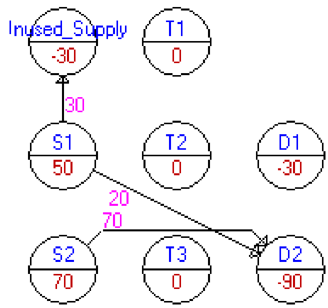
Table 4. Consequence of pareto optimal solutions

Iter.	Solution	C[m.u.]	T [min]	t [min]	q [t]	Properties
1	$x^{(4.1.1)}$	1,190	894	$t_{36} = 300$	$x_{36} = 30$	min C = 190 ⇒ max t = 300
	$x^{(4.1.2)}$		912			
2	$x^{(4.2.1)}$	1,220	460	$t_{47} = 120$	$x_{47} = 90$	$C^{(4.2)} > C^{(4.1)} \Rightarrow t^{(4.2)} < t^{(4.1)}$
	$x^{(4.2.2)}$		474			
3	$x^{(4.3.1)}$	1,400	330	$t_{14} = 108$	$x_{14} = 120$	$C^{(4.3)} > C^{(4.2)} \Rightarrow t^{(4.4)} < t^{(4.2)}$
	$x^{(4.3.2)}$		354		$x_{14} = 50$	
4	$x^{(4.4)}$	1,550	306	$t_{24} = t_{47} = 90$	$x_{24} + x_{47}$ 120+90	min t = 90 ⇒ max C = 1,550

Source: authors

Iteration 5. $c_{ij}^{(4.5)} = \propto$ for $t_{ij} \geq t^{(4)} = 90$

From \ To	S1	S2	T1	T2	T3	D1	D2	Supply
S1		1						50
S2	2				7			70
T1								0
T2							9	0
T3								0
D1								-30
D2								-90
Demand	0	0	0	0	0	0	0	



Inadmissible solution is determined $x^{(4.5)}$ that uses prohibited solutions
$S_1 \rightarrow D_2$
$S_2 \rightarrow D_2$

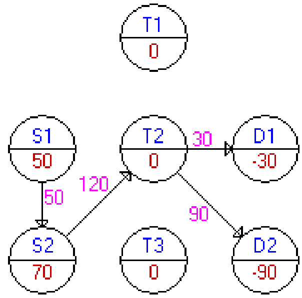
The solution process has been completed. It is noted that the software does not use prohibited communications up to D_1 and thus does not download 30[t] from the supply 50[t] at S_1.

(5) $\min q(x) = \sum_{(i,j)} (x_{ij} \mid t_{ij} = t^* = t^{(4.4)} = 90)$

Modification of the model in order to divert transport as much as possible to communications with times less than t*:

$c_{ij}^{(5)} = 1$ for $t_{ij} = t^*$, $c_{ij}^{(5)} = 0$ za $t_{ij} < t^*$, $c_{ij}^{(5)} = \propto$ for $t_{ij} > t^*$
It is determined $x^{(4)}$ solution $t^* = t^{(4)} = t_{24} = t_{47} = 90$[min]
$q^* = q^{(4)} = x_{24} + x_{47} = 120+90 = 210$[t]

From \ To	S1	S2	T1	T2	T3	D1	D2	Supply
S1		0						50
S2	0			1	0			70
T1								0
T2						0	1	0
T3								0
D1								-30
D2								-90
Demand	0	0	0	0	0	0	0	



Remark 1 Communication restrictions or other additional conditions may impair the criteria values shown. Illustrations for alternative solutions with $C^{(1)*}$ = 1,190 [m.u.].

a) It is optimal to transport $x_{13} \in [30,50]$ from Od S_1 to T_1. If the capacity of this communication is limited to 10 [t] it is obtained then $x_{13} = 10$ [t] and C = 1,210 [m.u.]>$C^{(1)*}$.
b) Communication $T_2 \rightarrow D_1$, is not used because $x_{36} = 0$. Condition $x_{36} \geq 25$ [t] sets $x_{36} = 25$ [t] and increases $C^{(1)*}$ to 1.215 [m.u.].

Remark 2 It is useful to consider the Range of Optimality and the Range of Feasibility (in the environment of the optimal solution), as well as Perform Parametric Analysis (considering the permissible values of c_{ij} unit cost or the characteristics p_i for the nodes in the model).

CONCLUSION

This paper outlined five criteria for the general problem of network transport and presented CNF solution algorithms from WinQSB software by considering a hypothetical example of a closed problem. The exhibited models need to be expanded to cover at least two types of cargo to more effectively describe practical problems. It can also be justified that, in practice, more important are the problems with vehicles, which are more complex to model and solve using Linear and Integer Programming. Moreover, if further consideration is given to vehicle types of different capacities, which is also more prevalent in practice.

REFERENCES

Barsov, A. (1959). *Čto takoe linejnoe programmirovanie?* . Moskva: Fizmatik.

Celis, H. M. (2003). MANUAL DE USO DEL WINQSB.

HAMER, P. (1969). Time-Minimizing Transportation Prolems. *Naval Research Logistics Quarterly*, 345-347.

Vinay Kumar. (2005). Minimum Cost Maximum Flow. Dostupno na: https://www.hackerearth.com/practice/algorithms/graphs/minimum-cost-maximum-flow/tutorial/ [24.03.2021.].

Kurtanović, O. (1998). *Matematske metode.* Bihać: Demfas Tuzla.

Kurtanović, O., & Destović, F. (2019). Optimisation of Project Duration and Costs Using Over-Working of Executors and Equipment When There are Limitations for Their Maximum Units. In: Karabegović I. (eds) New Technologies, Development and Application II. NT

20. *New Technologies, Development and Application II* (str. 332-341). Springer.

Kurtanović, O., Destović, F., & Kulenović, F. (2019). *Mathematical methods in economics using software.* Bihać: University of Bihać.

Long Chang, Y., & Desal, K. (2002). *WinQSB, Version 2.0.* Wiley.

Matthias, E. (2000). Multicriteria Optimization.

Nikolić, I. (2007). Total time minimizing transportation problem. *Yugoslav Journal of Operations Research*, 125-133.

Nikolić, I. (2007). Total time minimizing transportation problem. *Yugoslav Journal of Operations Research*, 125-133;.

Pašagić, H. (2003). *Mathematical models in transport, , Zagreb. 2003.* Zagreb: University of Zagreb, Faculty of Transport and Traffic Sciences.

Pašagić, H., & Kurtanović, O. (2017). *Mathematical methods in economics* . Bihać: University of Bihać.

Vukadinović, & Cvejić. (1995). *Mathematical programming*

Wikipedia (2021). Minimum-cost flow problem. Dostupno na: https://en.wikipedia.org/wiki/Minimum-cost_flow_problem [24.03.2021.].