Some examples of photorefractive oscillators

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Abstract—**The photorefractive effect presents a periodical change of the refractive index of an optical environment under the influence of a coherent light. An interesting phenomenon which occurs at this effect is optical phase conjugation (PC). Photorefractive oscillators, that is photorefractive mirrors present important decides in photorefractive optics and their function is based on photorefractive effect. In these oscillators, a phase-conjugated light beam occurs. The basic characteristics of photorefractive oscillators, such as reflectivity, the existance of the oscillation threshold and the threshold of the coupling strength are explained by the so-called grating-action method. This is analysed on a ring oscillator, semilinear mirror and linear mirror.**

***Index terms*—fotorefractive oscillators, grating-action method, ring oscillator, semilinear mirror, linear mirror.**

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I. INTRODUCTION

THE paper will analyse photorefractive oscillators [1] whose operation is based on the photorefractive effect. The photorefractive effect is based on the four-wave mixing of laser light beams (4TM) in some crystals [2]. In this effect, a periodical change of refraction index of an optic environment occurs under the activity of light. During the illumination of some crystals by coherent light, carriers of free charge are created by the transfer of the donor atom electrons to the conduction zone. The number of electron transfers is proportional to the number of donor atoms and the light intensity. Carriers diffuse to the place with lower light intensity, and the opposite sign charges remain in their positions, so-called holes. As the consequence of this rearrangement of charge, an internal electric field occurs, i.e. the phenomenon of change in local refraction index occurs, that is, a diffraction grate is formed in the crystal at which additional incident beams can be scattered (Figure 1).

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An interesting effect which causes light in a photorefractive environment is optical phase conjugation (PC). In this situation a simultaneous rotation of the phase and the direction of the light beam wave propagation. So, the crystal is illuminated with three laser beams: two oppositely directed pumps whose amplitudes are A_1 and A_2 and an amplitude signal A_4. As the result, the fourth wave occurs, with the amplitude A_3 which presents a phase-conjugated copy of the beam A_4. Unlike the known law of reflection of light in geometrical optics, the reflected beam A_3 returns by the same path of the incident beam A_4. A PC beam is interesting because of great opportunities for its practical application.

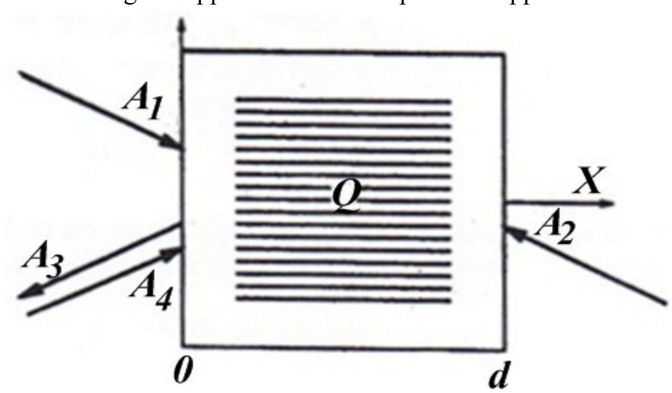

Fig. 1. Four-wave mixing in photorefractive crystals.

The basic equations which describe the process are [3] :

$$\frac{dA_1}{dx} = QA_4 - \alpha A_1, \qquad \frac{dA_4^*}{dx} = -QA_1^* - \alpha A_4^*, \tag{1}$$

$$\frac{dA_2^*}{dx} = -QA_3^* + \alpha A_2^*, \qquad \frac{dA_3}{dx} = QA_2 + \alpha A_3. \tag{2}$$

In the named equations, A denotes the amplitudes of waves, α is the absorption coefficient, Q presents the so-called amplitude of the transmission diffraction grating, and the "asterisk" denotes the conjugated-complex units. The amplitude of the transmission diffraction grating satisfies the following equation:

$$\tau \frac{dQ}{dt} + \varepsilon Q = \frac{\gamma}{I}\left(A_1 A_4^* + A_2^* A_3 \right), \tag{3}$$

where τ is the relaxation time, I is the total intensity of waves $I = \sum |A_i|^2$, while ε and γ are the parameters which depend on the electric fields formed in the crystal [4]. The solving of these equations is performed numerically ans may lead to numerical instabilities, but that is not going to be discussed in this paper [5], [6].

