

Stabilizing Ferroresonance Oscillations in Voltage Transformers Using Limiter Circuit

Hamid Radmanesh and Seyed Hamid Fathi

Abstract—This paper employs the multiple scales method and chaos theory for analyzing chaotic behavior of the voltage transformer (VT) with linear core loss model. It is shown that ferroresonance phenomenon in VTs can be classified as chaotic dynamics, including a sequence of bifurcations such as period doubling bifurcation (PDB), saddle node bifurcation (SNB), Hopf Bifurcation (HB) and chaos. Bifurcation diagrams and phase plane diagrams are drawn using a continuation method for linear core loss model and Lyapunov exponents are obtained using the multiple scales method. At first an overview of the subject in the literature is provided. Then, ferroresonance phenomenon is introduced and its various types in a VT are simulated. Finally the effects of ferroresonance suppression circuit on stabilizing these oscillations are studied. The proposed approach is implemented using MATLAB, and simulation results are presented. The results show connecting the ferroresonance suppression circuit to the system configuration, causes great controlling effect on ferroresonance overvoltage.

Index Terms—Ferroresonance oscillation, stabilizing, chaos control, voltage transformer, new ferroresonance suppression circuit.

I. INTRODUCTION

FERRORESONANCE is typically initiated by saturable magnetizing inductance of a transformer and capacitance of the distribution cable or transmission line connected to the transformer. In most practical situations, ferroresonance results in dominant currents, but in some operating “mode”, may cause significant high value distorted winding voltage waveform, which is typically referred to as ferroresonance. For example, when circuit breaker between the main and reserve busbars is opened, ferroresonance can occur in voltage transformer connected to the reserve busbar. Ferroresonance generates overvoltages, overcurrents and finally chaos with different frequencies in power system. Magnetic core of transformers is nonlinear, resulting in variable inductance,

while the line capacitance and grading capacitance of the circuit breaker are constant [1]. Therefore, unlike the ordinary resonance, Ferroresonance has multiple fixed points, which lose and regain their stability due to changes in the system parameters. This behavior depends, not only on the frequency, but also on the amplitude of the source voltage, initial conditions and core losses [2]. Although methods such as harmonics balance can be used for analyzing nonlinear differential equations, but solving these equations leads to a set of complex algebraic equations [3]. Thus, it is often preferred to use other methods to solve nonlinear dynamic equations; one of which is the bifurcation theory [4], [5]. Bifurcation theory enables us to describe and analyze qualitative properties of solutions (fixed points) when system parameters change. Evaluation of route to chaos in VT, considering linear core loss, circuit breaker model complexity and damping effects of the system has been carried out in [6], [7]. Early interest to the problem returns to 1907, when it was shown that the use of series capacitors for voltage regulation could cause ferroresonance in distribution systems [8]. Ferroresonance behavior of a 275 kV potential transformer, fed from a sinusoidal supply via circuit breaker grading capacitance, has been studied in [9]. The potential transformer ferroresonance from an energy transfer point of view has been presented in [10]. A systematic method for suppressing ferroresonance at neutral-grounded substations has been studied in [11]. A sensitivity study on power transformer ferroresonance in a 400 kV double circuit line has been performed in [12]. The impact of the transformer core hysteresis on the stability domain of ferroresonance modes has been studied in [13]. A new modeling of transformers enabling more accurate simulation of slow transients than the existing models in Simulink/MATLAB is presented in [14]. Controlling ferroresonance oscillations in potential transformer, considering nonlinear core losses and the circuit breaker shunt resistance effect, has been investigated in [15]. Impacts of hysteresis and magnetic couplings on the stability domain of ferroresonance in asymmetric three-phase three-leg transformers have been investigated in [17]. In this paper, a newly developed and accurate time-domain transformer model, capable of simulating dynamic and transient operating conditions is implemented. Using multiple scales method, Lyapunov exponents are obtained and the attraction behavior in route to chaos is analyzed by chaos theory. However, the

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effect of scaling method and suppression circuits on ferroresonance oscillations in VT has not been studied yet. In this paper, it is shown that the system is greatly affected by ferroresonance suppression circuit. The presence of the suggested limiter results in clamping of ferroresonance overvoltage. Using this method results in improving the voltage waveform and, consequently, protecting insulations, fuses and switchgears against high voltages. In this study, MATLAB program is employed to simulate the system and plot the related phase plane and bifurcation diagrams. The results of case study indicate that bifurcation occurs and the system states lead to chaos in the absence of ferroresonance limiter. The presence of the proposed suppression circuit, however, causes to clamp the ferroresonance oscillations and successfully decreases the chaotic region.

