An Improved Design of Gallager Mapping for LDPC-coded BICM-ID System

Lin Zhou, Weicheng Huang, Shengliang Peng, Yan Chen and Yucheng He

***Abstract*—Gallager mapping uses different signal points with different probabilities by assigning several labels for one signal point, and thus provides a promising approach to achieving shaping gains. An important issue in Gallager mapping is how to assign labels for signal points. In this paper, two optimized design rules for Gallager mapping of bit-interleaved coded modulation scheme with iterative decoding (BICM-ID) are proposed, where the Hamming distance among the labels for one signal point should be minimized. The extrinsic information transfer (EXIT) technique is utilized to design and analyze the proposed mapping patterns. Compared with conventional Gallager mapping, our proposed method provides extra shaping gain for the LDPC-coded BICM-ID system. And our proposed method supplies better performance than conventional uniform mapping with the same spectrum efficiency.**

***Index Terms*—low-density parity-check (LDPC) codes, Gallager mapping, quantization mapping, bit-interleaved, iterative decoding.**

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I. Introduction

In classical communication theory, coding and modulation had been treated as two separate parts. However, in recent decades, some new techniques [1]–[5] which integrate coding and modulation were devised to improve the performance of system.

The thinking of combined coding and modulation design was first suggested by Massey in [6] and then developed by Ungerboeck (trellis-coded modulation, TCM) [1] and Imai (multilevel coding, MLC) [2]. TCM scheme performs good over the Additive White Gaussian Noise (AWGN) channels but not so good over fading channels. Moreover, the complexity of MLC was very high because the multistage decoder costs too much. In 1992, a scheme named bit-interleaved coded modulation (BICM) was first introduced by Zehavi [3], and then developed by Li [7] as BICM with iterative decoding (BICM-ID). In past years, BICM-ID scheme with turbo-like decoding had been found very good performance over both AWGN and fading channels [4].

By Shannon’s Information Theory, the shaping gain with non-uniform input signal and multi-dimensional signal constellation can asymptotically approach 1.53 dB at most [8]. As so far, a few approaches [10]–[14] have been developed to provide this kind of shaping gain. Among which, non-uniformly spaced signals [11] with equal probability were extensively studied [10], [12], while another kind of shaping method with non-equiprobable signals attracted little attention [9], [13]. Quantization mapping works by generating non-uniform distribution of signal probabilities to approach the optimum input distributions. For the first introduction by Gallager in [8], quantization mapping is also called Gallager mapping. In 2004, a scheme using Gallager mapping and maximum likelihood (ML) decoding was proposed by Bennatan [9], and it could provide some shaping gain. However, the mapping distribution he used in the mapper was not optimally designed and the ML decoding prohibited practical applications for long codes.

In this paper, an improved Gallager mapping scheme for the BICM-ID system based on low-density parity-check (LDPC) codes is presented, where the extrinsic messages are transferred between the LDPC decoder and the signal demapper. No extra shaping code is needed in the proposed scheme, and the improvement of Gallager mapping results in no extra complexity for communication systems.

II. System model

The system proposed in this paper is described in Fig. 1, where LDPC codes and quantization mapper are utilized. Assume a two-dimensional constellation $\mathcal{A}_X = \{a_i \mid 0 \le i \le M-1,\ a_i \in \mathbb{C}\}$ of size M is used. The input vector $\mathbf{u} = (u_0, u_1, \cdots, u_{K-1})$ of information symbols is first encoded by the LDPC encoder into a codeword $\mathbf{c} = (c_0, c_1, \cdots, c_{N-1})$, then interleaved and producing a bit sequence $\mathbf{v} = (v_0, v_1, \cdots, v_{N-1})$. The Gallager mapper generates signal vectors $\mathbf{x} = (x_0, x_1, \cdots, x_j, \cdots)$ with $x_j = \mathcal{M}(v'_j)$, where j denotes the index of the modulated symbols, $\mathcal{M}(\cdot)$ stands for the signal mapping function and

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$\mathbf{v}'_j = \left(v'_{j,1}, v'_{j,2}, \cdots, v'_{j,T}\right)$ is a *T*-bit-length vector which extracted from $\mathbf{v}$, and the value of *T* depends on the mapping function $\mathcal{M}(\cdot)$.