II. Photorefractive Oscillators

The most important devices in photorefractive optics are photorefractive oscillators, that is, mirrors. Their operation is based on the photorefractive effect. Photorefractive oscillators are forms when photorefractive crystals are illuminated with one or two laser beams. In that, a diffraction grating is formed in the crystal, on which the diffraction of laser beams occurs. The most interesting ones are the configurations of oscillators which create the phase-conjugated beams of incident signals.

Photorefractive oscillators with one incident beam are: linear, semilinear, ring and "cat" oscillators. A *Cat* oscillator has two internal interference region, and each of the other oscillators has one. In the others, external mirrors are used, and *cat* oscillators use internal total reflection.

Photorefractive oscillators with two incident beams are: double PC mirror, mutually incoherent beam-coupler, and the so-called *bird* and *frog* oscillators.

In this paper, the oscillators with one incident beam will be analyzed: ring, semilinear and linear photorefractive oscillator.

An oscillator operation analysis can be explained efficiently by the grating-action method [7]. On the basis of this method, the relations of laser beam connections at the entrance (*0*) and exit of a crystal (*d*) are given by the expressions:

$$\begin{bmatrix} A_{1d} \\ A_{4d} \end{bmatrix} = \tau(u) \begin{bmatrix} A_{10} \\ A_{40} \end{bmatrix}, \qquad \begin{bmatrix} A_{30} \\ A_{20} \end{bmatrix} = \tau(u) \begin{bmatrix} A_{3d} \\ A_{2d} \end{bmatrix}, \tag{4}$$

where the matrix $\tau(u)$:

$$\tau(u) = \begin{bmatrix} \cos u & -\sin u \\ \sin u & \cos u \end{bmatrix} \tag{5}$$

where u is the total grating action and calculated from the expression:

$$\tan u = \frac{A_{10}A_{40}^* + A_{2d}^* A_{3d}}{aI\coth(a\gamma d/2) + I_{40} - I_{3d} + I_{2d} - I_{10}}, \tag{6}$$

where a is the constant calculated from boundary conditions, γ is the coupling strength of the crystal, d is the thickness of the crystal, and I denotes the intensities of light beams. On the basis of this method, it is possible to construct an adequate photorefractive circuit which, in a general case, corresponds to the 4TM process (Figure 2).

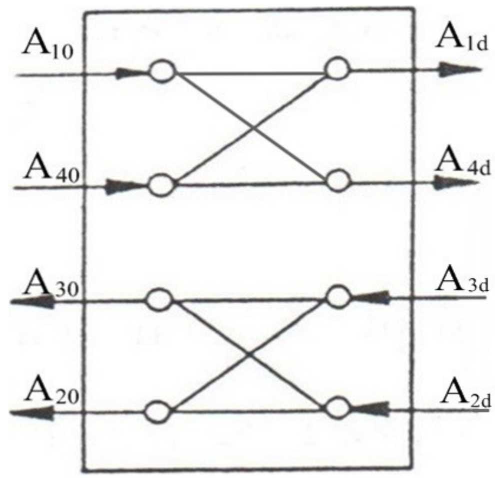

Fig. 2. Optical processor representation of the 4TM process.

III. Ring Oscillator

A ring oscillator has the possibility of self-starting at the oscillation threshold. This oscillator has two ordinary mirrors outside the crystal and one interaction region in the crystal (Figure 3.)

