A Hybrid Model Based on Chaos Theory and Artificial Immune Systems for the Analysis and Classification of Stock Market Anomalies

Marko M. Živanović[1], Emilija Kisić[2]
[1]*Faculty of Information Technology, University Metropolitan, Belgrade, Serbia, marko.zivanovic@metropolitan.ac.rs, https://orcid.org/0009-0005-9264-3314*
[2]*Faculty of Information Technology, University Metropolitan, Belgrade, Serbia, emilija.kisic@metropolitan.ac.rs, https://orcid.org/0000-0003-3059-2353*

Original scientific paper
https://doi.org/10.7251/JIT2501015Z UDC: 330.342.14:[34:124.1

Abstract: In this-paper, a system for analyzing chaotic patterns in financial markets has been developed by combining classical chaos metrics with artificial immune systems for anomaly detection. Implemented indicators include the Lyapunov exponent, correlation dimension, approximate entropy, Hurst exponent, and the distance from a reference Lorenz trajectory. These metrics enable the detection of changes in market stability and predictability over time. An adaptive algorithm inspired by artificial immune systems was developed for identifying anomalous behaviors, adjusting detectors based on detected deviations. The results are presented through a series of interactive visualizations, including 3D plots, time series, and anomaly density maps. In addition to standard analysis, the system supports false alarm detection through controlled parameter variations. This approach provides deeper insights into the complex dynamics of financial markets and can serve as a tool for forecasting periods of instability.

Keywords: anomaly detection, artificial immune systems, chaos metrics, financial markets, lorenz attractor, lyapunov exponent

Introduction

The intricate and nonlinear dynamics of financial markets have long challenged researchers seeking to model, predict, and understand their behavior [1]. In particular, the emergence of chaotic patterns [2], characterized by sensitivity to initial conditions and underlying structural complexity, necessitates the development of sophisticated analytical frameworks. Within this context, quantifying chaos using dynamical system metrics—such as the Lyapunov exponent, correlation dimension, approximate entropy, and the Hurst exponent—has proven instrumental in revealing hidden order within seemingly stochastic market behavior [3,4]. This study introduces an integrated computational framework for the detection and analysis of chaotic phenomena in financial time series. By employing a combination of classical chaos theory metrics and novel bio-inspired anomaly detection techniques—specifically, artificial immune system algorithms—this work offers a robust methodology for identifying critical transitions and stability fluctuations in financial markets. The innovative incorporation of Lorenz attractor trajectory comparisons further enhances the model's sensitivity to nonlinear deviations, providing an enriched perspective on temporal evolution and emergent anomalies [5]. The proposed system facilitates both qualitative and quantitative exploration through interactive, multidimensional visualizations, encompassing 3D scatter plots, temporal evolution graphs, and anomaly density heatmaps. Within this context, two distinct adaptive immune detection models are employed to simulate varying market surveillance scenarios—one of which incorporates stochastic false alarm mechanisms to emulate noisy and unpredictable detection environments. The second model operates without false alarm mechanisms, thereby reflecting a more idealized and deterministic surveillance framework for comparative analysis. By integrating traditional chaos theory with quantitative classification based on dynamical system indicators—such as the Lyapunov exponent and the Hurst exponent—the presented approach aims to enhance early warning systems and predictive analytics in financial engineering.

Methodology

In this work, an innovative methodology was developed for analyzing the chaotic characteristics of capital markets by combining mathematical models, chaos-based metrics (such as the Lyapunov and Hurst exponents), and adaptive immune system-inspired detection frameworks. The analysis was carried out through a series of functional components that enable quantitative measurement of nonlinear dynamics in stock price time series, as well as anomaly detection in market behavior. Stock price data were obtained using the Yahoo Finance service, ensuring the timeliness and relevance of the time series for the purposes of the analysis. Each method used is described in detail below. The Lorenz system is a classic mathematical model that describes chaotic behavior. It was created in 1963 when meteorologist Edward Lorenz tried to model atmospheric convection [6]. A particularly notable feature of this system is its extreme sensitivity to initial conditions, where even minimal changes can lead to vastly different outcomes—a hallmark of chaotic behavior.

The Lorenz system is defined by three coupled nonlinear differential equations:

$$\frac{dx}{dt} = \sigma(y - x) \tag{1}$$

$$\frac{dy}{dt} = x(\rho - z) - y \tag{2}$$

$$\frac{dz}{dt} = xy - \beta z \tag{3}$$

Where:
x – position in space (can be seen as the system's state),
y – second coordinate (e.g., rate of change),
z – third coordinate (could represent heat or altitude in atmospheric modeling) [7];

Parameters that control the system's behavior:
σ=10(Prandtl number – measures the ratio of viscosity to thermal diffusivity),
ρ=28(Rayleigh number – measures temperature difference),
β=83(geometric factor – depends on the system's shape);

When these parameters are set to these values, the Lorenz system exhibits pure chaotic behavior — the famous "Lorenz attractor"[8].

Numerical solutions were obtained using the variable-step integration method via the `solve_ivp` function, with initial conditions (x_0, y_0, z_0) = (1.0, 1.0, 1.0) and a time step of dt = 0.01. The resulting trajectory consists of state vectors (*x(t), y(t), z(t)*) at each discrete time point, allowing the creation of a representative pattern of chaotic behavior. After generating the Lorenz trajectory, a function was developed to quantify the similarity between the real-time series of market prices and the reference chaotic trajectory. The market price time window and the x-component of the Lorenz trajectory were independently normalized using standard Z-score normalization:

$$u_{norm} = \frac{u - \mu_u}{\sigma_u + 10^{-8}}, \tag{4}$$

where represents the mean, and the standard deviation of the observed series [9]. Normalization removes the influence of absolute scale, enabling a focus purely on fluctuation patterns.

The similarity between the normalized sequences was then measured using the Euclidean norm:

$$d(u, v) = \sqrt{\sum_{i=1}^{m} (u_i - v_i)^2}, \tag{5}$$

where m is the length of the shorter of the two compared sequences. This metric quantifies the global distance between the two signals, where lower distance values indicate a higher degree of similarity, i.e., a stronger chaotic resemblance between the market window and the Lorenz attractor [10]. In this way, a robust method was created for detecting latent chaotic dynamics within time series of market prices [11]. The choice of the Lorenz system as a reference model is justified by its ability to exhibit extremely sensitive and nonlinear behavior despite its deterministic nature, providing a valid benchmark for comparison with real-world market processes [12].

In this study, four key metrics were applied to quantify chaotic behavior in time series: Approximate Entropy, Hurst Exponent, Maximal Lyapunov Exponent, and Correlation Dimension. Each of these metrics provides a specific perspective on the internal complexity and predictability of temporal processes.

Approximate Entropy (ApEn) measures the regularity and unpredictability of fluctuations in a time series [13]. Formally, ApEn is defined as:

$$ApEn(m, r) = \phi(m) - \phi(m+1) \tag{6}$$

where:

$$\phi(m) = \frac{1}{N-m+1} \sum_{i=1}^{N-m+1} ln\, C_i^m(r) \tag{7}$$

Here, $C_i^m(r)$ represents the proportion of vectors of length mmm that are within a distance *r* from the reference vector *x(i)*. The threshold *r* is usually chosen as a percentage of the standard deviation of the time series.

