Modular Environment for Development and Characterization of Tunable Energy Harvesting Systems

Javier Casatorres-Agüero and Octavio Nieto-Taladriz

***Abstract*—This paper presents the design and development process for an electromagnetic self-tuned vibrational energy harvester prototype. Most state-of-the-art publications present non-tunable or manually tunable vibrational energy harvesters, even the market provides some commercial models of these categories for specific applications. On the other hand, self-tuned energy harvesters are yet rarely seen on the research community. The presented work follows the complete process of designing a prototype to work as a second-order oscillatory system in the form of a cantilever. Three different approaches to tune the resonant frequency of the harvester were considered, each based in changing a property of the cantilever that modifies its resonant frequency. Firstly, it was changed the effective vibrating length of the cantilever. Secondly it was introduced an axial load to the system. Then, the use of a dual cantilever wishbone structure was studied as it allows changing the equivalent stiffness of the system. Finally a prototype based on the first strategy was built and tested, including control algorithms for the maximum electrical energy harvesting point tracking which are presented.**

***Index Terms*—Energy harvesting, self-tuned vibrational energy harvester, Internet of Things, sensor autonomous nodes.**

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I. Introduction

Even though currently there is no "killer application" that demands the very low energy produced by common vibrational energy harvesters (with the notable exception of the automotive industry mainly in the United States, where incorporating tire pressure monitoring systems within the tires of any new vehicle is mandatory [1]), in the context of low-energy nodes that the Internet of Things trend will soon bring, this kind of energy-harvesting devices and technologies are most likely to play a major role as enablers. By providing enough energy for a low power node to operate, autonomy of such nodes would be greatly improved, with the corresponding reduction in the operational costs of a wireless sensor network. However, as there is not a significant demand for such systems nowadays, there exist few commercial solutions yet [2] [3], though many state-of-the-art investigations and prototypes have shown their effectiveness.

Energy harvesting can be achieved in a wide variety of ways, each suiting a different application. While solar modules have the largest energy density (energy per unit volume or mass), they rely on an intrinsically unpredictable source of energy, and are obviously not adequate for indoor applications. Aero-generators lack predictability, too, and are not "down-scalable": there is a minimum size for such systems to operate efficiently. Vibrational energy-harvesters, conversely, are able to become a predictable low-power source even in small scales, as long as the external vibration continues. Besides its wide projected use in automotive applications, this feature makes "*wearables"* a perfect target/perfect targets for mechanical energy harvesters, along with any other general application where motion is available.

There exist three different families of vibrational energy harvesters. Electrostatic vibrational energy harvesters rely on variations in capacitance due to changes in their geometry that appear when a vibration occurs. Their major drawback is their low energy density, and the fact that their capacitance must be pre-charged with an external power supply to operate correctly. Conversely, piezoelectric energy harvesters use the voltage that appears between their plates when submitted to a mechanical effort as a means to produce energy. They are the ones with the highest energy density at a macro and even microscopic level, but don't seem to operate well in the Nano scale, where are outperformed by their electrostatic and electromagnetic counterparts. Furthermore, the piezoelectric materials required are expensive and the resulting harvester is quite complex.

Finally, electromagnetic devices are based on Faraday's Law of Induction: they work by moving a coil within a magnetic field. As coils and permanent magnets have a non-despicable size, electromagnetic harvesters are difficult to miniaturize to micron scales, but have proven to be a simple and promising solution at a macroscopic level.