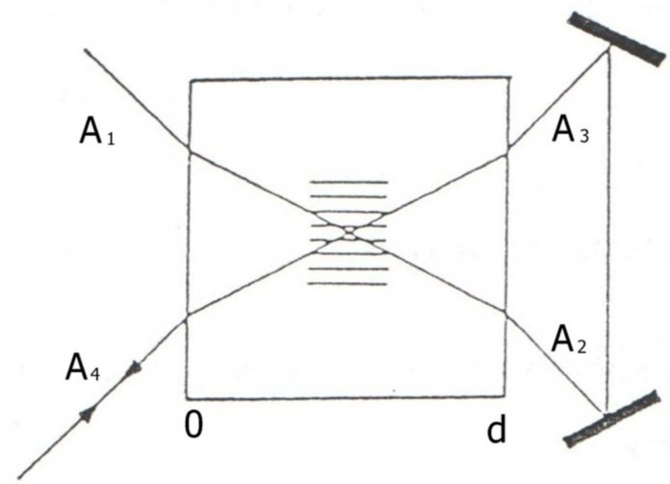

Fig. 3. Photorefractive ring mirror.

The analysis of operation of this oscillator can be efficiently explained by the mentioned grating-action method. Figure 4 shows an adequate photorefractive circuit of this oscillator. By applying the expressions (4) and (5), for a ring oscillator is $A_{10} = 0$ so it follows that the test signal $P = A_{40}$, among the others, generates the output signal $S = A_{30}$, i.e.

$$A_{30} = -\sqrt{M} A_{40} \sin 2u \,, \tag{7}$$

where M is the product of reflectivity of external ordinary mirrors.

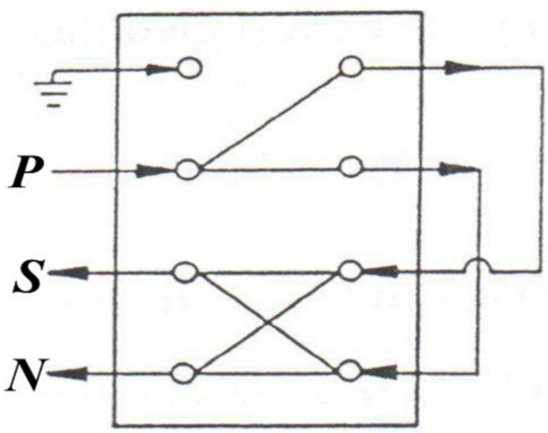

Fig. 4. Photorefractive circuits related to the ring mirror.

The reflectivity of beam in this oscillator is given by the expression $R = I_{30} / I_{40} = A_{30}^2 / A_{40}^2$, so, from (7) it follows:

$$R = M \sin^2 2u \,, \tag{8}$$

while the action u is calculated from (6) and given by the expression:

$$\cos^2 u = \frac{M - 1 - (1+M)\, a \coth(a\gamma d/2)}{4M} \tag{9}$$

Figure 5 presents the solutions which follow from (8) and (9), defining the dependence of the oscillator refection in the function of the coupling parameter, where M is given as a parameter, and $d = 1$.

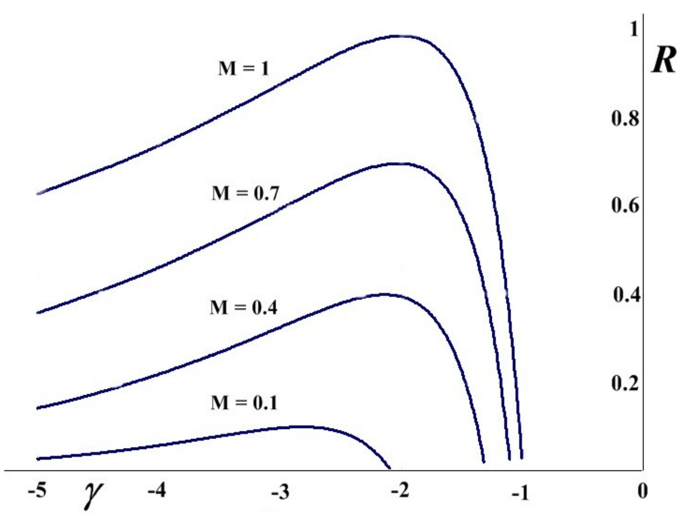

Fig. 5. Oscillator reflection in the function of coupling constant.