The distance between two vectors is measured by the maximum absolute difference between their respective components [14].

$$d(x(i), x(j)) = \max_{k=1,2,\dots,m} |x(i+k-1) - x(j+k-1)| \tag{8}$$

Higher values of Approximate Entropy indicate lower predictability and greater chaos within the system.The Hurst Exponent is a measure of long-term memory in a time series [15]. Its interpretation is as follows:

- H=0.5: The process is a random walk (memoryless),
- H>0.5H: Positive autocorrelation (trending behavior),
- H<0.5H: Negative autocorrelation (mean-reverting behavior).

Hurst's relation is expressed through the rescaled range analysis:

$$E[R(n)/S(n)] \propto n^H \tag{9}$$

where *R(n)* is the range of cumulative deviations, *S(n)* is the standard deviation, and nnn is the length of the subseries.

The Maximal Lyapunov Exponent measures the rate of divergence between initially close trajectories in the phase space [16]. Formally:

$$\lambda max = \lim_{t\to\infty} \frac{1}{t} ln \frac{d(t)}{d(0)}, \tag{10}$$

where *d(0)* and *d(t)* are the initial and evolved distances between two nearby points, respectively.

The Correlation Dimension estimates the fractal complexity of a system [17]. It is defined through the correlation function *C(r)* as:

$$C(r) = \lim_{N\to\infty} \frac{2}{N(N-1)} \sum_{i<j} \Theta(r - \|x_i - x_j\| \tag{11}$$

where Θ is the Heaviside step function, and *r* is the distance threshold. In practice, the correlation dimension D_2 is approximated as:

$$D_2 \approx \frac{dlnC(r)}{dlnr} \tag{12}$$

For calculation, the distance matrix between reconstructed phase space vectors is generated, and the number of vector pairs with distances less than *r* is counted, providing an insight into the complexity of the dynamical system [18].

The Artificial Immune System (AIS) is inspired by the biological immune system and is utilized for anomaly detection in complex datasets. Two versions of the AIS algorithm were used here: without false alarms and with false alarms, both based on reactive cloning of detectors [19]. For a dataset *X* = {*x*1, *x*2, ... , *xN*} where each vector instance is defined as:

$$xi = (Lyapunovi, CorrDimi, ApproxEntropyi, Hursti, Lorenz\ Disti) \tag{13}$$

the features are first standardized:

$$z_i = \frac{x_i - \mu}{\sigma} \tag{14}$$

where and are the vectors of mean values and standard deviations of the individual features.

The formulation with false alarms is:

$$Anomaly = (\min_i di > \theta) \vee (rand() < pfalse) \tag{15}$$

where *rand ()* is a uniformly random value from the interval [0,1].

Results

In this study, we analyzed the chaotic dynamics of the stock prices of major technology companies (AAPL, MSFT, GOOGL, NVDA, INTC, AMD, and IBM) [20, 21] using a set of nonlinear time series metrics. The analysis covered the period from January

1, 2020, to April 3, 2025. For each company's closing price time series, a sliding window approach was used with a window size of

$$W = 200 \quad (16)$$

samples and a step size of S = 20. Within each window, the following metrics were calculated:

- Maximum Lyapunov Exponent (λmax), indicating sensitivity to initial conditions.
- Correlation Dimension (D_2), measuring the fractal complexity of the trajectory.
- Approximate Entropy (ApEn), evaluating the unpredictability of the system.
- Hurst Exponent (H), indicating long-term memory and trend persistence.
- Lorenz Distance (d_{Lorenz}), comparing the real data to the reference Lorenz attractor.

The calculated features were then passed through the Artificial Immune System (AIS) algorithm for anomaly detection. Two AIS versions were tested: Normal AIS without induced false alarms, AIS with False Alarms, which introduces 5% random anomalies to simulate realistic detection imperfections. Additionally, each time window was classified into one of several market states based on threshold conditions over the Lyapunov exponent and Hurst exponent. Classification of Market States Based on Lyapunov and Hurst Exponents (Table 1) [22].

Table 1. *Summary of Quantitative Data for Apple Inc.*

Lyapunov	Hurst	Market State
λ>0.3	H<0.3	Very Chaotic
0.1<λ≤0.30	H<0.4	Chaotic
λ<0.05	H>0.7	Highly Predictable
λ<0.1	0.5≤H≤0.70	Stable
0.05≤λ≤0.2	0.4≤H≤0.6	Semi-Stable
otherwise	otherwise	Highly Unstable

A series of visualizations was generated to illustrate the behavior and evolution of chaotic metrics across time for selected stock market symbols. These include time-series plots of individual metrics (Lyapunov exponent, correlation dimension, approximate entropy, Hurst exponent, and Lorenz distance), a 3D scatter plot of the Lyapunov–Correlation Dimension–Lorenz Distance space, as well as anomaly detection visualizations such as heatmaps and scatter diagrams. These visual analyses reveal transitions between different market states and highlight the artificial immune system's effectiveness in identifying anomalies, even when false alarms are introduced, such as during periods of heightened volatility (e.g., the 2020 pandemic shock).

The following images show the results for Apple (Figure1, Figure 2, Figure 3, Figure 4 and Figure 5)