A typical electromagnetic mechanical energy harvester consists on a cantilever fixed on one end, and with a mass on the other. External vibrations produce a forced oscillation on the cantilever, so that its non-fixed end behaves like a 2nd order spring-mass system. By fixing a permanent magnet in the non-fixed end of the cantilever and a coil in the vibrating base (or vice versa: a coil on the non-fixed end and a permanent magnet on the vibrating base), the relative displacement between the two elements induce an electromotive force in the coil's terminals as predicted by Faraday's Law of Induction. As far as the

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Casatorres-Agüero J. is with the Research Group "B105 Electronic Systems Lab", Departamento de Ingeniería Electrónica, E.T.S.I. de Telecomunicación, Universidad Politécnica de Madrid, SPAIN

Nieto-Taladriz, O. is the Head of the Research Group "B105 Electronic Systems Lab", Departamento de Ingeniería Electrónica, E.T.S.I. de Telecomunicación, Universidad Politécnica de Madrid, SPAIN

external vibration does not stop, the device keeps generating energy, which makes it very adequate to power sensors situated in indoor areas that are difficult to reach, as those in common industrial environments or ship engines.

However, systems as the one described above have a very high quality factor (Q). What this means is if the external vibration frequency doesn't perfectly match the one the energy-harvester has been designed for, the energy production falls dramatically to near-zero values, which makes the device useless, as depicted in Fig.1. Consequently, if the frequency of the external vibration changes over time, the energy production drops to zero. Various strategies are then to overcome the problem, ranging from devices that are manually tunable so they can be manually adjusted to any application, to non-linear harvesters designed to have a much lower Q, so they can produce energy in a wider frequency range. While the former strategy is obviously not suited to those "difficult to reach areas", the latter has the problem of having a much lower power density: devices following that strategy produce much less energy than a linear harvester correctly tuned, as long as the external vibration has one dominant frequency, and is not a broadband signal or white noise.

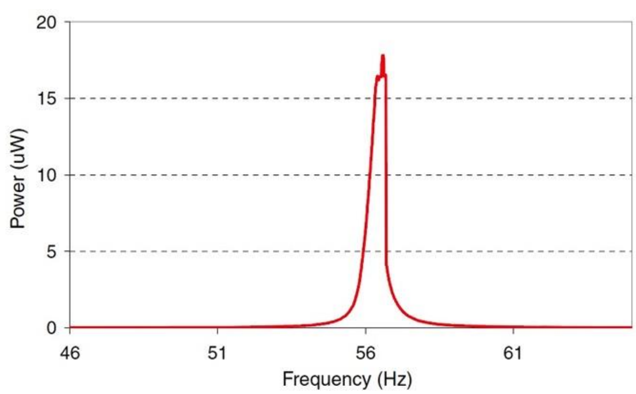

Fig.1. Example of power generation vs. frequency in our test platform

The most promising solution seems to be that harvesters are automatically tuned, without any kind of human intervention. However, this implies both a mechanical tuning mechanism and a closed-loop control system that ensures that the device is operating in its maximum power point, with its subsequent increase in complexity. Furthermore, it leads to the need for some power auto-consumption: some of the energy produced is used to power the tuning mechanism, reducing the amount of energy produced.

In this paper, some strategies for mechanical tuning are presented, along with some algorithms for the control system. A prototype is made and measured, and its problems are shown as guidelines for future research work.

II. Theoretical basis: 2nd order systems and energy harvesting

As has been said, the simplest way to create a vibrational energy harvester is by using a cantilever: a thin plate of material which is clamped to a rigid base on one of its ends, and free at the other, where an inertial mass is fixed. The equations that govern the behavior of such systems are well known, and a full development can be found in [4]. However, some facts should be highlighted:

When the system is under an external vibration, it is forced to oscillate at the same frequency that the external vibration, which is called forced oscillation. The only difference is the phase of the oscillation that appears in the free-end.