Threshold of the coupling strength γ_{th} is calculated from (9) (for $u \to 0$) and from that it follows:

$$\tanh(a\gamma_{th} d/2) = \frac{M-1}{3M+1} \,, \tag{10}$$

what is equivalent to the results in [8] but obtained in a simpler way. It is obvious that the coupling threshold has the value $\gamma_{th} d \to -1$, when $M \to 1$ (Figure 6).

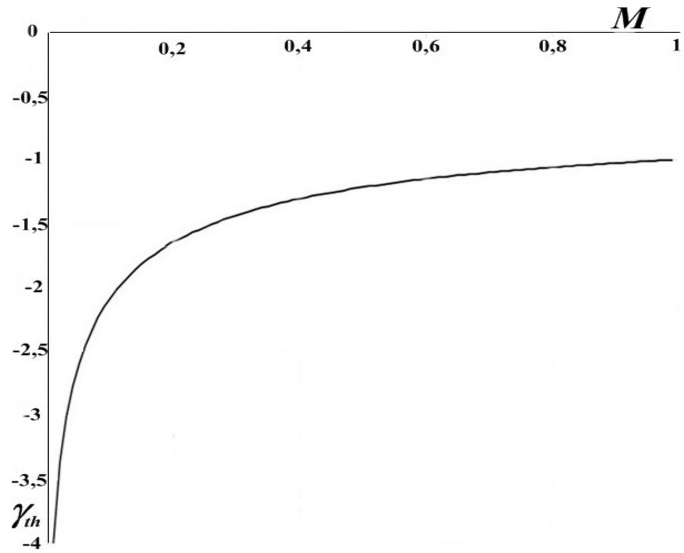

Fig. 6. Threshold of the coupling strength in the function of reflectivity *M*.

At the oscillation threshold, it is $a = (1-M)/(1+M)$ and from that, on the basis of (10) it follows that the relation between the threshold of the coupling strength and the parameter a given by the expression;

$$\gamma_{th} d = \frac{1}{a} \ln(1-a) \,, \tag{11}$$

what is presented in Figure 7 (for $d = 1$).

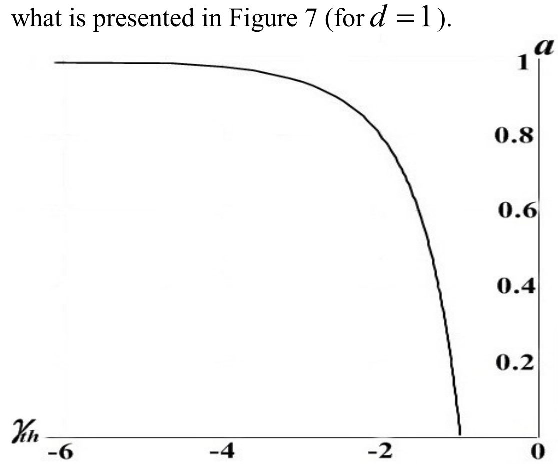

Fig. 7. Constant a in the function of the threshold of the coupling strength.

Ring oscillators can have an interesting application. For example, two couples ring oscillators whose operation is based on 4TM present a combination which is analogous to an electronic flip-flop circuit [9], [10]. Such a flip-flop oscillator consists of two ring oscillators which are from the outside pumped by the light intensities which may be different. In that, only one oscillator oscillates and the other shuts down and vice versa.