Suppose the complex signal vector $\mathbf{x}$ is transmitted over the AWGN channel. The received vector $\mathbf{y} = \left(y_0, y_1, \cdots, y_j, \cdots\right)$ is then given by

$$y_j = x_j + n_j \tag{1}$$

where $n_j \sim CN\left(0, N_0\right)$ are independent and identically distributed complex Gaussian random variables with zero mean and variance $N_0 / 2$ per dimension.

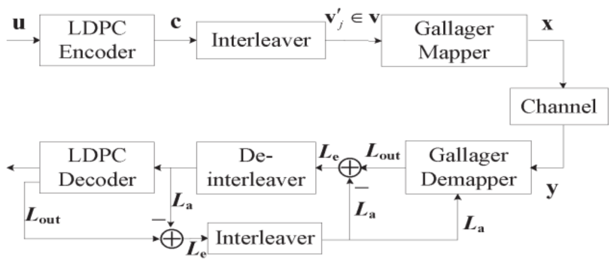

Fig.1 Block diagram of the system model

At the receiver, the received signal y from the channel is firstly processed by the Gallager demapper, and the extrinsic information L_e is delivered to the LDPC decoder after the interleaving. Then the output extrinsic information of the demapper is de-interleaved and fed back to the LDPC decoder as the *a priori* information L_a. This iterative process continues until certain condition is satisfied.

The Gallager demapper processes the received symbol y_j and the corresponding a priori log-likelihood ratio (LLR)

$$L_a\left(v'_{j,t}\right) = \log\left(\frac{P\left(v'_{j,t} = 0\right)}{P\left(v'_{j,t} = 1\right)}\right) \tag{2}$$

where $j = 0, 1, 2, \ldots,$and $t = 1, 2, \ldots, T$ to generate the extrinsic information L_e as follows,

$$\begin{aligned} L_e\left(v'_{j,t}\right) &= L_{out}(y_j, L_a(v^{'}_{j,t})) - L_a\left(v'_{j,t}\right) \\ &= \log\left(\frac{P\left(v'_{j,t} = 0 \mid y_j, L_a\left(\mathbf{v}'_j\right)\right)}{P\left(v'_{j,t} = 1 \mid y_j, L_a\left(\mathbf{v}'_j\right)\right)}\right) - L_a\left(v'_{j,t}\right) \end{aligned} \tag{3}$$

which is then delivered to the LDPC decoder and taken as the *a priori* information for decoding. Next, the extrinsic information of the LDPC decoder should be fed back to the Gallager demapper as the *a priori* information. Apparently, the demapper and decoder exchange the extrinsic information in an iterative manner.

III. IMPROVED GALLAGER MAPPING

A. Quantization Mapping

Let $\left\{P\left(x\right) \mid x \in \mathcal{A}_X\right\}$ be a probability mass function set associated with constellation $\mathcal{A}_X$. For a conventional signal mapper, the signal points are used with equal possibility, and the corresponding mapping function can be expressed as $P\left(x\right) = 1/M$.

Definition 1. As described in [8], A quantization $Q_{P(x)}\left(\mathbf{v}'\right) : \left\{0,1\right\}^T \to \mathcal{A}_X$ is a mapping from labels $\mathbf{u}$ of length T to $x \in \mathcal{A}_X$, such that the number of labels mapped to x is $2^T P\left(x\right)$. For brevity, we use $Q\left(\mathbf{v}'\right)$ for $Q_{P(x)}\left(\mathbf{v}'\right)$.

Note that, $P\left(x\right)$ can be a non-uniform distribution. Thus when applying the quantization mapping distribution to input sequences, the resulted signals $\left\{x_0, x_1, \cdots, x_j, \cdots\right\}$ over the constellation space has the potential to provide shaping gain.

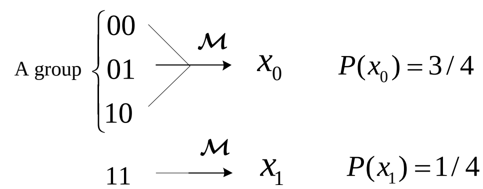

Fig.2 An example of quantization mapping

Example 1. Quantization mapping for 2-bit-length symbols

An example is presented to help understanding the definition of quantization mapping. The two points of the constellation are used with the unequal probability of $3/4$, $1/4$, respectively. In this paper, all symbol sequences which are corresponding to a same point are called labels within a group, and are shown in Fig. 2.

B. Optimal Input Distribution

Let X denotes the transmitted symbol, Y denotes the received symbol. The channel capacity will be approached when the mutual information $I\left(X;Y\right)$ is maximized. For a discrete-time memoryless AWGN channel, according to [16], X should be subjected to Gaussian distributed to maximize the mutual information $I\left(X;Y\right)$.