- A 2nd order system such as the one described above, when submitted to a punctual excitation, has a response that highly depends on the damping coefficient. For a typical cantilever system, the first mode response is under damped, which means that it tends to oscillate at a given frequency called *damped natural frequency*, with amplitude that decays exponentially over time. If the damping coefficient became zero, the system would oscillate indefinitely at a frequency called *un-damped natural frequency*. Higher order modes are also excited, but their relative power to the first mode is very low. Analysis is usually carried out only for the first mode, as it will often be the only one excited in the frequencies of interest.
- When an under-damped 2nd order system is forced to oscillate at a frequency that matches its un-damped natural frequency, it occurs what is usually called *resonance*, which means that the energy transmitted from the environment to the system has a maximum. It is this state of resonance that frequency tuning seeks in an energy harvester, as produced power is maximum when the un-damped natural frequency matches the external vibration frequency, and nearly zero when it doesn't. The un-damped natural frequency is often called *resonant frequency*, and it does not depend on the damping of the system.
- When the 2nd order system is purely mechanical, the only damping that appears is the mechanical damping (which is function of the characteristics and shape of the cantilever and its free-end mass, as a result of its friction with air). However, when the system produces energy by means of moving a coil in a magnetic field, with a load connected to the terminals of the coil, an electrical damping coefficient appears, to be added to the mechanical damping. It shall, however, not be seen as an undesirable effect, as it is precisely because of that damping that the system generates energy.
- Of the total generated power, characterized by that electrical damping, part is dissipated as electrical losses, and part is dissipated on the load, being the actual generated power, which is what it is intended to maximize. Furthermore, there exists an optimal load that produces the exact electrical damping to extract maximum power from the energy harvester. This is an additional degree of freedom for this kind of systems, where not only the resonant frequency but also the load should be tuned to ensure maximum power is extracted. In the simplest approach, both parameters are addressed independently, as

changes in the value of the load should not affect the resonant frequency of the system.

There is confusion in which the value of the optimal load should be. It is many times stated that the optimal load is the one that matches the value of the conjugated output impedance of the harvester (equal to the impedance of the coil). This maximizes the power dissipated on the load out of the overall electrical power generated, but does not maximize the electrical power.

On the other hand, it is sometimes said that the optimal load makes the electrical and mechanical damping equal. Again, this maximizes the electrical power generated, but most of it is dissipated in non-desirable ways, becoming electrical losses. In [4] it is shown that the optimal load matches the coil impedance plus the electrical equivalent impedance of the mechanical damping. This means that the mechanical damping is modeled as an additional electrical element, with a resistive impedance capable of dissipating power in series with that of the coil. Therefore, the optimal load should be the conjugate of the sum of both impedances, as in any other impedance matching problem.

III. Mechanical solutions for frequency tuning

It has previously been mentioned the extreme importance that matching the resonant frequency of the system with that of the external vibration has in terms of the generated power. The un-damped natural frequency of an under-damped second order system can be computed as in Eq.1.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad with \quad K = \frac{Ywh^3m}{4L^3(m+0,24m_c)}$$

Eq.1

Where m is the equivalent mass, which can be approximated by that of the free end of the cantilever; m_c is the mass of the cantilever, *Y* is its Young's modulus, *L* is its length, *W* is its width and *h* is its thickness.

By simple observation of the above formulas, some techniques for changing the resonant frequency can be suggested. While changes in the mass on the free end of the cantilever, or in its position are extremely difficult to implement effectively, and dynamically modifying the cantilever's width or thickness is nearly impossible, there are not many strategies left. It is possible to change the resonant frequency by changing the *effective length* of the cantilever, meaning the length that vibrates freely. This can be simply achieved by moving the point where the cantilever is attached to the vibrating base, as described in [5].

Another way of changing the resonant frequency can be by changing the *equivalent stiffness* of the system. A method for doing so is presented in [6], where adjusting the distance between the clamped ends of two parallel cantilevers attached in their free-end allows tuning the frequency at which the system resonates.

In addition, if an axial load is applied to the cantilever, its resonant frequency can be shifted [7] according to the expression:

$$f'_r = f_r \cdot \sqrt{1 - \frac{F}{F_b}} \qquad with \ F_b = \frac{\pi^2 \cdot Y \cdot w \cdot h^3}{48 \cdot l^2}$$

Eq.2

Where F_b is the compressive force necessary to buckle the cantilever, or the tensile force that makes it behave like a string.