II. MODELING OF SYSTEM INCLUDING VOLTAGE TRANSFORMER

The Fig. 1 shows single line circuit diagram of the power system at a 275 kV substation. VT is isolated from sections of bus bars by disconnectors. Ferroresonance conditions may occur upon closure of disconnector DS_1 with circuit breaker (CB) and DS_2 open, leading to a system fault and failure in the VT's primary winding [15] and [18].

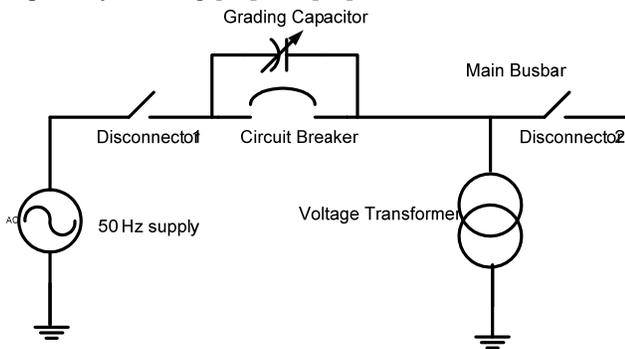


Fig. 1. Single line diagram of the power system including VT.

Fig. 2 shows the basic ferroresonance equivalent circuit used in this analysis. Resistor R represents the transformer core losses. In [15] and [18] accurate model for magnetization curve of the core, considering hysteresis, has been introduced, but in the present paper the nonlinear transformer magnetization curve is modeled by a single valued seventh order polynomial [15] and [18].

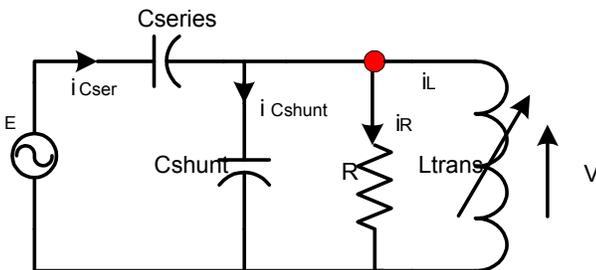


Fig. 2. Reduced equivalent ferroresonance circuit of the power system.

In Fig. 2, E is the power system rms phase voltage, C_{series} is the total grading capacitance of circuit breaker and C_{shunt} is the equivalent line-to-earth and line-to-line capacitance of the arrangement. The resistor R represents the transformer core losses. In the peak current range, the flux-current characteristic becomes highly nonlinear. Here, λ - I characteristic of the voltage transformer is modeled, as in [3], by the polynomial

$$i = a\lambda + b\lambda^7 \tag{1}$$

where, $a=3.14$, $b=0.41$ and $\lambda=N\phi$ (N is the coil turns of the transformer). The polynomial order of seven and the value of coefficients in equation (1) are obtained by dick and Watson [16], for the best fit of the saturation region to the true magnetization characteristic. It was found that for a fairly accurate representation of the saturation characteristics of a VT core, the exponent q may acquire a value of 7 [16]. Fig. 3 shows simulation of this iron core characteristic ($\phi - i$) for $q=7$.

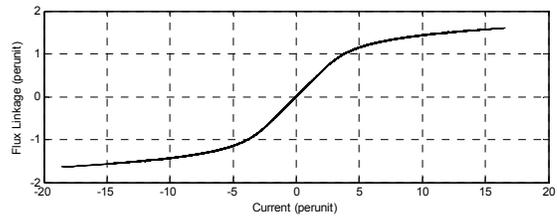


Fig. 3. Flux-current characteristic of the transformer core.

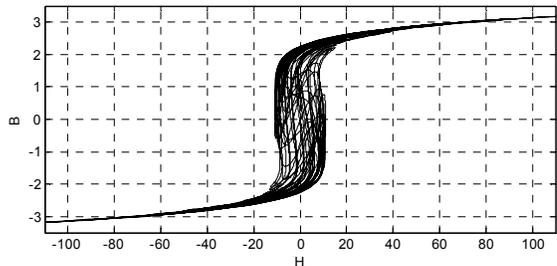


Fig. 4. Hysteresis loop of the transformer core.

The basic ferroresonance circuit as shown in Fig. 2 is analyzed by differential equations. Because of the nonlinear nature of the transformer magnetizing characteristics, the behavior of the system is more sensitive to change in system parameters and initial conditions [15]. A small change in the value of the system voltage, capacitance or losses may cause to dramatic changes in its behavior. A more suitable mathematical approach for studying ferroresonance and other nonlinear phenomena is provided by nonlinear dynamic methods. Nonlinear dynamical analysis of the equivalent circuit by applying KVL and KCL results in the following equations:

$$e = \sqrt{2}E \sin(\omega t) \tag{2}$$

$$v_L = \frac{d\lambda}{dt} \tag{3}$$

$$\left(\sqrt{2}E \cos \omega t\right) = \frac{(C_{series} + C_{shunt})}{\omega C_{series}} \frac{d^2 \lambda}{dt^2} + \frac{1}{\omega R C_{series}} \frac{d\lambda}{dt} + \frac{1}{\omega C_{series}} (a\lambda + b\lambda^7) \quad (4)$$