Time	Lyapunov	CorrDim	proxEntro	Hurst	LorenzDist	Anomaly	AnomalyS	MarketSta	nomalyNu
2020-10-1	0,48218	-2,1876	0,54375	0,79571	457,082	FALSE	Normal	Highly Un	0
2020-11-1	0,55553	-2,3507	0,56797	0,78121	457,082	FALSE	Normal	Highly Un	0
2020-12-1	0,59424	-2,2752	0,5324	0,80563	457,082	FALSE	Normal	Highly Un	0
2021-01-1	0,56597	-2,318	0,55487	0,85973	457,082	FALSE	Normal	Highly Un	0
2021-02-1	0,55065	-2,4556	0,40524	0,80464	457,082	FALSE	Normal	Highly Un	0
2021-03-1	0,53503	-2,6871	0,55564	0,70303	457,082	FALSE	Normal	Highly Un	0
2021-04-1	0,42995	-2,8484	0,69205	0,6153	457,082	FALSE	Normal	Highly Un	0
2021-05-1	0,56505	-3,2855	0,79915	0,57556	457,082	FALSE	Normal	Highly Un	0
2021-06-0	0,27624	-3,706	0,88505	0,44629	457,082	FALSE	Normal	Highly Un	0
2021-07-0	0,21742	-3,4778	0,85138	0,50773	457,082	FALSE	Normal	Highly Un	0
2021-08-0	0,25271	-3,2195	0,75521	0,51714	457,082	FALSE	Normal	Highly Un	0
2021-09-0	0,30766	-3,0205	0,7026	0,53207	457,082	FALSE	Normal	Highly Un	0
2021-09-1	0,19298	-2,9285	0,69064	0,52385	457,082	FALSE	Normal	Semi-Stat	0
2021-10-1	0,16061	-2,8654	0,64575	0,57001	457,082	FALSE	Normal	Semi-Stat	0
2021-11-1	0,18411	-2,604	0,57023	0,58575	457,082	FALSE	Normal	Semi-Stat	0
2021-12-1	0,31997	-2,3124	0,42368	0,68238	457,082	FALSE	Normal	Highly Un	0
2022-01-1	0,35081	-2,2495	0,41762	0,7124	457,082	FALSE	Normal	Highly Un	0
2022-02-1	0,38919	-2,2485	0,43006	0,70994	457,082	FALSE	Normal	Highly Un	0
2022-03-1	0,45047	-2,576	0,56611	0,63158	457,082	FALSE	Normal	Highly Un	0
2022-04-1	0,34665	-2,769	0,67216	0,5023	457,082	FALSE	Normal	Highly Un	0
2022-05-1	0,43679	-3,0959	0,75489	0,48007	457,082	FALSE	Normal	Highly Un	0
2022-06-1	0,53181	-3,1681	0,71412	0,50075	457,082	FALSE	Normal	Highly Un	0
2022-07-1	0,56283	-3,2028	0,72007	0,52776	457,082	FALSE	Normal	Highly Un	0
2022-08-1	0,60588	-3,3821	0,77552	0,4859	457,082	FALSE	Normal	Highly Un	0
2022-09-1	0,60027	-3,597	0,79971	0,47895	457,082	FALSE	Normal	Highly Un	0
2022-10-1	0,61928	-3,4412	0,81961	0,48585	457,082	FALSE	Normal	Highly Un	0
2022-11-0	0,63459	-3,6167	0,84956	0,48084	457,082	FALSE	Normal	Highly Un	0
2022-12-0	0,6276	-3,6899	0,87353	0,46443	457,082	FALSE	Normal	Highly Un	0
2023-01-0	0,66281	-3,4697	0,83213	0,51093	457,082	FALSE	Normal	Highly Un	0
2023-02-0	0,60189	-3,6642	0,87401	0,47972	457,082	FALSE	Normal	Highly Un	0
2023-03-0	0,50928	-3,5225	0,86502	0,47583	457,082	FALSE	Normal	Highly Un	0
2023-04-0	0,50767	-3,4444	0,81853	0,53353	457,082	FALSE	Normal	Highly Un	0
2023-05-0	0,50934	-3,3249	0,77097	0,53153	457,082	FALSE	Normal	Highly Un	0
2023-06-0	0,54495	-3,1005	0,68284	0,57694	457,082	FALSE	Normal	Highly Un	0
2023-07-0	0,60887	-2,7762	0,55603	0,65877	457,082	FALSE	Normal	Highly Un	0
2023-08-0	0,65724	-2,5791	0,42076	0,66876	457,082	FALSE	Normal	Highly Un	0
2023-09-1	0,63185	-2,5405	0,3582	0,72823	457,082	FALSE	Normal	Highly Un	0
2023-09-1	0,61843	-2,4995	0,38044	0,7616	457,082	FALSE	Normal	Highly Un	0
2023-10-1	0,51438	-2,4865	0,40677	0,75271	457,082	FALSE	Normal	Highly Un	0
2023-11-1	0,41393	-2,7284	0,53107	0,60626	457,082	FALSE	Normal	Highly Un	0
2023-12-1	0,35881	-2,8138	0,57995	0,63599	457,082	FALSE	Normal	Highly Un	0
2024-01-1	0,35151	-3,086	0,70485	0,50851	457,082	TRUE	Anomalou	Highly Un	1
2024-02-1	0,1458	-3,2609	0,78312	0,43129	457,082	FALSE	Normal	Semi-Stat	0
2024-03-1	0,15715	-3,597	0,81478	0,38629	457,082	FALSE	Normal	Chaotic	0
2024-04-1	0,17632	-3,2669	0,79853	0,38579	457,082	FALSE	Normal	Chaotic	0
2024-05-1	0,22648	-3,3909	0,79214	0,3784	457,082	FALSE	Normal	Chaotic	0
2024-06-1	0,25065	-3,0786	0,72035	0,47807	457,082	FALSE	Normal	Highly Un	0
2024-07-1	0,41159	-2,441	0,51908	0,5555	457,082	FALSE	Normal	Highly Un	0
2024-08-1	0,50241	-2,3166	0,47885	0,4517	457,082	FALSE	Normal	Highly Un	0
2024-09-1	0,33907	-2,2944	0,43347	0,60063	457,082	FALSE	Normal	Highly Un	0
2024-10-0	0,34965	-2,2526	0,42121	0,60915	457,082	FALSE	Normal	Highly Un	0
2024-11-0	0,40586	-2,3145	0,50217	0,58774	457,082	FALSE	Normal	Highly Un	0
2024-12-0	0,61316	-2,0327	0,35441	0,68222	457,082	FALSE	Normal	Highly Un	0
2025-01-0	0,72899	-2,3011	0,57517	0,75507	457,082	FALSE	Normal	Highly Un	0
2025-02-0	0,78423	-2,3541	0,5049	0,75551	457,082	FALSE	Normal	Highly Un	0
2025-03-0	0,66012	-2,8704	0,69222	0,58976	457,082	FALSE	Normal	Highly Un	0
2025-04-0	0,59573	-3,4196	0,84017	0,41427	457,082	FALSE	Normal	Highly Un	0