The easiest way to apply and adjust that axial force is by changing the distance between two permanent magnets, one on the free end of the cantilever and another one at exactly the same height, attached to the vibrating base. By moving the position of the latter, the axial force changes, modifying consequently the resonant frequency of the cantilever.

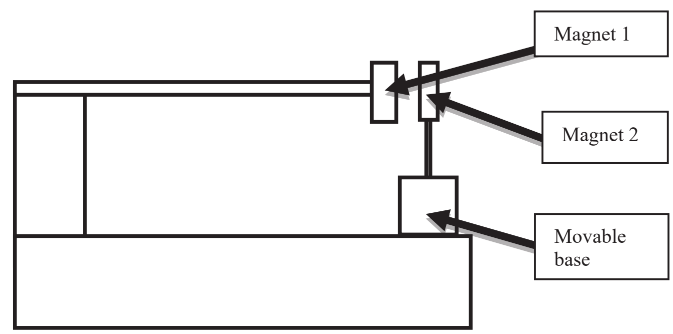

Fig.2. Magnet configuration for axial force tuning

Though the three strategies should be able to dynamically change the resonant frequency while being reasonably easy implementable, there is a major difference among them, namely the tuning range. The system is intended to work in the range of 30 to 80 Hz, as most engine vibrations fall within that interval. To do so, the following values are used to carry out the simulations:

Table 1. Parameters for simulations

Material	Stainless steel 301	
Young's modulus	$1.98 \cdot 10^{11}$	N/m^2
Density	$7.9 \cdot 10^3$	kg/m^3
Length	45	mm
Width	5	mm
Thickness	0.5	mm
Mass on free end	10	g

In the first approach, changing the vibrating length has a dramatic impact on the value of the resonant frequency. A simple plot of the formula shows that if the nominal resonant frequency –i.e. the one that the system has when the whole cantilever vibrates- is adjusted to ~30 Hz, changing the vibrating length to 50% is enough to reach the 80 Hz intended limit:

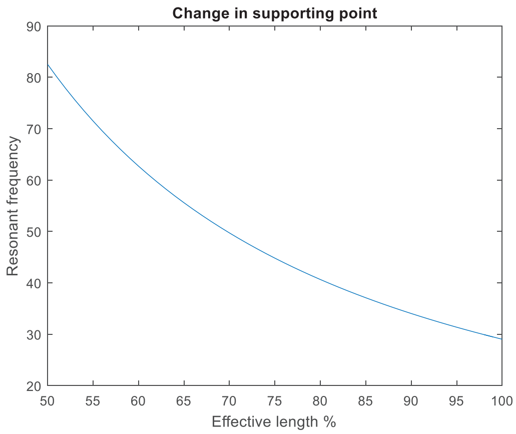

Fig.3. Resonant frequency dependence on vibrating length

One main problem the graph shows is that reducing the equivalent length makes the sensitivity quite large. Consequently, even small deviations in the correct position would yield non-despicable deviations in the desired resonant frequency. In very a high Q system as the one described, the output power would drop. If the total length of the cantilever is about 45 mm, changes of about 1 mm mean a frequency shift of more than 10 Hz.

The second strategy lacks a theoretical model, so simulations are not performed. Despite the curves given in [6], it is intended to build a prototype to evaluate the tuning range that the technique is capable of, setting a frequency of 30 Hz as the mandatory lower end of the interval. This, however falls of the scope of the present article and is under current development.

Finally, the third strategy has a major drawback: if the harvester has to be minimized, the permanent magnets size is not enough to give a reasonable tuning range. To illustrate so, another simulation was performed, using the work presented in [8] with the "*Matlab"* code in [9] to compute the forces between magnets, both in the tensile and the compressive cases. The nominal frequency was set to ~30 Hz in the tensile force case, and to ~40 Hz in the compressive force simulation.