State space formulation is done by selecting λ and V_L as the state variables x_1 and x_2 , respectively. For simplification, some new parameters are defined as bellows:

$$\varepsilon = \frac{1}{(C_{series} + C_{shunt})} \quad (5)$$

$$\mu = \frac{1}{R} \quad (6)$$

$$K = C_{series} \omega (\sqrt{2}E) \quad (7)$$

Substituting in (4), the following equation is obtained:

$$\ddot{X}_1 + \varepsilon (aX_1 + bX_1^7) + \mu \varepsilon \dot{X}_1 = K \varepsilon \cos \theta \quad (8)$$

where ε is a small positive parameter, μ is the damping coefficient and is a positive parameter, $K\varepsilon$ is amplitude of voltage source and ω is the frequency of voltage source. State equations are formed as follows:

$$\dot{X}_1 = X_2 \quad (9)$$

$$\dot{X}_2 = -\varepsilon (aX_1 + bX_1^7) - \varepsilon \mu X_2 + K \varepsilon \cos \theta \quad (10)$$

If state equations are considered as (11)

$$\dot{X} = AX + BU \quad (11)$$

where X and U are state variables and system inputs vectors and A and B represent coefficients matrices, thus:

$$\dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a\varepsilon & -\varepsilon\mu \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K\varepsilon \end{bmatrix} U(t) \quad (12)$$

$$\begin{bmatrix} 0 & 0 \\ -b\varepsilon & 0 \end{bmatrix} \begin{bmatrix} X_1^7 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ K\varepsilon \end{bmatrix} U(t)$$

where $U(t) = \cos \omega t$.

The fixed or equilibrium points are defined as the vanishing of the vector field; that is:

$$\dot{X} = 0 \quad (13)$$

For the initial conditions, we have:

$$\lambda(0) = 0.0; \quad v_l = \frac{d\lambda}{dt}(0) = \sqrt{2} \quad (14)$$

At an equilibrium point, since the right-hand side term of (11) becomes zero, its stability is dominated by the eigenvalues of the Jacobean $J = \frac{\partial f}{\partial x}$ evaluated at fixed point.

Multiple scales method can be used as simplification method for stability and bifurcation analysis [20]. By using the multiple scales method one obtains a first order approximation for the solution of (8) as:

$$X_1 = h \cos(\omega t - \gamma) + o(\varepsilon) \quad (15)$$

The parameters a , μ and k are independent of ε . Further, the frequency of system is such that

$$\omega = 1 + \varepsilon \delta \quad (16)$$

where δ is named external detuning. By using the multiple scales method, we seek for the first order uniform expansion of (8) in the form:

$$X_1(t; \varepsilon) = X_{1,0}(T_0, T_1) + \varepsilon X_{1,1}(T_0, T_1) + \dots \quad (17)$$

where $T = t_0$ and $T_1 = \varepsilon T_0$. In term of T_1 the time derivative becomes:

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \dots \quad (18)$$

Substituting (17) and (18) into (8), and equating coefficient of the similar powers of ε , we obtain:

$$O(\varepsilon^0): D_0^2 X_{1,0} = 0 \quad (19)$$

$$O(\varepsilon): D_0^2 X_{1,1} + 2D_1 D_0 X_{1,0} + \mu D_0 X_{1,0} + bX_{1,1}^q = K \cos \omega_0 t \quad (20)$$

The solution of Eq. (20) can be expressed as:

$$X_{1,0} = A(T_1)T_0 + A_0 \quad (21)$$

Substituting (21) in (20):

$$D_0^2 X_{1,1} + aX_{1,1} + bX_{1,1}^q = -2A' - \mu A + \frac{K}{2} e^{i5T} + cc \quad (22)$$

where cc is complex conjugate of preceding terms and the prime indicates the derivation with respect to T_1 . Using (8) in eliminating the lead to secular terms in $X_{1,1}$ from (19), we obtain:

$$2A' - \mu A + \frac{K}{2} e^{i5T} = 0 \quad (23)$$

If A is defined in the polar form, we have $A = \frac{1}{2} \alpha e^{i(\beta + \delta T)}$ where α, β are functions of T_1 , by separating real and imaginary parts in (20):

$$\alpha e^{i(\beta + \delta T)} + \alpha i(\beta + \delta) e^{i(\beta + \delta T)} + \frac{1}{2} \alpha e^{i(\beta + \delta T)} - \frac{k}{2} e^{i\delta T} = 0 \quad (24)$$