Lemma 1. If a random variable satisfy the following restriction

$$\sigma_v^2 = \int_{-\infty}^{+\infty} p\left(v\right)\left(v - m\right)^2 dv < \infty \tag{4}$$

, where σ_v and m are the covariance and mean value of v, respectively. Then

$$H\left(V\right) \le \log\sqrt{2\pi e}\sigma_v \tag{5}$$

with equality if and only if v is subject to Gaussian distribution, where V is the set of v, and $H(V)$ denotes the entropy of V.

Theorem 1. For a discrete-time memoryless AWGN channel, the optimal input distribution of X, which maximizes the mutual information $I(X;Y)$ is subject to Gaussian distribution.

Let $C \triangleq MaxI(X;Y)$, where C denotes channel capacity, since

$$I(X;Y) = H(Y) - H(Y \mid X) = H(X+N) - H(N) \quad (6)$$

According to *Lemma 1*, X should be Gaussian distributed to achieve the channel capacity.

In 2004 [9], Bennatan found some good non-uniform input distributions close to optimal distribution for the discrete-time AWGN channel. However, the mapping labels in natural (ascending or descending) order were used in his scheme, which resulted performance loss for practical schemes.

C. Improved Gallager Mapping

As pervious description, for Gallager mapping, every constellation point may correspond to one or more coded symbol sequences. The problem of finding good mapping patterns is how to design the probabilities of every point and which labels should be assigned into a same group. Based on the theory of optimal input distribution and the characteristic of overlapped labels in Gallager mapping, we propose two mapping rules to solve the problem and improve the performance of BICM-ID system with Gallager mapping. And the academic explanations will be given in section IV.

Rule 1: Let the probability mass function of the signal points in the employed constellation approach the discrete-time Gaussian distribution;

Rule 2: Try to minimize the Hamming distance among the different labels within every group.

Note that the mathematical demonstration of *Rule 1* has been given in the foregoing Section III B for a discrete signaling over an AWGN channel, the output of the mapper should follow the discrete Gaussian distribution as much as possible. While according to *Rule 2*, the number of reliable coded bits is guaranteed to be as large as possible when recovering the coded bits sequences from the signal points. In other words, the number of bits recovered with low reliability is as few as possible by *Rule 2*. Unfortunately, it is hardly to analysis *Rule 2* by mathematical equations, so computer-based analysis and simulations will be provided in the next sections.

IV. Analysis of Proposed Gallager Mapping

In the previous section, two rules for the improved Gallager mapping method based on BICM-ID system are presented. To further illustrate the effect of the proposed mapping rules and find good mapping patterns, the technique of EXIT chart [15] is employed to analyze the iterative system.

As is known, the LDPC decoder can be seen as concatenation of variable-node decoder (VND) and check-node decoder (CND). Thereby, we could expressly partition the receiver in Fig. 1 into two blocks, which is depicted in Fig. 3, where block **A** mainly consists of the demapper and variable-node decoder, and block **B** solely comprises the check-node decoder. I_A, I_B and I_D denote the mutual information at the output end of block **A**, **B**, and demapper, respectively.

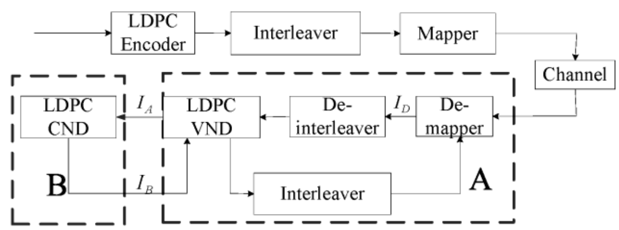

Fig. 3. EXIT Chart analysis model of our scheme

At the receiving end, the Soft-In-Soft-Out (SISO) demapper processes the channel-corrupted sequences, and generates the posteriori log-likelihood ratios (LLRs) to the LDPC decoder for reconstruction. Indeed, the information exchange not only exists between the SISO demapper and the decoder, but also inside the LDPC decoder. Hence, based on Fig. 3, the overall decoding process can be formulated as follows.

Step 1. The SISO modulator computes LLRs and sends them to the LDPC VND after de-interleaving, and the output of LDPC VND is what block **B** received from block **A**.