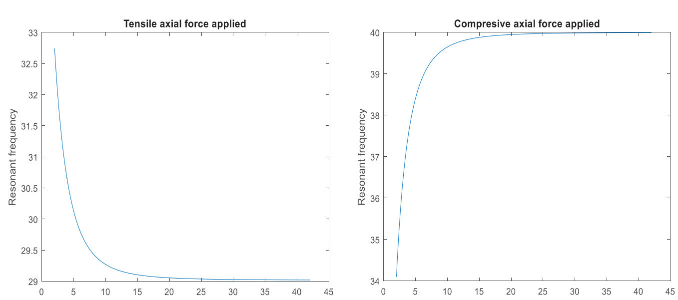

Fig.4. Resonant frequency vs. distance between tuning magnets

The simulated magnets sizes were 5 x 5 x 5 mm for the one on the cantilevers free end, and 5 x 5 x 10 mm for the moving magnet (being the third dimension the one along the movement axis), and they were type N52 magnets, the category that produces the highest magnetic flux per unit volume. As stated before, the tuning range is way too small to be useful. Furthermore, the permanent magnets presence does (though slightly) affect the magnetic flux through the harvester's coil, in a difficult to predict manner

IV. The design environment

In the process of designing a vibrational energy harvester, a design and testing environment has been developed. Its modular nature allows separating the physical design of the harvester from the algorithms to be implemented, and the test-bench used to measure the prototypes.

The physical design has been made through two separate stages. The first stage consists of the mechanical simulation of the tuning strategies where the behavioral model – i.e. the equations that describe its behavior - was available. The simulations have been carried away in "*Matlab"*, and are indispensable to define the parameters of the model (dimensions, mass, material…) to a first degree, which is intended to be refined after measuring the corresponding prototypes. Furthermore, they are useful to discard the strategies that seem less promising, as happened with the axial-load approach. Some of the resulting graphs have been shown above.

Once determined the correct dimensions and topologies to be tested, the second stage starts, where the models are 3D modeled using the software "*SolidWorks®*" so they can be 3D-printed afterwards. Examples of the prototypes that are currently under development can be seen in Fig.5. 3D printing offers a relatively cheap way of optimizing the design, as some parameters that were coarsely determined in the simulations can be finely tuned through measures.

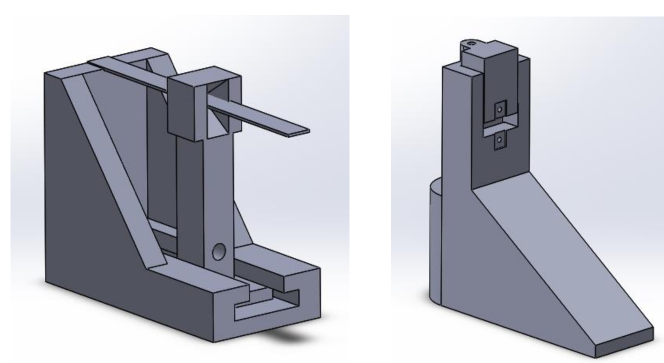

Fig.5. 3D Modeled prototypes. (a) based in changing vibrating length, (b) based in changing equivalent stiffness

As the prototype was designed, a test-bench for measuring was also built. A vibration-generator was acquired and characterized, and some electronics for power conditioning had to be done to allow proper functioning. The design of such test-bench, however, falls of the scope of the present paper, though is of extreme usefulness for the designing process and testing of different algorithms.

V. The prototype

The first prototype, which is presented in this article, uses the first described frequency tuning technique. In order to mitigate the high sensitivity problem described, it is designed so that its length is slightly larger than the one simulated simulate. In particular, its length is 55 mm. The prototype base is built using a 3D printer, with a special piece –tuner from now on-designed to be movable along the prototype's axis, effectively changing the vibrating length. A motor connected to a worm screw maxes this longitudinal movement possible, while some other pieces are printed for support.