From (24), we obtain (25) and (26):

$$\alpha' \cos \beta - \alpha \beta' \sin \beta - \alpha \delta \sin \beta + \frac{1}{2} \alpha \cos \beta = 0 \quad (25)$$

$$\alpha' \sin \beta - \alpha \beta' \cos \beta - \alpha \delta \cos \beta + \frac{1}{2} \alpha \sin \beta = 0 \quad (26)$$

Multiplying $-\sin \beta$ in (25) and $\cos \beta$ in (26) we have:

$$\alpha \beta' + \alpha \delta + \frac{k}{2} \sin \beta = 0 \quad (27)$$

With multiplying $\cos \beta$ in (27) and $\sin \beta$ in (28) we have:

$$\alpha' \frac{1}{2} \alpha - \frac{k}{2} \cos \beta = 0 \quad (28)$$

Setting $\alpha' = 0$ and $\beta' = 0$ in (27) and (28) we find that their fixed points are given by:

$$\alpha_0 \delta + \frac{k}{2} \sin \beta_0 = 0 \quad (29)$$

$$\frac{1}{2} \alpha_0 - \frac{k}{2} \cos \beta_0 = 0 \quad (30)$$

Squaring and adding (29) and (30) yield the frequency

response equation:

$$\alpha_0 \delta^2 + \frac{1}{4} \alpha_0^2 = \frac{1}{4} k^2 \quad (31)$$

The stability of the fixed points depends on eigenvalues of Jacobean matrix which are given in (32).

$$A = \begin{bmatrix} \frac{1}{2} & \frac{k}{2} \sin \beta \\ \frac{k}{2\alpha^2} \sin \beta & -\frac{k}{2} \cos \beta \end{bmatrix} \quad (32)$$

Determinant of $[\lambda I - A]$ yields eigenvalues:

$$\lambda^2 + \left(\frac{k}{2\alpha} \cos \beta \right) \lambda - \frac{k}{4\alpha} \cos \beta - \frac{k^2}{4\alpha^2} \sin^2 \beta = 0 \quad (33)$$

Where λ is eigenvalue. Substituting the polar form of A into (18) and substituting result into (19), we find that, to first approximation X_1 is given by:

$$X_1 = \alpha \cos(\omega t + \beta) + \dots \quad (34)$$

$$\text{If } k = 0 \rightarrow \begin{cases} \alpha\beta = -\alpha\delta \\ \alpha' = -\frac{1}{2}\alpha \end{cases} \quad (35)$$

For nontrivial solutions, $\alpha \neq 0$ and its follows from (35) that:

$$\beta = -\delta T_1 + \beta_0, T_1 = \varepsilon t \rightarrow \beta = -\varepsilon \delta t + \beta_0 \quad (36)$$

Substituting (36) into (34), we find that to the first approximation, the free oscillations of (36) are given by:

$$X_1 = \alpha \cos(\omega_0 t + \beta_0) + \dots \quad (37)$$

Where α is given by (35) which has the normal form of a supercritical pitchfork bifurcation. Equation of eigenvalue is as follows:

$$\lambda^2 + \left(\frac{1}{2} \alpha_0 - \frac{1}{2} \right) \lambda - \frac{1}{4} \delta^2 = 0 \quad (38)$$

We obtain first order approximation of (8) by multiple scale method and by using the chaos theory we discuss its stability. For more details about chaos theory see [19] and [20]. Table I shows the base values used in the analysis; and the parameters' value resulting in two possible types of ferroresonance are given in Table II.

TABLE I
BASE VALUES OF THE SYSTEM USED FOR SIMULATION

Base value of voltage	275 / $\sqrt{3}$ KV
Base value of volt-amperes	100 VA
Base angular Frequency	2 π 50 rad/sec

TABLE II
PARAMETERS' VALUE USED IN SIMULATION [9]

Parameters	C_{shunt} (nf)	C_{series} (nf)	R_{core} (M Ω)	R_{FLR} (M Ω)	ω (rad/sec)	E (KV)
First case	0.19	0.5	225	2	314	275
Second case	0.1	3	1900	2	314	275

III. DESCRIPTION OF SYSTEM WITH FERRORESONANCE SUPPRESSION CIRCUIT

Power system including ferroresonance suppression circuit

is shown in Fig. 5.

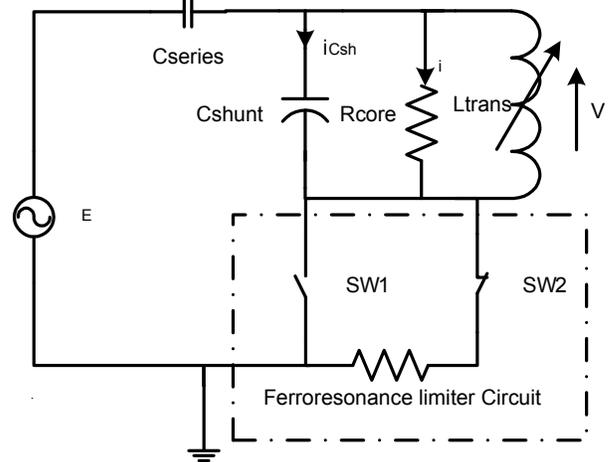


Fig. 5. (a) System equivalent circuit when ferroresonance suppression circuit is connected.