IV. Semilinear mirror

Semilinear mirror is, also, an oscillator with one incident light beam and one interaction region. It has one external mirror and does not have the possibility of self-restarting (Figure 8 and Figure 9).

We will repeat the procedures as in the previous case [11]. The incident signal $P = A_{40}$ generates the beams and on the basis of (4) and (5) we get the beams at the exit:

$$A_{1d} = -A_{40} \sin u \, , \; A_{4d} = A_{40} \cos u \, , \qquad (12)$$

and, in that, $A_{10} = 0$.

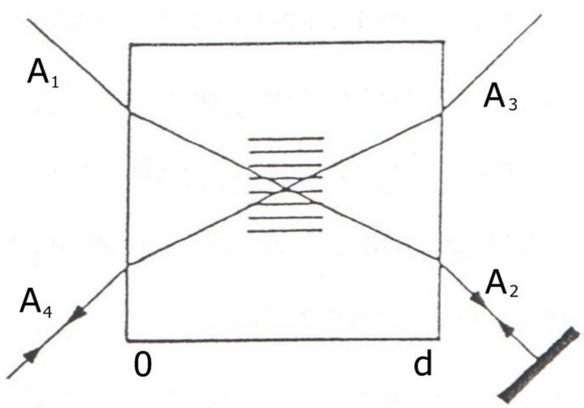

Fig. 8. Photorefractive semilinear mirror.

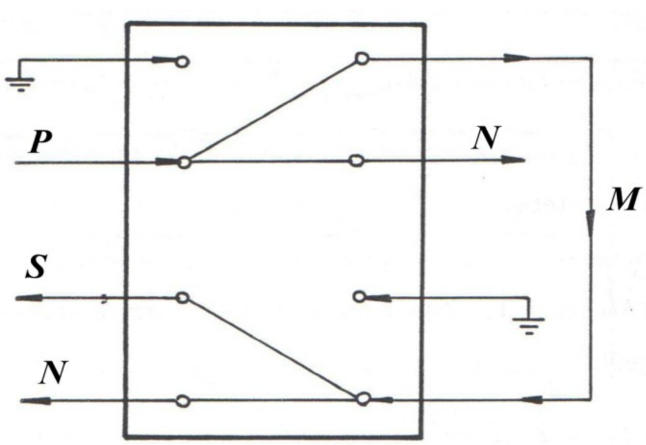

Figure 9. Photorefractive circuits related to the semilinear mirror.

The beam A_{1d} via the external mirror with reflection coefficient M presents the entrance $A_{2d} = \sqrt{M} A_{1d}$, while $A_{3d} = 0$. By applying the relations (4) and (5) it follows:

$$A_{30} = -\sqrt{M} A_{1d} \sin u \, , \quad A_{20} = -\sqrt{M} A_{1d} \cos u \, , \qquad (13)$$

The constant a is calculated from the expression $a = \sqrt{T_d} / I_d$, where $T_d = 4 I_{1d} I_{4d} + (I_{1d} + I_{2d} - I_{4d})^2$, I_{1d}, I_{2d} are I_{4d} light beam intensities for $z = d$ and I_d is their sum. On the basis of the calculation for the constant a, and also from (12) an (13) for the reflection of this mirror $R = I_{30} / I_{40}$ it is obtained:

$$R = \left(\frac{\sqrt{M}\left(1+a^2\right) + 2\left[a^2\left(M+1\right)-1\right]^{1/2}}{M(1-a^2)+4} \right)^2 . \qquad (14)$$

By the arranging from the expression (6) for this case, it follows:

$$\gamma d = \frac{1}{a} \ln \frac{1-a}{1+a} . \qquad (15)$$

On the basis of (14) and (15) in Figure 10 the dependence $R = f(\gamma)$ is presented, where M is the parameter, and $d = 1$.