Figure 1. *Summary of Quantitative Data for Apple Inc.*

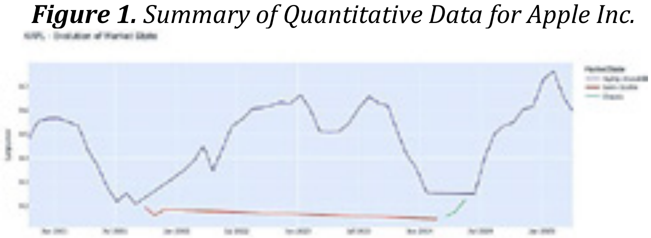


Figure 2. *APPL Evaluation of Market State*

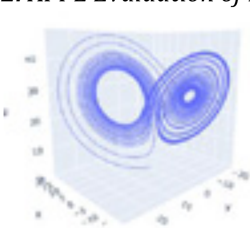


Figure 3. *Lorenz Attractor Reference*

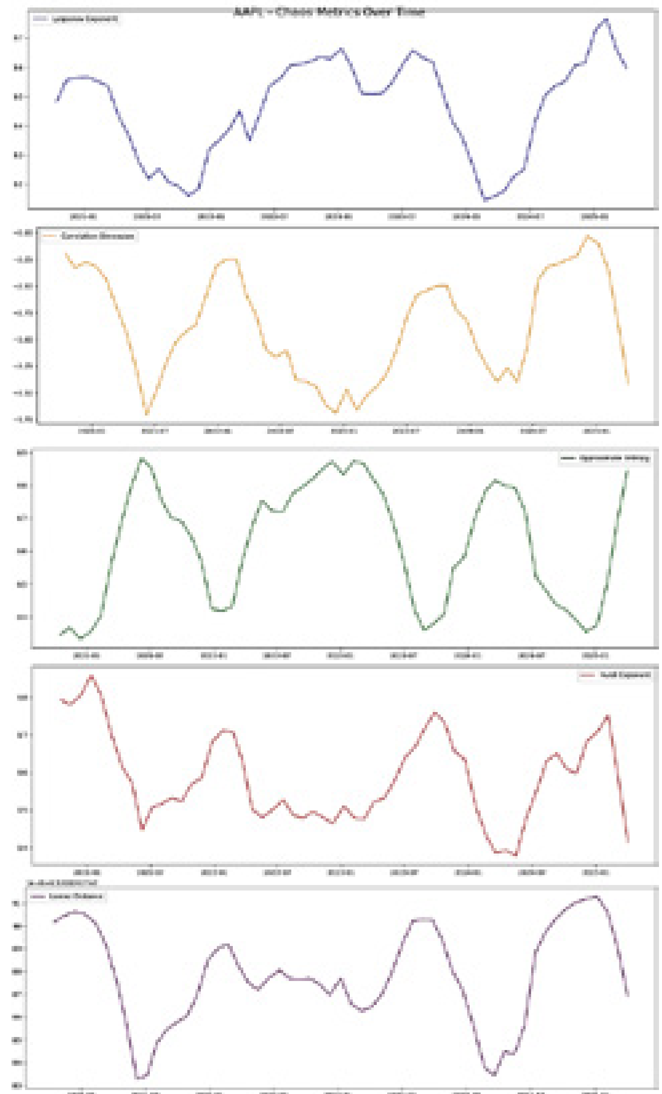


Figure 4. *Chaos Metrics Over Time*

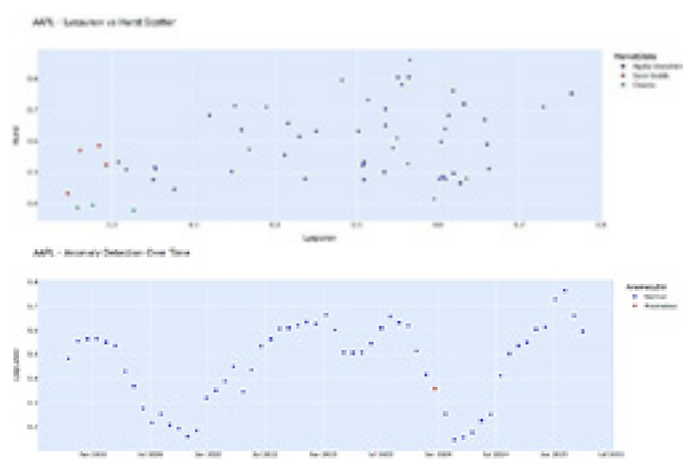


Figure 5. *Lyapunov vs Hurst Scatter and Anomaly Detection over Time for Apple Inc.*

The following images show the results for Microsoft. (Figure 6, Figure 7, Figure 8, Figure 9 and Figure 10).

	Time	Lyapunov	CorrDim	ApproxEntropy	Hurst	LorenzDist	Anomaly	AnomalyStr	MarketState	AnomalyNum
0	2020-10-18	0.864208	-2.682017	0.570056	0.585099	457.082478	False	Normal	Highly Unstable	0
1	2020-11-13	0.908497	-2.701109	0.611944	0.505761	457.082478	False	Normal	Highly Unstable	0
2	2020-12-14	0.820548	-2.513469	0.570249	0.648108	457.082478	False	Normal	Highly Unstable	0
3	2021-01-13	0.751093	-2.704224	0.675080	0.661012	457.082478	False	Normal	Highly Unstable	0
4	2021-02-11	0.669098	-2.934167	0.724048	0.598357	457.082478	False	Normal	Highly Unstable	0
5	2021-03-12	0.701065	-3.142494	0.804076	0.515003	457.082478	False	Normal	Highly Unstable	0
6	2021-04-12	0.650608	-3.184795	0.825943	0.484882	457.082478	False	Normal	Highly Unstable	0
7	2021-05-10	0.712818	-2.921089	0.729886	0.476538	457.082478	False	Normal	Highly Unstable	0
8	2021-06-08	0.753561	-2.964718	0.669436	0.517207	457.082478	True	Anomalous	Highly Unstable	1
9	2021-07-07	0.767830	-2.710929	0.530177	0.615295	457.082478	False	Normal	Highly Unstable	0
10	2021-08-04	0.795441	-2.491429	0.436158	0.671100	457.082478	False	Normal	Highly Unstable	0
11	2021-09-01	0.853120	-2.412409	0.536486	0.756178	457.082478	False	Normal	Highly Unstable	0
12	2021-09-30	0.901489	-2.485607	0.596349	0.756367	457.082478	False	Normal	Highly Unstable	0
13	2021-10-28	0.915244	-2.460702	0.566667	0.752828	457.082478	False	Normal	Highly Unstable	0
14	2021-11-26	0.951850	-2.527647	0.285807	0.760972	457.082478	False	Normal	Highly Unstable	0
15	2021-12-27	0.986164	-2.510089	0.502287	0.784551	457.082478	False	Normal	Highly Unstable	0
16	2022-01-25	0.988751	-2.450215	0.584780	0.790161	457.082478	False	Normal	Highly Unstable	0
17	2022-02-23	1.037871	-2.500981	0.502917	0.708918	457.082478	False	Normal	Highly Unstable	0
18	2022-03-23	0.981474	-2.900877	0.646848	0.619013	457.082478	False	Normal	Highly Unstable	0
19	2022-04-21	0.894979	-3.145974	0.648158	0.456487	457.082478	False	Normal	Highly Unstable	0
20	2022-05-19	1.019042	-3.141138	0.779852	0.481120	457.082478	False	Normal	Highly Unstable	0
21	2022-06-17	1.145721	-3.060681	0.701568	0.553545	457.082478	False	Normal	Highly Unstable	0
22	2022-07-19	1.185809	-3.074215	0.707257	0.544714	457.082478	False	Normal	Highly Unstable	0
23	2022-08-15	1.216830	-3.041215	0.719884	0.553879	457.082478	False	Normal	Highly Unstable	0
24	2022-09-14	1.225081	-3.280755	0.785187	0.505804	457.082478	False	Normal	Highly Unstable	0
25	2022-10-12	1.226250	-3.530770	0.794602	0.538620	457.082478	False	Normal	Highly Unstable	0
26	2022-11-09	1.234830	-3.442801	0.778267	0.514102	457.082478	False	Normal	Highly Unstable	0
27	2022-12-08	1.210647	-3.460152	0.769962	0.542845	457.082478	False	Normal	Highly Unstable	0
28	2023-01-09	1.162100	-3.334152	0.756008	0.555709	457.082478	False	Normal	Highly Unstable	0
29	2023-02-07	1.049384	-3.598479	0.843838	0.459608	457.082478	False	Normal	Highly Unstable	0
30	2023-03-08	0.995462	-3.650621	0.868806	0.471113	457.082478	False	Normal	Highly Unstable	0
31	2023-04-05	1.000873	-3.466106	0.850058	0.406799	457.082478	False	Normal	Highly Unstable	0
32	2023-05-04	1.074886	-3.141106	0.742662	0.514795	457.082478	True	Anomalous	Highly Unstable	1
33	2023-06-02	1.149076	-2.682748	0.623110	0.607050	457.082478	False	Normal	Highly Unstable	0
34	2023-07-05	1.202705	-2.521515	0.489533	0.699241	457.082478	False	Normal	Highly Unstable	0
35	2023-08-01	1.290793	-2.340547	0.422838	0.721780	457.082478	False	Normal	Highly Unstable	0
36	2023-08-29	1.290200	-2.310918	0.395311	0.748010	457.082478	False	Normal	Highly Unstable	0
37	2023-09-27	1.265569	-2.299340	0.386651	0.756183	457.082478	False	Normal	Highly Unstable	0
38	2023-10-25	1.206163	-2.230704	0.432721	0.750061	457.082478	False	Normal	Highly Unstable	0
39	2023-11-22	1.202094	-2.340027	0.452886	0.747497	457.082478	False	Normal	Highly Unstable	0
40	2023-12-21	1.175137	-2.418034	0.454924	0.765458	457.082478	False	Normal	Highly Unstable	0
41	2024-01-23	1.111447	-2.441160	0.470068	0.714646	457.082478	False	Normal	Highly Unstable	0
42	2024-02-21	1.124517	-2.511174	0.488566	0.650300	457.082478	False	Normal	Highly Unstable	0
43	2024-03-20	1.130141	-2.208139	0.437644	0.656068	457.082478	False	Normal	Highly Unstable	0
44	2024-04-18	1.175874	-2.182864	0.366677	0.682658	457.082478	False	Normal	Highly Unstable	0
45	2024-05-16	1.149041	-2.120889	0.348884	0.706774	457.082478	False	Normal	Highly Unstable	0
46	2024-06-14	1.148405	-2.200154	0.380518	0.712386	457.082478	True	Anomalous	Highly Unstable	1
47	2024-07-16	1.255697	-2.285518	0.382470	0.759077	457.082478	False	Normal	Highly Unstable	0
48	2024-08-13	1.234613	-2.610136	0.519382	0.753848	457.082478	False	Normal	Highly Unstable	0
49	2024-09-11	1.085106	-2.881065	0.676735	0.601805	457.082478	False	Normal	Highly Unstable	0
50	2024-10-09	1.087597	-3.270827	0.823788	0.501899	457.082478	False	Normal	Highly Unstable	0
51	2024-11-06	0.974505	-3.611554	0.902828	0.461063	457.082478	False	Normal	Highly Unstable	0
52	2024-12-05	0.949707	-3.613845	0.935565	0.468815	457.082478	False	Normal	Highly Unstable	0
53	2025-01-06	0.972905	-3.587126	0.876578	0.466124	457.082478	False	Normal	Highly Unstable	0
54	2025-02-05	1.010847	-3.609144	0.862471	0.439554	457.082478	False	Normal	Highly Unstable	0
55	2025-03-06	0.988234	-3.529515	0.849170	0.420023	457.082478	False	Normal	Highly Unstable	0
56	2025-04-05	1.111312	-3.280087	0.797710	0.476838	457.082478	False	Normal	Highly Unstable	0