Step 2. The LDPC CND, i.e., block **B** operates on the received extrinsic information to compute what to be passed to block **A**.

Note that only extrinsic information is exchanged in between. Hence, it is possible to draw EXIT curves for both blocks. According to [17], [18] the approximate formulas to compute I_A and I_B are given as follows.

$$I_B = 1 - \sum_j \rho_j J\left(\sqrt{j-1} J^{-1}(1 - I_A)\right) \quad (7)$$

$$I_A = \sum_j \lambda_j J\left(\sqrt{(i-1)(J^{-1}(I_B)^2) + (J^{-1}(I_D)^2)}\right) \quad (8)$$

, where the details of the $J(\bullet)$ function was defined in [17], and $\lambda(x)$, $\rho(x)$ express the degree distributions of LDPC codes.

Then given a regular LDPC code assemble $C(3,6)$ over the AWGN channel, the design of good Gallager mapping patterns will be presented by EXIT Chart analysis. As is known, the best performance will be achieved when the curves of variable-node decoder and signal demapper

(VND+DE) properly match the curves of CND. There is no relationship between the CND curve and the channel state information, that is to say, we just need to design a good mapping pattern to make sure that the VND+De curve must not cross the fixed CND curve at a certain SNR as low as possible.

According to the previous two Gallager mapping rules and the analyses of EXIT Chart, two mapping examples using 1-dimensional and 2-dimensional signaling are presented to clarify the proposed improved Gallager mapping method.

Example 2. 8-PAM Gallager mapping

Fig. 4 shows the conventional Gallager mapping and the improved Gallager mapping for an 8-ary Pulse Amplitude Modulation (8-PAM) constellation. Here, 16 length-4 coded symbol sequences are mapped to the 8-PAM constellation with unequal probabilities. Obviously, for conventional Gallager mapping, the maximum Hamming distance D (D_{max}) within every group is 2 or 3, and the average Hamming distance (D_{ave}) within every group is 2 or 4/3. For the improved mapping method, D_{max} within every group is 2 and D_{ave} within each group is 4/3, which are equal to or less than that of conventional mapping method.

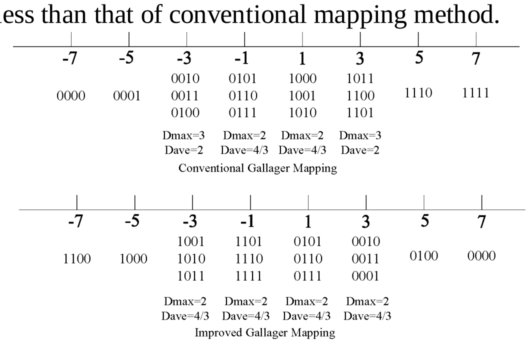

Fig. 4. Gallager mapping for 8-PAM constellation

Example 3. 16-QAM quantization mapping

Fig. 5 provides another example of 2-dimensional signaling. Here, 32 length-5 coded symbol sequences are mapped to traditional 16-ary Quadrature Amplitude Modulation (16-QAM) constellation. The D_{max} and D_{ave} are also reduced by using the proposed mapping rules, which thus provides improvement for extra shaping gain. Due to lack of space, only the improved mapping constellation is provided here. In the following, the corresponding EXIT Chart analysis curves of the two examples will be provided, too.

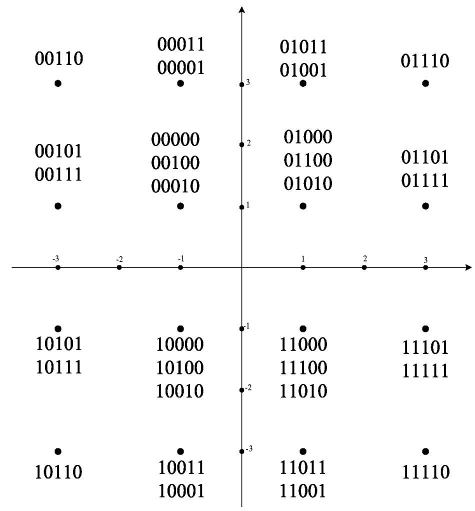

Fig. 5. Improved Gallager mapping for 16-QAM constellation

Fig. 6 and 7 show the EXIT Chart analyses of the improved Gallager mapping and the conventional Gallager mapping and some traditional mappings (such as Gray mapping and Anti-Gray mapping). It can be seen that there is an open and narrow tunnel between the improved VND+DE curve and the CND curve in each chart. On the contrary, the conventional curves cross each other which bring in incorrigible errors and result in performance loss.