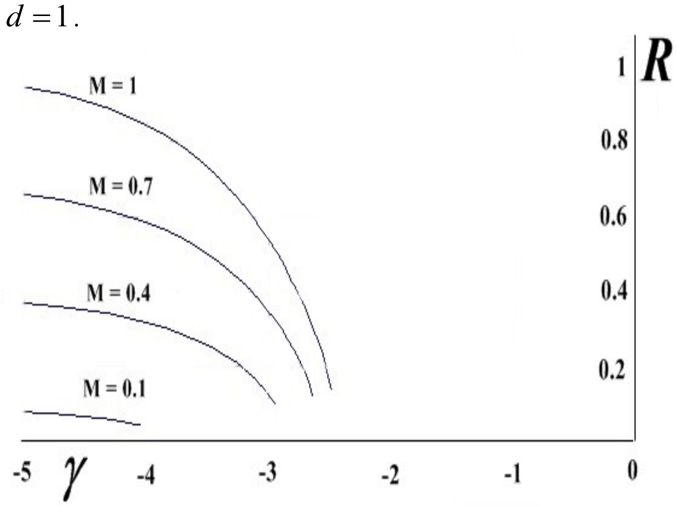

Fig. 10. Oscillator reflection in the function of the coupling constant.

Minimal reflectivity value follows from the equation (14) for the value of the constant a:

$$a^2 = \frac{1}{M+1} . \qquad (16)$$

The threshold of the coupling strength follows from (15) and (16):

$$\gamma_{th} d = \sqrt{1+M}\,\ln\left(\frac{\sqrt{1+M}-1}{\sqrt{1+M}+1}\right). \qquad (17)$$

This result is identical to the result from the paper [8] where it has been obtained in a completely different, but a more complicated way. For the case of maximal reflectivity $M \to 1$, $\gamma_{th} \to -2.49$. (Figure 11)

Also, the solutions given in Figure 12 follow from the expression (15).

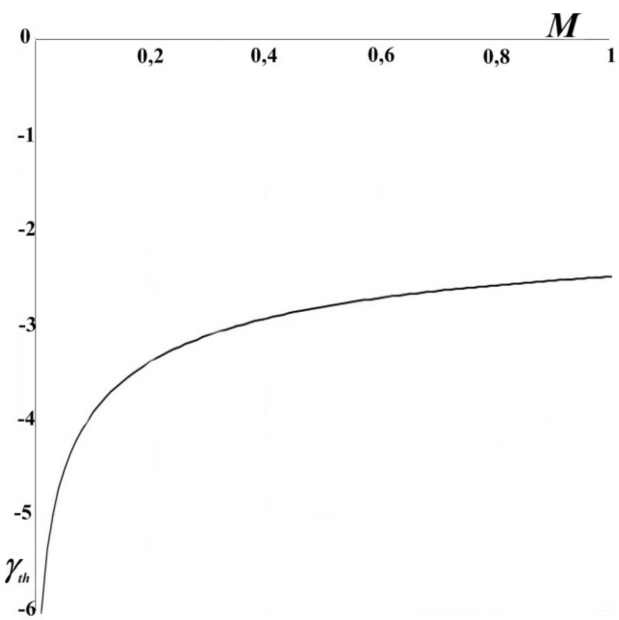

Fig. 11. Threshold of the coupling strength in the function of reflexivity *M*.

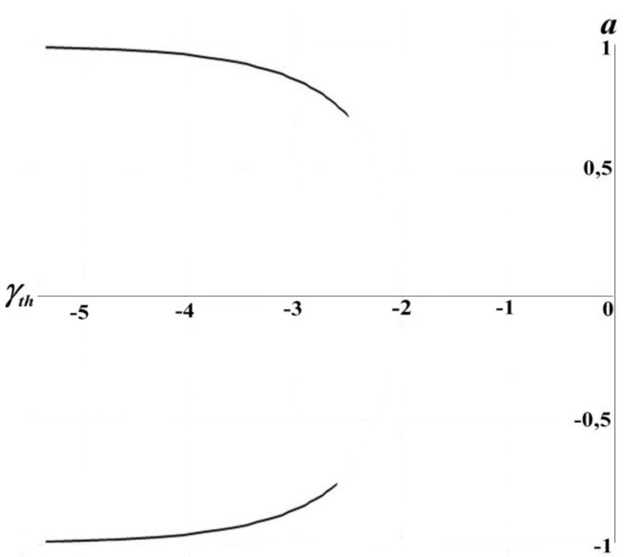

Fig. 12. Constant *a* in the function of the threshold of the coupling strength.