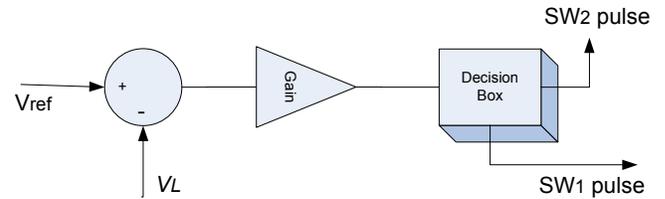


Fig. 5. (b) Switching process diagram.

The primary purpose of inserting ferroresonance limiter between the neutral point of the transformer and the earth is to limit the ferroresonance overvoltage and overcurrent. Low impedance earthing is conventionally defined as impedance that limits the prospective ferroresonance current to the full load current of the transformer. The value of the impedance required is easily calculated to a reasonable approximation by dividing the rated phase voltage by the rated phase current of the transformer. The ferroresonance limiter impedance is conventionally achieved using resistors rather than inductors, to limit the tendency for the fault arc to persist due to the inductive stored energy. The suggested ferroresonance limiter consists of a compact circuit including one resistor, power electronic switches and control circuit. This circuit is placed in the grounding path of the voltage transformer. During normal operation the resistor is bypassed, while at ferroresonance occurrence, it is inserted into the circuit via power electronic switches, activated by control circuit. Regarding the above explanation, in Fig. 5 (a), R_{FLR} is the ferroresonance limiter resistance. The power electronic switches need proper switching pulses, in order to connect and disconnect the ferroresonance limiter resistance to the grounding point of the transformer. Fig. 5 (b) shows the switching process diagram with one decision box. V_{ref} is adjusted between 0.9-1.2 pu of the source voltage and this reference voltage is compared with the measurement voltage on the transformer coil. When reference and measurement voltages are equal, then the decision box generates a pulse for SW1, otherwise, this circuit activates the second switch. To protect the power electronic switches against overvoltages, the step down transformer is

used here. The values for various system parameters, considered for simulation, are kept the same as previous case, while ferroresonance limiter circuit is added to the power system configuration and its value is given below:

$$R_{FLR} = 2M\Omega$$

The differential equation for the circuit in Fig. 5 can be presented as follows:

$$\begin{aligned} (R_{FLR}R_{core}C_{ser}C_{shunt})\frac{d^2v_L}{dt^2} &= R_{core}C_{ser}\sqrt{2}E\omega\cos\omega t - \\ (1+R_{FLR}R_{core}C_{ser}a+R_{FLR}R_{core}C_{ser}7b\lambda^6)v_L \\ -(R_{core}C_{sh}+RC_{ser}+R_{FLR}C_{ser})\frac{d^2\lambda}{dt^2} &- R_{core}(a\lambda+b\lambda^7) \end{aligned} \quad (39)$$

IV. SIMULATION RESULTS

A. Ferroresonance Study of System

For better results, time domain simulations are performed using fourth order Runge-Kutta-fehlberg method and validated by Matlab-Simulink. The major analytical tools, used to study the chaotic ferroresonance, are phase plane, time domain oscillation, and bifurcation diagram. The phase plane analysis is a graphical method, in which the time behavior of a system is represented by the movement of state variables of the system in the state space coordinates versus time. As time evolves, the initial point follows a trajectory. If a trajectory closes on itself, then the system produces a periodic solution. In the chaotic system, the trajectory will never close to itself to shape cycles. A bifurcation diagram is a plot that displays single or multiple solutions (bifurcations) as the value of the control parameter is increased. The results are obtained by numerical solutions and preliminary analysis is based on the mathematical theories. Bifurcation diagrams are given by using Matlab software. Note in these bifurcation diagrams each route has special color. The routs that have same color have similar frequency. In this section, behavior of the VT is studied where there is no controlling circuit in the system configuration. Simulations are performed based on the two sets of the parameters value given in Table II. The following results are obtained. Simulation results are shown in Figs. 6 and 7. Fig. 6 (a) shows time domain behavior of the transformer terminal voltage which depicts sub-harmonic oscillation. Fig. 6 (b) is phase plane diagram when $V_{in} = 3.5p.u$ and $q = 7$. Sub-harmonic ferroresonance oscillations in these figures can be observed. The resulting waveform has sub-harmonic, with a period of 40 ms, i.e. twice the period of the supply. It can be found that there are sharp peaks in flux waveform. This type of flux can damage the insulation of voltage transformer.