Figure 6. *Summary of Quantitative Data for Microsoft Inc.*

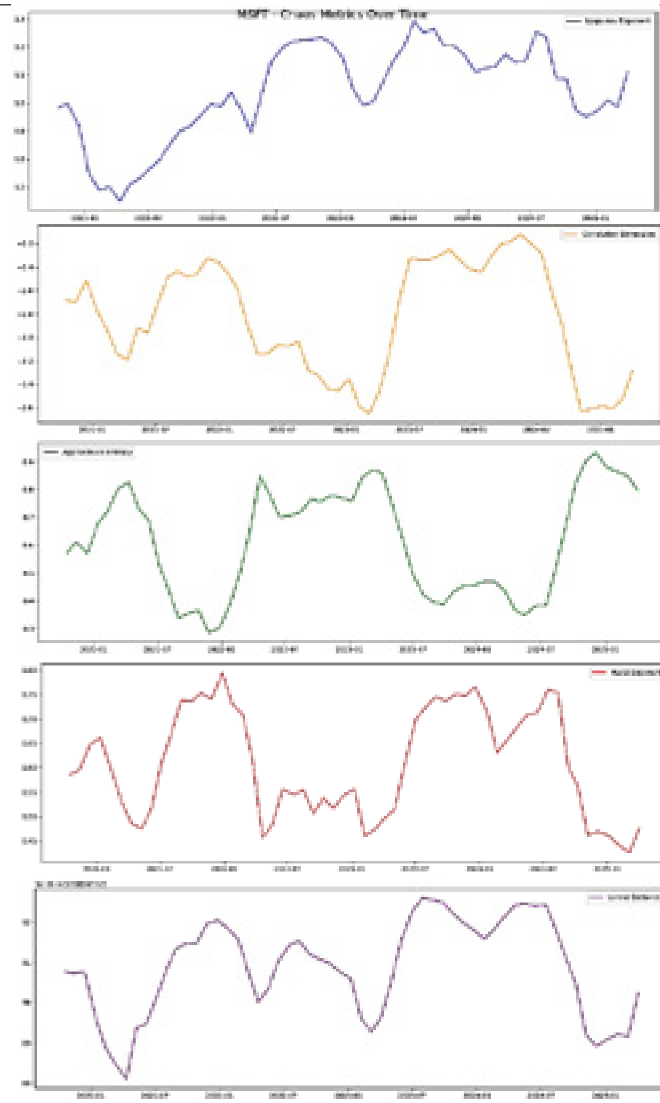


Figure 9. *Chaos Metrics Over Time*

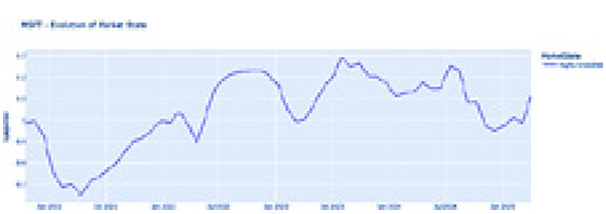


Figure 7. *Evaluation of Market State*

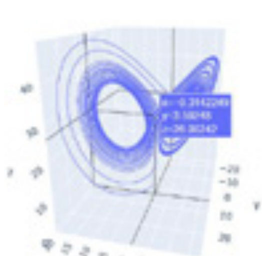


Figure 8. *Lorenz Attractor Reference*

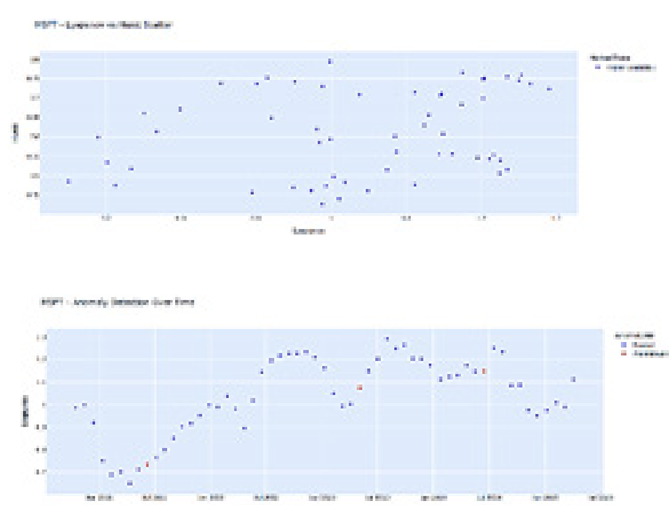


Figure 10. *Lyapunov vs Hurst Scatter and Anomaly Detection over Time for Microsoft Inc.*

The following images show the results for Google
(Figure 11, Figure 12, Figure 13, Figure 14 and Figure 15)