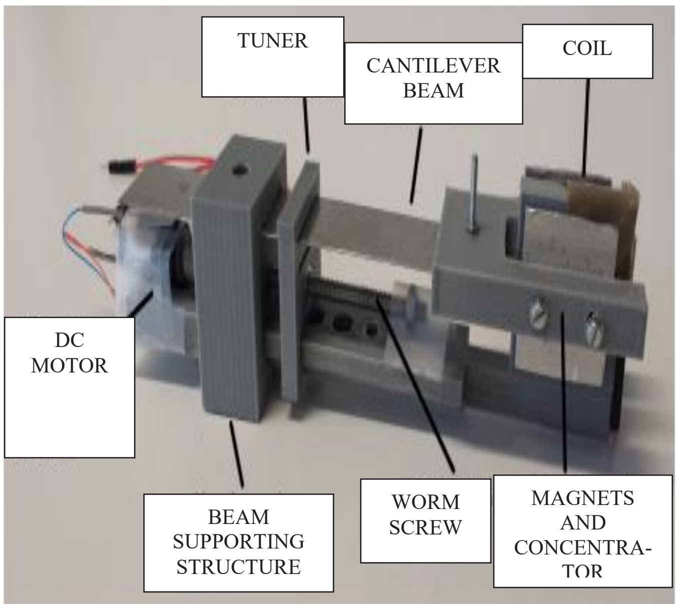

Fig.6. The prototype

The chosen topology for electrical power generation consists of 4 small N52 magnets attached to a u-shaped piece clamped in the free end of the cantilever. A coil is then fixed to the vibrating base so that is between the two arms of the u-shaped piece. The magnets are glued to a pair of metal pieces that act both as support and as a concentrator of the magnetic flux, maximizing the flux that goes through the coil. Fig.7 shows a front view.

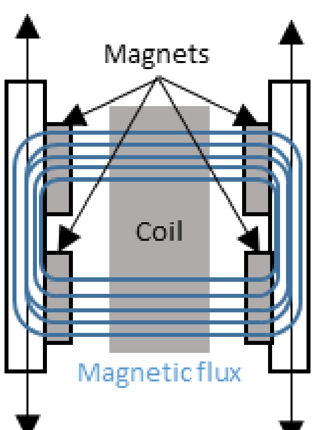

Fig.7. Coil-magnets configuration

The reason why this topology is used, instead of others such as moving the coil while a magnet is inside, or using only one magnet while the coil is directly underneath it, is because it has been proven to maximize the generated power. This experimental result while comparing various coupling topologies can be found in [10].

The obtained output voltage value is very small (~0.5 V peak in the best case), and not constant, which makes it useless for most electronic applications. A voltage multiplier in used to get a rectified voltage high enough to work with. It is used an 8-or-4-stage Villard multiplier, with a switch to change the multiplying factor. "*Schottky"* diodes are used instead of regular PN diodes because the output voltage value without multiplication is lower than a regular diode forward voltage, so the multiplier would not work if using regular diodes. As can be seen from Fig.8., the chosen configuration gives the predicted results, both rectifying and multiplying the output voltage.

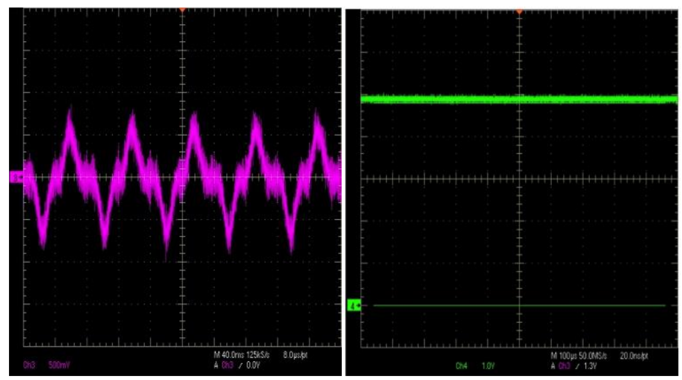

Fig.8. Output voltage (a) before the Villard multiplier (b) after the Villard multiplier

The microcontroller used to implement the tuning algorithm is a Microchip's PIC 16LF1503 [11], whose 30 uA consumption at 1.8 V (54 uW) make it perfectly suitable for controlling the system. Finally, the DC motor that has been chosen to tune the harvester is the model 212-008 of Precision Microdrives, which can take up to 2.5 V [12]. An H bridge is used to change its rotation direction, so that it can both increase and decrease the vibrating length.