V. LINEAR MIRROR

We will mention the linear mirror, too. This oscillator is, also, a mirror with one incident beam, one interaction region and has the possibility of self-starting at the threshold. This mirror also has two ordinary external mirrors (Figure 13).

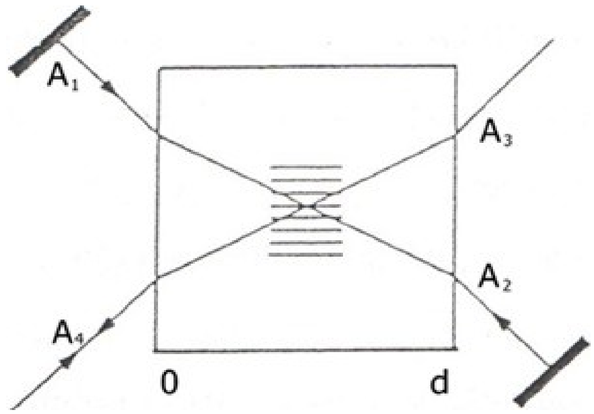

Fig. 13. Photorefractive linear mirror.

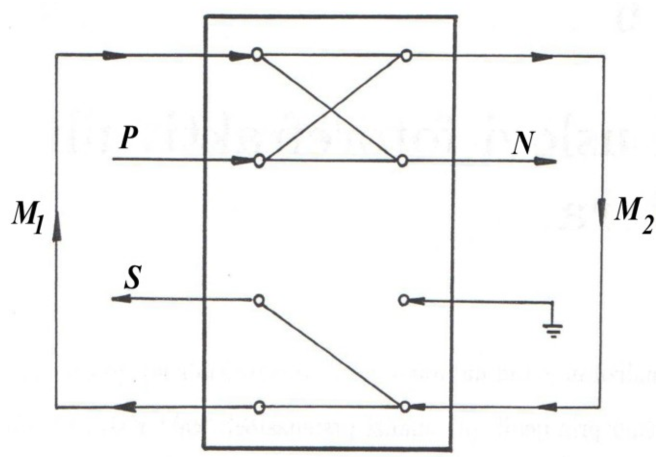

Fig. 14. Photorefractive circuits in relation to the linear mirror.

We will repeat the same procedures as for the previous examples. The incident signal $P = A_{40}$ and beam A_{10} generate the beams at the exit. The beam A_{1d} via the external reflection coefficient M_2 presents the entrance $A_{2d} = \sqrt{M_2} A_{1d}$,while $A_{3d} = 0$. From (4) and (5) among the other, it follows:

$$A_{30} = -\sqrt{M_2} A_{1d} \sin u, \quad A_{20} = \sqrt{M_2} A_{1d} \cos u \qquad (18)$$

The beam A_{20} returns via the external mirror of reflection M_1, i.e. $A_{10} = \sqrt{M_1} A_{20}$ and thus the process is repeated.

It follows from the calculation that the minimal value of the threshold of coupling is:

$$\gamma_{th}d = \ln\sqrt{M_1 M_2} \tag{19}$$

It is obvious that the threshold of the coupling strength tends to zero $\gamma_{th}d \rightarrow 0$, when $M_1 M_2 \rightarrow 1$.