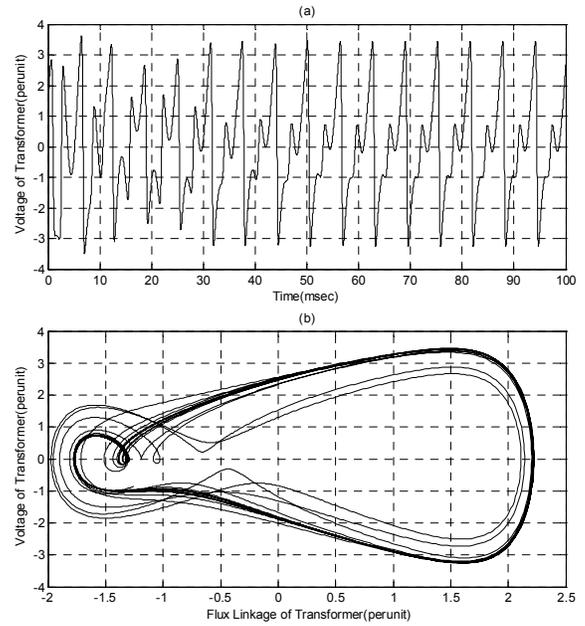


Fig. 6. Subharmonic ferroresonance oscillation a) Time domain simulation b) Phase plane diagram.

The results also indicate subharmonic resonances of the power system, with amplitude of the overvoltages reaching about 3.5p.u which is very dangerous for the power system equipments and may cause VT failure. In the second part of the ferroresonance simulation, parameters value are changed to those presented in the second row of Table II and the results are illustrated in Fig. 7 parts (a) and (b). According to the phase plan diagram, it can be deduced that the amplitude of the ferroresonance oscillation is increased up to 5pu, and behavior of the oscillation is completely chaotic. This overvoltage certainly can cause VT failure.

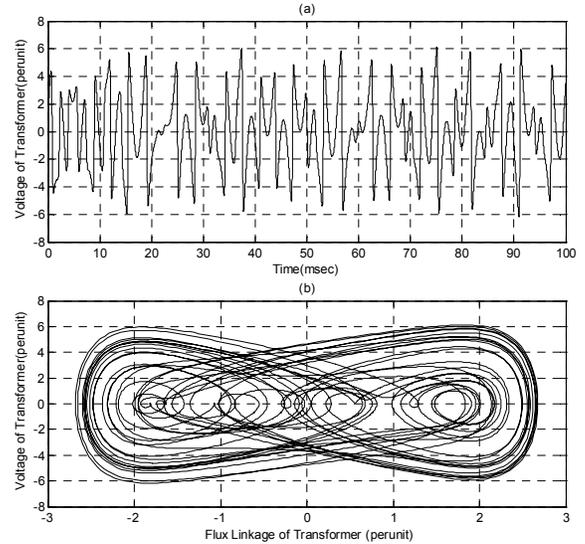


Fig. 7. Chaotic ferroresonance oscillation a) Time domain simulation b) Phase plane diagram.

Phase plan diagram shows the chaotic behavior of the flux linkage and voltage of the transformer. Trajectory of the system indicates nonlinear oscillation with amplitude of the

ferroresonance overvoltages reaching to 5p.u. Although duration of the ferroresonance overvoltages is short, but due to the high amplitude, it can cause serious failure in the power equipments.

B. Adding Ferroresonance Suppression Circuit to the System

This section shows the effect of ferroresonance limiter circuit on the ferroresonance overvoltages by proper nonlinear dynamical tools such as bifurcation and phase plan diagrams. It is considered that all parameters are the same as the previous case, in which ferroresonance occurred, and only parameter value of the ferroresonance limiter circuit are added to the power system. The simulation results indicate that subharmonic and chaotic ferroresonance are changed to the periodic oscillations. By considering ferroresonance limiter circuit effect, amplitude of the overvoltages “3.5p.u” is decreased to 1.5p.u. Voltage waveform, phase plane diagram of normal sinusoidal oscillation are shown in Fig. 8 parts a, and b, respectively. The resulting waveform is periodic, ferroresonance limiter circuit successfully clamps the ferroresonance overvoltages and changes it to the closed trajectory with the frequency equal to that of the source.