	Time	Lyapunov	CorrDim	ApproxEntropy	Hurst	LorenzDist	Anomaly	AnomalyStr	MarketState	AnomalyNum
0	2020-10-16	-0.182609	-2.651788	0.613614	0.595597	457.062478	True	Anomalous	Stable	1
1	2020-11-13	-0.059585	-2.704930	0.601582	0.603935	457.062478	False	Normal	Stable	0
2	2020-12-14	-0.000017	-2.509485	0.499421	0.666283	457.062478	False	Normal	Stable	0
3	2021-01-13	-0.134695	-2.403047	0.401234	0.763910	457.062478	False	Normal	Highly Predictable	0
4	2021-02-11	-0.142109	-2.365701	0.418772	0.723040	457.062478	False	Normal	Highly Predictable	0
5	2021-03-12	-0.139782	-2.173072	0.407186	0.688217	457.062478	False	Normal	Stable	0
6	2021-04-12	-0.068037	-2.132210	0.393831	0.730600	457.062478	False	Normal	Highly Predictable	0
7	2021-05-10	0.055418	-2.110107	0.328080	0.750482	457.062478	False	Normal	Highly Predictable	0
8	2021-06-08	0.082162	-2.147935	0.309623	0.798282	457.062478	False	Normal	Highly Unstable	0
9	2021-07-07	0.056661	-2.085258	0.251369	0.854776	457.062478	False	Normal	Highly Unstable	0
10	2021-08-04	0.110605	-2.104349	0.289528	0.858547	457.062478	False	Normal	Highly Unstable	0
11	2021-09-01	0.118805	-2.115483	0.229827	0.839183	457.062478	False	Normal	Highly Unstable	0
12	2021-09-30	0.167918	-2.207957	0.241523	0.858904	457.062478	False	Normal	Highly Unstable	0
13	2021-10-28	0.214939	-2.232360	0.299095	0.845906	457.062478	False	Normal	Highly Unstable	0
14	2021-11-26	0.184622	-2.324064	0.332202	0.762957	457.062478	False	Normal	Highly Unstable	0
15	2021-12-27	0.163780	-2.384666	0.382658	0.777586	457.062478	False	Normal	Highly Unstable	0
16	2022-01-25	0.121765	-2.553293	0.524667	0.698136	457.062478	False	Normal	Highly Unstable	0
17	2022-02-23	0.154115	-2.840011	0.642971	0.653592	457.062478	False	Normal	Highly Unstable	0
18	2022-03-23	0.089488	-3.270627	0.816995	0.481072	457.062478	True	Anomalous	Semi-Stable	1
19	2022-04-21	0.077148	-3.660292	0.856515	0.386691	457.062478	False	Normal	Highly Unstable	0
20	2022-05-19	0.209986	-3.142494	0.817436	0.413630	457.062478	False	Normal	Highly Unstable	0
21	2022-06-17	0.344420	-2.794834	0.698093	0.567227	457.062478	False	Normal	Highly Unstable	0
22	2022-07-18	0.409609	-2.829794	0.687636	0.564705	457.062478	False	Normal	Highly Unstable	0
23	2022-08-16	0.432481	-2.812161	0.687605	0.546816	457.062478	False	Normal	Highly Unstable	0
24	2022-09-14	0.452792	-2.938879	0.729222	0.484275	457.062478	False	Normal	Highly Unstable	0
25	2022-10-12	0.522211	-2.946466	0.666497	0.528426	457.062478	False	Normal	Highly Unstable	0
26	2022-11-09	0.579185	-2.898132	0.657696	0.665700	457.062478	False	Normal	Highly Unstable	0
27	2022-12-08	0.518690	-2.847743	0.631665	0.688875	457.062478	False	Normal	Highly Unstable	0
28	2023-01-09	0.483165	-2.794834	0.623524	0.690086	457.062478	False	Normal	Highly Unstable	0
29	2023-02-07	0.346948	-3.180236	0.818538	0.523094	457.062478	False	Normal	Highly Unstable	0
30	2023-03-08	0.329206	-3.249960	0.768630	0.487515	457.062478	False	Normal	Highly Unstable	0
31	2023-04-05	0.307241	-3.278759	0.796628	0.517127	457.062478	False	Normal	Highly Unstable	0
32	2023-05-04	0.246099	-3.318066	0.795554	0.518511	457.062478	False	Normal	Highly Unstable	0
33	2023-06-02	0.233535	-3.129835	0.706110	0.533106	457.062478	False	Normal	Highly Unstable	0
34	2023-07-03	0.305169	-2.748482	0.630636	0.576441	457.062478	False	Normal	Highly Unstable	0
35	2023-08-01	0.278019	-2.610010	0.560556	0.596976	457.062478	False	Normal	Highly Unstable	0
36	2023-08-29	0.281827	-2.453699	0.479480	0.603628	457.062478	False	Normal	Highly Unstable	0
37	2023-09-27	0.331250	-2.361539	0.388517	0.669029	457.062478	False	Normal	Highly Unstable	0
38	2023-10-25	0.328604	-2.352901	0.396910	0.685439	457.062478	False	Normal	Highly Unstable	0
39	2023-11-22	0.268484	-2.350737	0.411866	0.709585	457.062478	False	Normal	Highly Unstable	0
40	2023-12-21	0.197467	-2.416054	0.512175	0.703915	457.062478	False	Normal	Highly Unstable	0
41	2024-01-23	0.131185	-2.666069	0.606219	0.587083	457.062478	False	Normal	Semi-Stable	0
42	2024-02-21	0.228705	-3.005278	0.665771	0.590969	457.062478	False	Normal	Highly Unstable	0
43	2024-03-20	0.167769	-3.274785	0.769261	0.514490	457.062478	False	Normal	Semi-Stable	0
44	2024-04-18	0.235625	-3.061538	0.718282	0.513848	457.062478	False	Normal	Highly Unstable	0
45	2024-05-16	0.286685	-2.626565	0.602744	0.513137	457.062478	False	Normal	Highly Unstable	0
46	2024-06-14	0.361842	-2.321513	0.450115	0.568431	457.062478	False	Normal	Highly Unstable	0
47	2024-07-16	0.508250	-2.265028	0.371844	0.662423	457.062478	False	Normal	Highly Unstable	0
48	2024-08-13	0.599238	-2.512581	0.309950	0.688069	457.062478	False	Normal	Highly Unstable	0
49	2024-09-11	0.596113	-2.729148	0.481335	0.634588	457.062478	False	Normal	Highly Unstable	0
50	2024-10-09	0.550892	-2.836094	0.559799	0.623188	457.062478	False	Normal	Highly Unstable	0
51	2024-11-06	0.535919	-2.830642	0.562920	0.624930	457.062478	False	Normal	Highly Unstable	0
52	2024-12-05	0.522284	-2.866021	0.606662	0.652309	457.062478	False	Normal	Highly Unstable	0
53	2025-01-06	0.475645	-3.026707	0.661605	0.521930	457.062478	False	Normal	Highly Unstable	0
54	2025-02-05	0.523576	-2.869540	0.618642	0.567765	457.062478	False	Normal	Highly Unstable	0
55	2025-03-06	0.544741	-3.011334	0.684325	0.584754	457.062478	False	Normal	Highly Unstable	0
56	2025-04-03	0.582370	-2.846466	0.698706	0.577617	457.062478	False	Normal	Highly Unstable	0