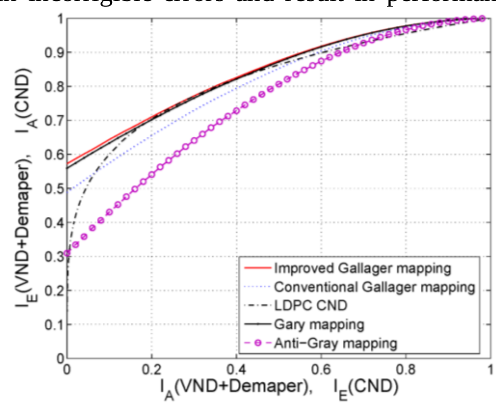

Fig. 6. EXIT Chart analysis for 8-PAM Gallager mapping, $E_b / N_0 = 9.6\,dB$

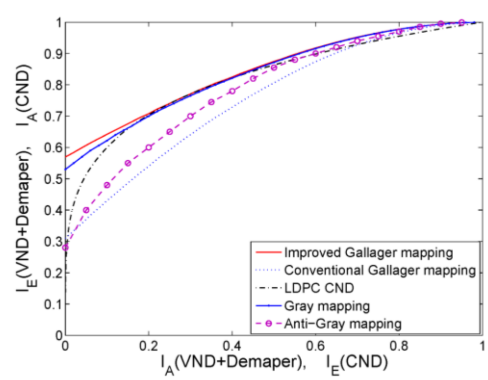

Fig. 7. EXIT Chart analysis for 16-QAM Gallager mapping, $E_b / N_0 = 5.4\,dB$

By EXIT Chart analyses, we also get the threshold of the proposed scheme. That is to say, when E_b / N_0 is larger than 9.6 dB or 5.4 dB, respectively, the two proposed schemes can successfully work without any errors by using good enough channel codes.

V. SIMULATION RESULTS

In this section, a rate-1/2, length-9216 regular binary (3,6) LDPC code from China Mobile Multimedia Broadcasting (CMMB) [19] and a rate-2/3 length-9216 regular (3,9) LDPC code will be used to demonstrate the effectiveness of our proposed mapping method for the LDPC-coded BICM-ID system over AWGN channels. As known, the standard sum- product decoding algorithm (SPA) is the most popular algorithm for LDPC decoding [20], and the most common number of iteration should be 50. To ensure the fairness of comparison between the proposed Gallager-mapping-based system and the traditional system, the maximum inner iteration number of our proposed LDPC decoder is set to be 10, and furthermore, the outer iteration number between the demapper and LDPC decoder is set to be 5. So the computational complexity of our proposed scheme is almost the same as the traditional LDPC- coded BICM-ID scheme.

Fig. 8 shows the performance of length-4 symbols which are mapped to the conventional 8-PAM (shown in Fig. 4) and conventional 16-PAM set by using the rate-1/2 LDPC code. At the BER of 10^{-5}, almost 0.5 dB and 0.2 dB of extra shaping gain are attained by proposed Gallager mapping over the conventional Gallager mapping and traditional 16-PAM mapping, respectively. The spectral efficiency of the proposed scheme is 2 bit/s/Hz. Fig. 9 shows the performance of length-4 symbols which are mapped to the conventional 8-PAM set(shown in fig. 4) and conventional 16-PAM set by using the rate-2/3 LDPC code. Similar results will be obtained. At the BER of 10−5, almost 1.2 dB and 0.2 dB of extra shaping gain are attained by proposed Gallager mapping over the conventional Gallager mapping and traditional 16-PAM mapping, respectively. The spectral efficiency of the proposed scheme is 2.67 bit/s/Hz.