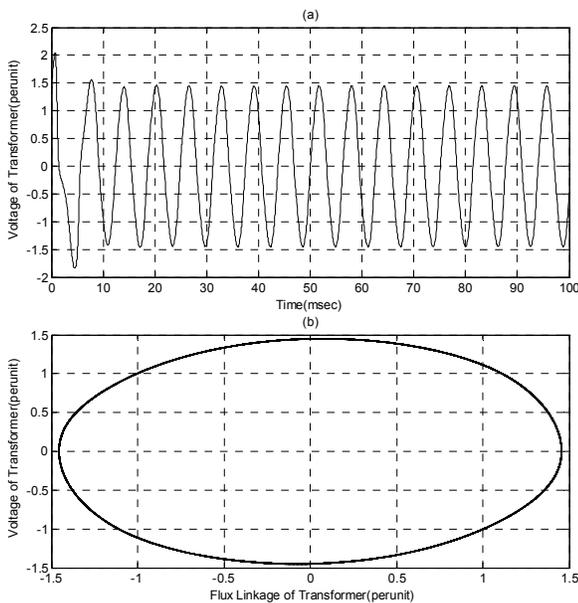


Fig. 8. Periodic oscillation a) Time domain simulation b) phase plane diagram.

Phase plan diagram in Fig. 8 (b) clearly shows the periodic behavior of the voltage of the transformer, which is the effect of limiter circuit. Phase plan shows the closed trajectory of the system, in which there is no extra harmonic. For confirming the effect of suggested ferroresonance limiter circuit on controlling ferroresonance overvoltages, new set of the power system parameters are considered. The simulation results in this case corresponds to the last ferroresonance oscillation of the previous case, In these results, also, the effect of ferroresonance limiter circuits is obvious. High amplitude of the ferroresonance overvoltages is decreased to the lower amplitude as shown in Fig. 9 parts (a) and (b). Chaotic

ferroresonance changes to the quasiperiodic resonance by connecting ferroresonance limiter circuit.

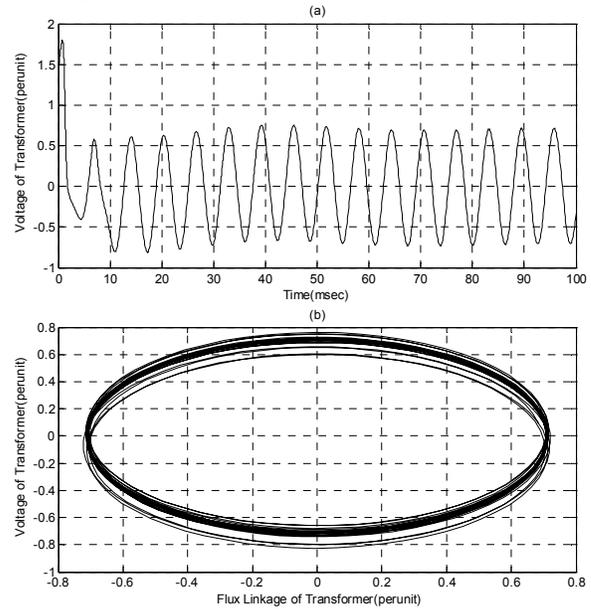


Fig. 9. Quasiperiodic oscillation behavior a) Time domain simulation b) phase plane diagram.

This plots show the qasiperiodic oscillation with some fundamental oscillations included. Amplitude of this resonance is decreased to 0.6p.u. Time domain simulation shows this quasiperiodic behavior while extra resonances are in it. The behavior of this case is better when compared with the chaotic case. Ferroresonance limiter circuit successfully changes the chaotic oscillation into periodic and quasiperiodic oscillation as shown here.

C. Bifurcation Diagram Analysis

Number In this section, the effect of variation in the voltage of the system on ferroresonance overvoltage in the VT, and also the effect of applying ferroresonance limiter circuit on this overvoltage is studied by the bifurcation diagrams. Table III shows parameters value of the power system equipments which are considered for simulation.

TABLE III
POWER SYSTEM PARAMETERS USED FOR PLOTTING THE BIFURCATION DIAGRAM [15]

Power system Parameters	C_{shunt} (nf)	C_{series} (nf)	R_{core} (M Ω)	R_{FLR} (K Ω)	E (KV)
Parameters value	0.5	0.1	1900	50	275-1375

In the bifurcation diagrams, horizontal axis is the power system input source voltage, and vertical axis is the voltage of the transformer terminals. By using this dynamical tool, it is shown that if due to some natural event or switching interrupts, voltage of the system is increased, ferroresonance occurs. Fig. 10 clearly shows the ferroresonance overvoltage in VT when voltage of the system is increased up to 5 p.u. According to this plot, the behavior of the system is completely chaotic. In $E=0.8p.u$ period-3 appears and amplitude of the overvoltage is reached to 2p.u. By increasing

the source voltage, at $E=2$ p.u period doubling starts, and by PDB logic, system behavior goes into the chaotic region. For finding the beginning point of the chaotic region Lyapunov exponent is used. In this plot, effect of the variation in the input voltage on ferroresonance occurring is shown by the Lyapunov exponent. If the exponent remains below zero line, its oscillation have a periodic behavior, otherwise it has a chaotic behavior. According to this explanation, ferroresonance is controlled by connecting ferroresonance limiter and this exponent stays below zero line all the region. This means ferroresonance limiter successfully causes to control ferroresonance phenomena.