Whilst the microcontroller is perfect for the application, the DC motor consumes an amount of power various orders of magnitude above that the harvester can produce, making the total produced energy balance negative unless tuning is very rarely done. Any contemporary macroscopic general-purpose DC motor is likely to have that high consumption problem, so their use for mechanical tuning in this kind of devices won't be a competitive solution until application-specific low power actuators are developed, as those used in common wristwatches.

VI. Tuning algorithms

As has been stated many times before, a second order mechanical energy harvester has a very large Q, so precise self-tuning of its resonant frequency is mandatory. Two stages integrate the tuning algorithm: monitoring to detect if the system is out of tune, and adjusting its resonant frequency if it is. There are two main strategies to follow: those that imply a constant monitoring and adjustment of the operating point while always consuming, and those that rely on periodic sampling and adjusting, thus only consuming power within a fraction of the operating time of the harvester. It is the second strategy which is followed as very tight power consumption constraints must be considered in mechanical energy harvesters.

When monitoring the operating point and detecting whether or not it is the most desirable, both operating frequency and output power can be used. Monitoring the output power – or equivalently the output voltage - has the advantage of being very simple while not needing additional sensors. The voltage level could be measured both before and after the voltage multiplier. If measured before, the algorithm ought to be aware of the operating frequency, or it would detect that the system is out of tune on every cycle. If measured after, it should be taken into account that the response would be slow, as the multiplier capacitors prevent the signal from changing abruptly. Despite the point where the voltage is measured, detecting a voltage drop might trigger an unnecessary tuning process if that drop was caused by a reduction in the amplitude of the external vibration. Using the frequency as an indicator, on the other hand, avoids that problem but requires additional hardware and computing power. A particularly interesting method for detecting the optimal working point lies on the resonance phenomena: when the harvester is perfectly tuned there appears a phase difference between its free end and the vibrating base (or the external vibration) of exactly $\pi/2$. This difference appears in all position, velocity and acceleration signals, which makes possible to use a pair of accelerometers to detect the phase shift. If it is different from $\pi/2$ an adjustment process could be triggered. As it seems to be a promising solution, it is a present line of work.

Once detected a deviation in the operating point, the adjusting algorithm starts. This process is highly dependent on the architecture used for mechanical tuning. For the motor driven tuning used in the prototype, it means powering the motor until the correct vibrating length is reached. Some different approaches can be programmed on the microcontroller. If the environment is well known, so that its different vibration frequencies are characterized, these can be saved in the system's memory so it can power the motor the exact amount of time to get to the correspondent vibrating length. The output voltage is then measured and compared to a threshold to determine whether the frequency is correct, restarting the process if it is not. This would mean that all the frequencies are progressively "checked" according to their probability of occurrence. For the architecture used in the prototype, the time needed for a given displacement pf the tuner can be found as $t = L \cdot \frac{60}{P \cdot RPM}$ being L the displacement, P the pitch of the worm screw (0.7 mm in the prototype) and RPM the angular speed the motor has in *revolutions per minute* (42 in the prototype). This way the time that the motor has to be powered to switch to the next frequency can be computed, as long as both the equivalence between length and resonant frequency, and the current position of the tuner are known. A calibration period is therefore needed to give the microcontroller information about the tuner's initial position, which also needs additional (possibly passive) hardware.

However, in the most generic case there is no knowing at all about the environment. This means that the whole range of vibrating lengths must be gone over when tuning, by implementing a *sweeping* algorithm. The easiest way do this is with a *linear* sweep, by starting at the last resonant frequency known and then increasing the vibrating length until a new resonant frequency, characterized by an increase in the output voltage, is found. If it is not, the sweep should restart from the last resonant frequency, but reducing the vibrating length. No resonant frequency should be found, the system would go to sleep mode to perform a new analysis later.