Figure 11. *Summary of Quantitative Data for Google Inc.*

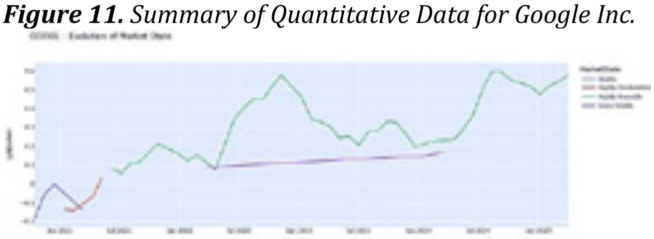


Figure 12. *Evaluation of Market State*

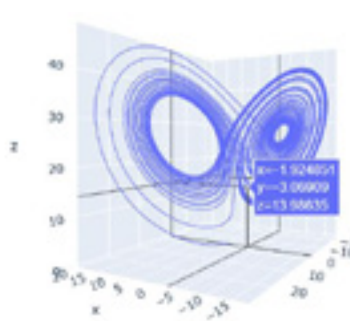


Figure 13. *Lorenz Attractor Reference*

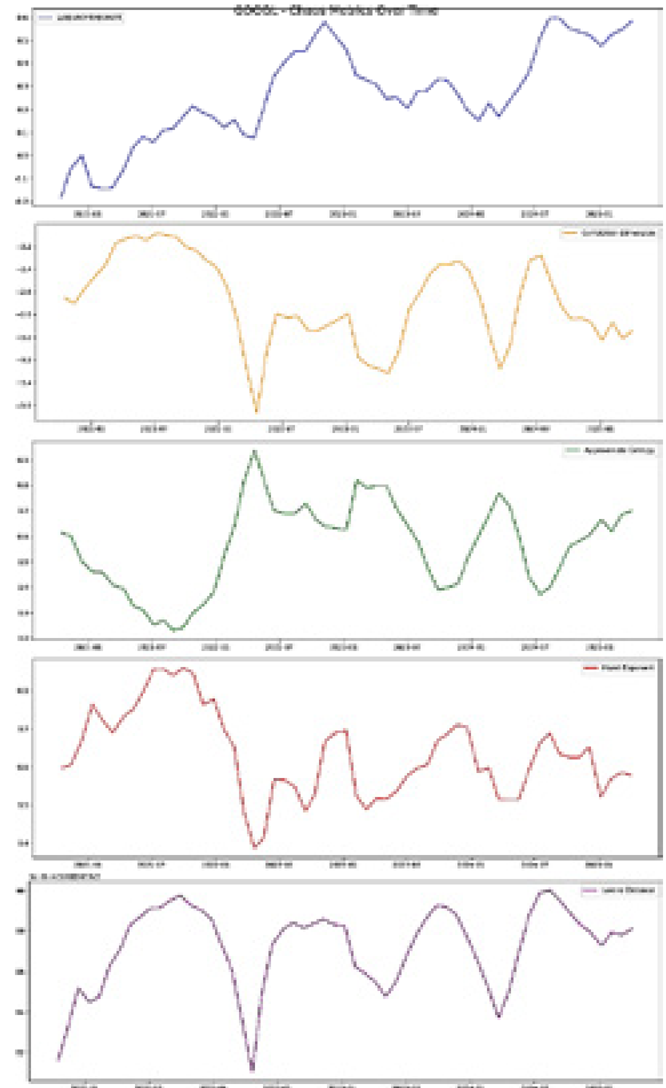


Figure 14. *Chaos Metrics Over Time*

Figure 15. *Lyapunov vs Hurst Scatter and Anomaly Detection over Time for Google Inc.*

The following images show the results for Nvidia (Figure 16, Figure 17, Figure 18, Figure 19 and Figure 20).

Figure 16. *Summary of Quantitative Data for Nvidia Inc.*

	Time	Lyapunov	Corrdim	ApproxEntropy	Hurst	[illegible]	Anomaly	AnomalyStat	MarketState	AnomalyFlag
0	2020-10-16	-1.516418	-2.228038	0.308307	0.809063	457.082478	False	Normal	Highly Predictable	0
1	2020-11-13	-1.414973	-2.367390	0.350006	0.812027	457.082478	True	Anomalous	Highly Predictable	1
2	2020-12-14	-1.454880	-2.300318	0.330000	0.880766	457.082478	False	Normal	Highly Predictable	0
3	2021-01-15	-1.595819	-2.158701	0.348882	0.855106	457.082478	False	Normal	Highly Predictable	0
4	2021-02-11	-1.675495	-2.162606	0.441890	0.785548	457.082478	False	Normal	Highly Predictable	0
5	2021-03-12	-1.607840	-2.483807	0.583010	0.676208	457.082477	False	Normal	Stable	0
6	2021-04-12	-1.728801	-2.848900	0.751219	0.621260	457.082477	False	Normal	Stable	0
7	2021-05-10	-1.704675	-3.220751	0.798819	0.530165	457.082477	False	Normal	Stable	0
8	2021-06-08	-1.727176	-3.350267	0.814041	0.460051	457.082477	False	Normal	Highly Unstable	0
9	2021-07-07	-1.593036	-2.288009	0.627718	0.578618	457.082477	False	Normal	Stable	0
10	2021-08-04	-1.309227	-1.998020	0.479614	0.672799	457.082478	False	Normal	Stable	0
11	2021-09-01	-1.393244	-1.907573	0.374701	0.710051	457.082478	False	Normal	Highly Predictable	0
12	2021-09-30	-1.281487	-2.104549	0.365668	0.728695	457.082478	False	Normal	Highly Predictable	0
13	2021-10-28	-1.159940	-2.291398	0.379151	0.740410	457.082478	False	Normal	Highly Predictable	0
14	2021-11-26	-0.942138	-2.120002	0.256859	0.825054	457.082478	False	Normal	Highly Predictable	0
15	2021-12-27	-0.836678	-2.154812	0.289842	0.911811	457.082478	False	Normal	Highly Predictable	0
16	2022-01-25	-0.778259	-2.258765	0.345118	0.845460	457.082478	False	Normal	Highly Predictable	0
17	2022-02-23	-0.705317	-2.454865	0.425344	0.805969	457.082478	False	Normal	Highly Predictable	0
18	2022-03-23	-0.746869	-2.595280	0.574134	0.641656	457.082478	False	Normal	Stable	0
19	2022-04-21	-0.729123	-2.787499	0.674740	0.608298	457.082478	False	Normal	Stable	0
20	2022-05-19	-0.625910	-2.865189	0.684790	0.595312	457.082478	False	Normal	Stable	0
21	2022-06-17	-0.565070	-2.971528	0.694871	0.677859	457.082478	False	Normal	Stable	0
22	2022-07-19	-0.485207	-2.974460	0.605125	0.673475	457.082478	False	Normal	Stable	0
23	2022-08-16	-0.305065	-2.798017	0.584445	0.685281	457.082478	False	Normal	Stable	0
24	2022-09-14	-0.317700	-2.708675	0.555452	0.705523	457.082478	False	Normal	Highly Predictable	0
25	2022-10-12	-0.600521	-2.714697	0.517695	0.706714	457.082478	False	Normal	Highly Predictable	0
26	2022-11-09	-0.685207	-2.704930	0.566237	0.751330	457.082478	False	Normal	Highly Predictable	0
27	2022-12-08	-0.765016	-2.691362	0.585979	0.768341	457.082478	False	Normal	Highly Predictable	0
28	2023-01-09	-0.879905	-2.797291	0.640982	0.771222	457.082478	False	Normal	Highly Predictable	0
29	2023-02-07	-1.029035	-3.502400	0.818050	0.592310	457.082478	False	Normal	Stable	0
30	2023-03-08	-0.930866	-3.075317	0.703175	0.620882	457.082478	False	Normal	Stable	0
31	2023-04-05	-0.822256	-2.600002	0.502254	0.738480	457.082478	False	Normal	Highly Predictable	0
32	2023-05-04	-0.785176	-2.378657	0.415309	0.807818	457.082478	False	Normal	Highly Predictable	0
33	2023-06-02	-0.630976	-2.136012	0.291772	0.882097	457.082478	False	Normal	Highly Predictable	0
34	2023-07-03	-0.510774	-2.018346	0.190828	1.007162	457.082478	False	Normal	Highly Predictable	0
35	2023-08-01	-0.389524	-2.032749	0.174076	1.058478	457.082478	False	Normal	Highly Predictable	0
36	2023-08-29	-0.383713	-1.990481	0.205969	0.950089	457.082478	False	Normal	Highly Predictable	0
37	2023-09-27	-0.310673	-2.078242	0.226382	0.959898	457.082478	False	Normal	Highly Predictable	0
38	2023-10-25	-0.347659	-1.999317	0.261049	0.942420	457.082478	False	Normal	Highly Predictable	0
39	2023-11-22	-0.420100	-2.012467	0.302969	0.819059	457.082478	False	Normal	Highly Predictable	0
40	2023-12-21	-0.457296	-2.058835	0.353051	0.748656	457.082478	True	Anomalous	Highly Predictable	1
41	2024-01-23	-0.405625	-2.207048	0.414155	0.735788	457.082478	False	Normal	Highly Predictable	0
42	2024-02-21	-0.248144	-2.028939	0.339522	0.780970	457.082478	False	Normal	Highly Predictable	0
43	2024-03-20	-0.132351	-1.538388	0.240259	0.799397	457.082478	False	Normal	Highly Predictable	0
44	2024-04-18	-0.053887	-1.414903	0.208091	0.804111	457.082478	False	Normal	Highly Predictable	0
45	2024-05-16	0.094800	-1.595805	0.237021	0.791629	457.082478	False	Normal	Highly Unstable	0
46	2024-06-14	0.315525	-1.691326	0.193055	0.858122	457.082478	False	Normal	Highly Unstable	0
47	2024-07-16	0.512795	-1.828583	0.195848	0.956858	457.082478	False	Normal	Highly Unstable	0
48	2024-08-13	0.667450	-2.092527	0.272451	0.972230	457.082478	False	Normal	Highly Unstable	0
49	2024-09-11	0.751216	-2.273755	0.340229	0.898762	457.082478	True	Anomalous	Highly Unstable	1
50	2024-10-09	0.811011	-2.437546	0.426100	0.812314	457.082478	False	Normal	Highly Unstable	0
51	2024-11-06	0.852775	-2.583954	0.537027	0.735076	457.082478	False	Normal	Highly Unstable	0
52	2024-12-05	0.823278	-2.671116	0.589112	0.704049	457.082478	False	Normal	Highly Unstable	0
53	2025-01-06	0.820088	-2.803389	0.656433	0.662118	457.082478	True	Anomalous	Highly Unstable	1
54	2025-02-05	0.859716	-3.165928	0.711002	0.650640	457.082478	False	Normal	Highly Unstable	0
55	2025-03-06	0.752238	-3.718308	0.924813	0.426672	457.082478	False	Normal	Highly Unstable	0
56	2025-04-03	0.718797	-3.831598	0.914838	0.404673	457.082478	False	Normal	Highly Unstable	0