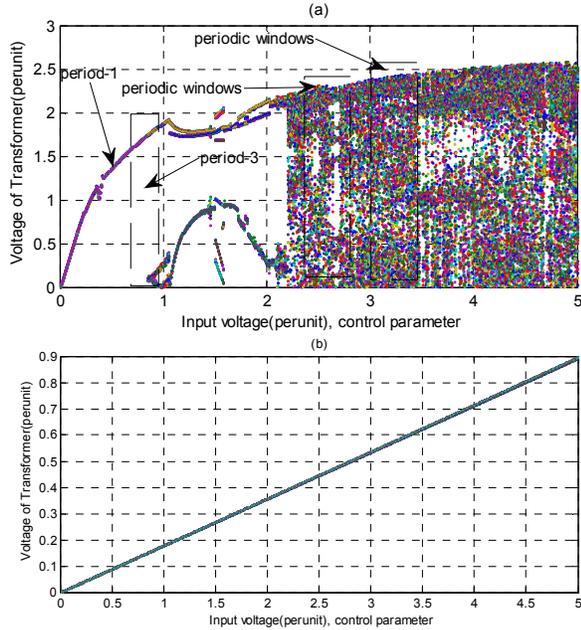


Fig. 10. Bifurcation diagram (a) chaotic behavior without controlling circuit, (b) periodic behavior causes by ferroresonance limiter circuit.

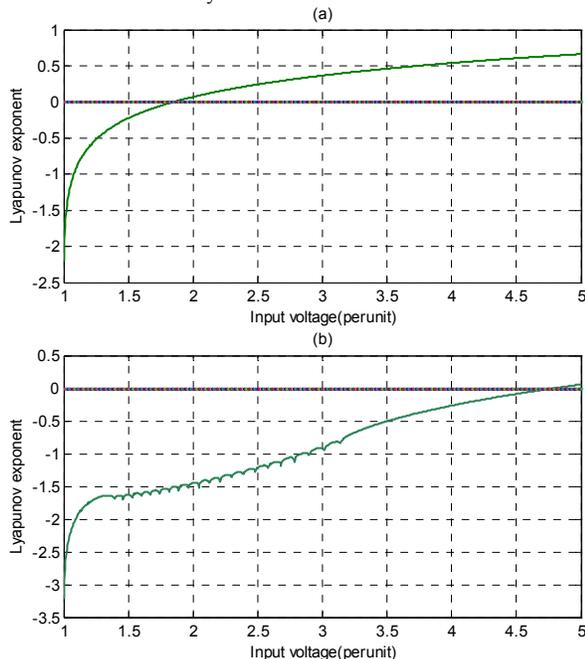


Fig. 11. Lyapunov exponent (a) without limiter effect, (b) when limiter circuit is added.

In part (a) and (b) of Fig. 11, ferroresonance condition and effect of the ferroresonance limiter on controlling nonlinear oscillation is shown via Lyapunov exponent. In the last part of the simulation, system behavior is shown by different set of the system parameters as shown in Table III. Effect of ferroresonance limiter circuit on decreasing ferroresonance over voltages is clearly obvious. In this plot, amplitude of the oscillation is limited to 0.9p.u, and chaotic ferroresonance is changed to the period-1 oscillation.

V. CONCLUSION

In this paper, chaotic ferroresonance oscillations of voltage transformer with considering linear core loss model have been described. Bifurcation and chaos analysis has been introduced and different types of bifurcation for ferroresonance phenomena have been observed using multiple scales method. Lyapunov exponents for different fix points in bifurcation diagrams have been obtained. It was shown that chaos occurs in voltage transformer from a sequence of PDB. It was found that nonlinear magnetization curve has a great influence on bifurcation diagrams and domains of ferroresonance occurrence. Linear core loss model has been used in dynamics equations. Border collision where system becomes suddenly chaotic in bifurcation diagram has been shown. In phase plane diagrams period-n windows have been observed. For fixed points periodic and non-periodic solution and also type of bifurcation are gained. It was shown that for some reasons chaotic ferroresonance can appear causing voltage transformer failure. By connecting the ferroresonance limiter circuit to the transformer ferroresonance phenomena is greatly affected. The presence of the ferroresonance limiter circuit results in clamping the ferroresonance overvoltages in the studied system. The ferroresonance limiter circuit successfully suppresses the chaotic behaviour of the system. Consequently, the system shows less sensitivity to initial conditions and amplitude of the chaotic ferroresonance is controlled successfully.

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