An *alternating sweep* is far more suited to situations where the external vibration frequency has changed to a slightly lower value. It would first increase the vibrating length slightly, and if no voltage peak were detected, it would start reducing it twice as much as it was previously increased. Again, if no resonant frequency were found, the vibrating length would be increased by 4 times the initial increase. In this two-times increase fashion, the algorithm would solve very quickly the situations where the new resonant frequency is very close (either larger or smaller) than the previous one. However, if the change is big, this algorithm would take an unreasonably large amount of time.

Approaches based on gradient descend would be nearly optimal if the Q of the system were lower, as it is a convex problem. However, trying to use such strategies would mean an enormous sensitivity to minimal voltage changes, as when far from the resonant frequency, the output voltage is extremely low, and the voltage difference between successive steps would be extremely low.

It should be noted, however, that this problems only occur when using the output power – or voltage - to detect if the system is correctly tuned. By using its frequency or the previously described phenomena of phase shift in resonance, it could be easily determined if the vibrating length of the harvester ought to be increased or decreased. Furthermore, if the phase shift is subtracted to $\pi/2$, the error value can be converted into an electrical signal to feed the tuning motor, possibly through a Pulse Width Modulation. A PID controller that optimizes the convergence speed would then be the best solution to solve the problem.

VII. Results

An operational prototype has been built and tested under a controlled environment. The system has been designed to be modular so that different modules and approaches can be built, characterized and tested, Fig.9.

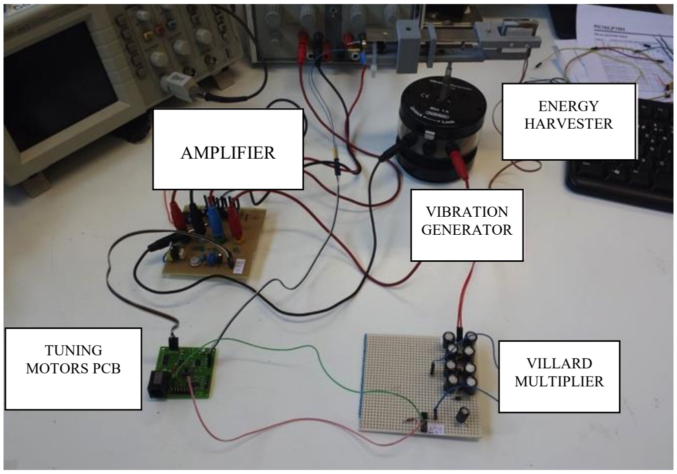

Fig.9. Complete testing environment

First result is that the available energy is quite low in most environments to get it as a usable power supply for the state of the art electronic technology, mainly regarding mechanical actuators. Specific cases as automotive applications where vibrating energy is high will be the first to use these technologies. However, as very low energy nodes are becoming available, this is a quite promising technology in many other fields

Several tuning algorithms have been programmed and tested, and a wide field for new developments is presented in terms of predicting the mechanical spectrum and deciding whether is better to continue in the same frequency or moving to another closer to the optimum operating point.

VIII. Conclusion

A platform for designing, testing and characterization of self-tunable mechanical energy harvesting systems has been designed and a first prototype has been used as validation of the principles presented. Some other mechanical, electronic and algorithmic approaches are under current development.

IX. Future work

Some lines of future work have been presented along the paper. The following are currently under development:

- Miniaturization of the vibrating-length approach prototype and correction of some mechanical issues related.
- Design, construction, testing and characterization of an energy harvester using the wishbone structure presented in [5]
- Optimization of the voltage-level-detection tuning algorithms presented before in terms of their convergence speed and robustness,
- Implementation of a phase-detection tuning algorithm to overcome the problems presented before.
- -Study and optimization of suitable energy storage options.

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