For the case when the reflectivities $M_1 = M_2$ it follows from (19) that

$$\gamma_{th}d = \ln M, \tag{20}$$

and that is presented in Figure 15.

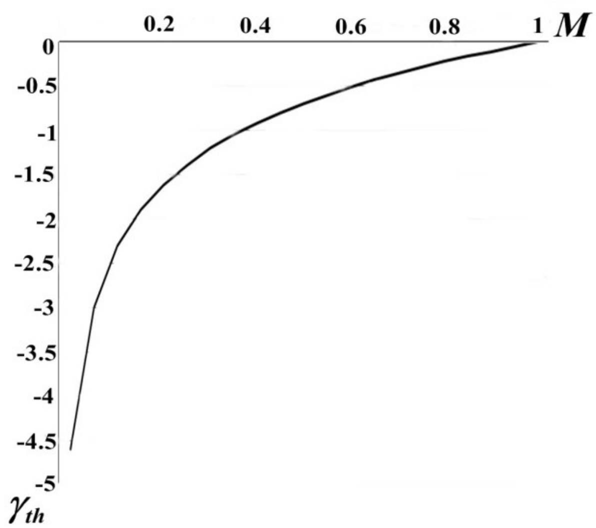

Fig.15. Threshold of coupling strength in the function of reflectivity *M*.

VI. CONCLUSION

The paper analyses the examples of photorefractive oscillators with one incident beam. The operation of this oscillators is based on the photorefractive effect, and in that a phase-conjugated light beam also appears, so they are also called phase conjugated mirrors. So-called grating-action method has been applied, by means of which are efficiently explained the reflectivity of oscillator, then the so-called oscillation threshold and the threshold of coupling strength. This is analysed on a ring oscillator, semilenear and linear mirror. Analytical calculations have been performed and characteristic units, such as reflectivity and characteristic units at the oscillation threshold have been presented graphically.

REFERENCES

[1] Z. Ljuboje " *Fotorefraktivni oscilatori sa jednim upadnim zrakom",* INFOTEH-JAHORINA, Vol. 14 , March 2015.

[2] P. Gunter and J.P. Huignard (eds), *Photorefractive Materials and Their Applications*, I and II (Springer, Berlin, 1988, 1989)

[3] W. Krolikowski, K. D. Shaw, M.Cronin-Golomb and A. Bledowski, J. Opt.Soc.Am.B 6 (1989); W. Krolikowski, M.R. Belić, M.Cronin-Golomb and A. Bledowski, J. Opt.Soc.Am.B 7 (1990).

[4] N. V. Kukhtarev, V. Markov and S. Odulov, "*Transient energy transfer during hologram formation in LiNbO3 in external electric field*", Opt. Commun. 23 338 (1977).

[5] M. Belić and Z. Ljuboje, "*Chaos in phase conjugation: physical vs numerical instabilities*", Opt. Quant. Electron. 24 745 (1992)

[6] Z. Ljuboje, O. Bjelica "*Numeričke nestabilnosti pri rješavanju nekih problema u fizici*", INFOTEH-JAHORINA, Vol. 13, March 2014.

[7] M. S. Petrović, M. R. Belić, and F. Kaiser, *"Photorefractive circuitry and optical transistors* ", Opt. Commun. 121 (1995).

[8] M. C. Golomb, B. Fischer, J.O. White and A. Yariv, *" Theori and aplication of four-wave mixing in photorefractive media*" IEEE J. Qant. Electron. QE-20, 12 (1984).

[9] M. Petrovic, M.R. Belic, M.V. Jaric and F. Kaiser, *"Optical photorefractive flip-flop oscilator*", Opt. Commun. 138, 349 (1997)

[10] Z. Ljuboje, M. Petrović, "*Optički fotorefraktivni flip-flop oscilator"* INFOTEH-JAHORINA, Vol. 3, March 2003.

[11] Z. Ljuboje, " *Numerički haos u fotorefraktivnoj optici*". neobjavljeno.