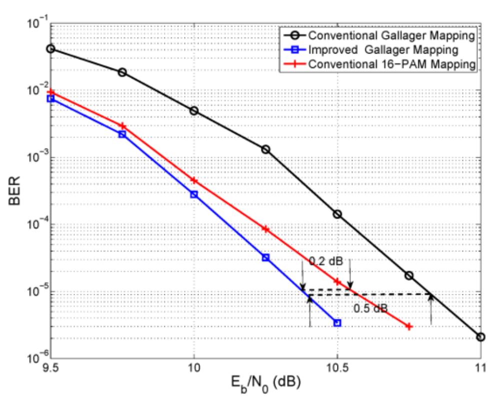

Fig. 8. Performance of 8-PAM Gallager mapping, R=1/2

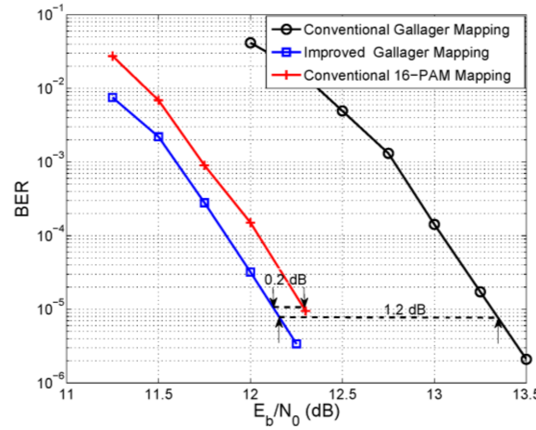

Fig. 9. Performance of 8-PAM Gallager mapping，R=2/3

As known, 16-QAM signaling is one of the most popular modulation types in modern digital communication system. Fig. 10 shows the performance of different modulation schemes with the same spectral efficiency of 2.5 bit/s/Hz. It can be observed that, at the BER of 10^{-5}, the proposed Gallager mapping outperforms the conventional Gallager mapping by 1dB and is 0.2 dB better than the conventional 32-QAM with outer iteration.

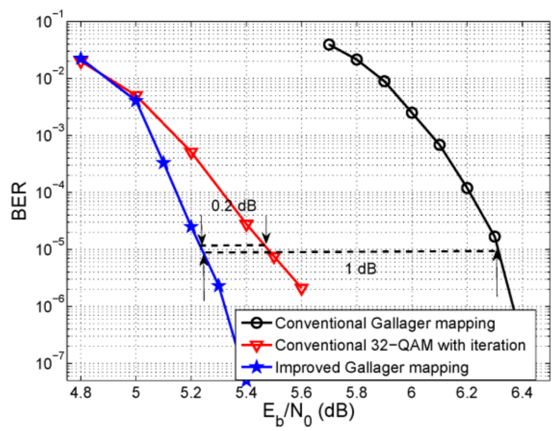

Fig. 10. Performance of 16-QAM Gallager mapping，R=1/2

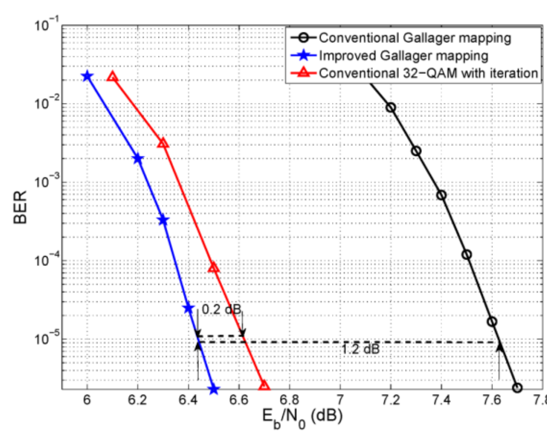

Fig. 11. Performance of 16-QAM Gallager mapping，R=2/3

Fig. 11 shows the performance of different modulation schemes with the same spectral efficiency of 3.33 bit/s/Hz. Comparing with the conventional Gallager mapping and the traditional 32-QAM mapping, our improved Gallager mapping obtain 1.2 dB and 0.2 dB extra shaping gain, respectively.

VI. Conclusion

In this paper, we presented an improved Gallager mapping method for the BICM-ID scheme based on LDPC codes, which could generate Gaussian-like input distributions and improved mapping within each group to obtain extra shaping gain without any other shaping codes. In fact, the proposed scheme could be generalized to any codes with iterative decoding, such as turbo-like codes.

The EXIT chart analyses were used to optimize the mapping patterns and supply some information-theoretic explanations for the proposed method. The threshold of the proposed scheme was improved obviously. Numerical results demonstrated that the proposed scheme were not only much better than conventional Gallager mapping, but also a little bit better than BICM-ID systems based on traditional uniform mapping with the same spectral efficiency.

The advantages of the proposed scheme are manifested in two aspects: 1) BER performance is improved because of the extra shaping gain and coding gain; 2) Higher spectral efficiency can be achieved by using lower-order modulation types, which can decrease the complexity of the signal mapper. The above advantages will make it to be an attractive candidate for future communication systems with turbo-principle-based receivers.

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