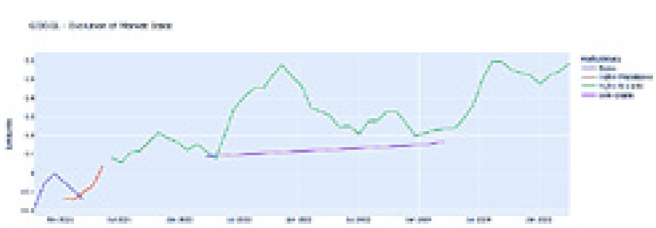


Figure 17. *Evaluation of Market State*

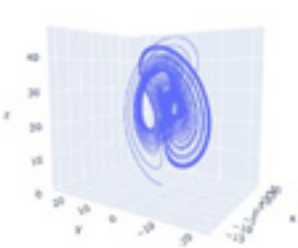


Figure 18. *Lorenz Attractor Reference*

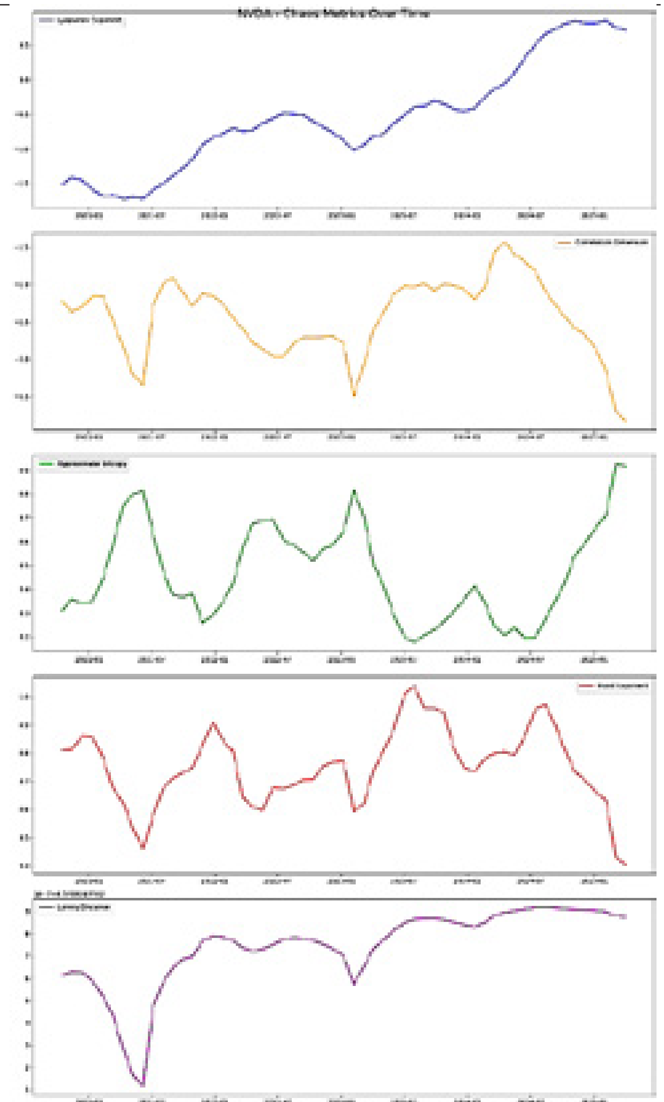


Figure 19. *Chaos Metrics Over Time*

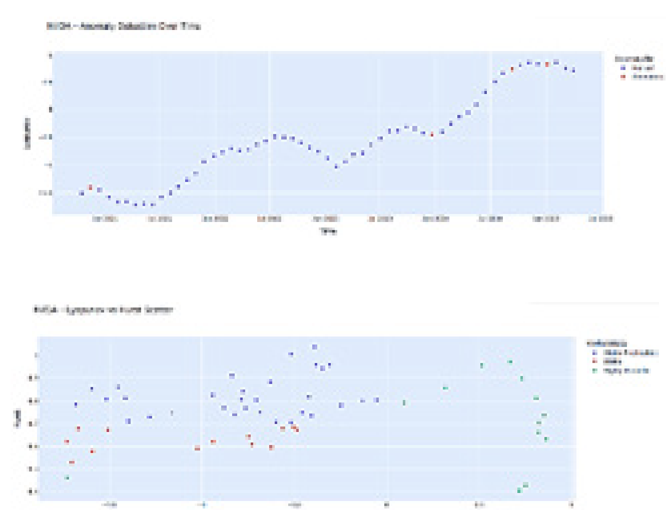


Figure 20. *Lyapunov vs Hurst Scatter and Anomaly Detection over Time for Nvidia Inc.*

The following images show the results for Intel. (Figure 21, Figure 22, Figure 23, Figure 24 and Figure 25).

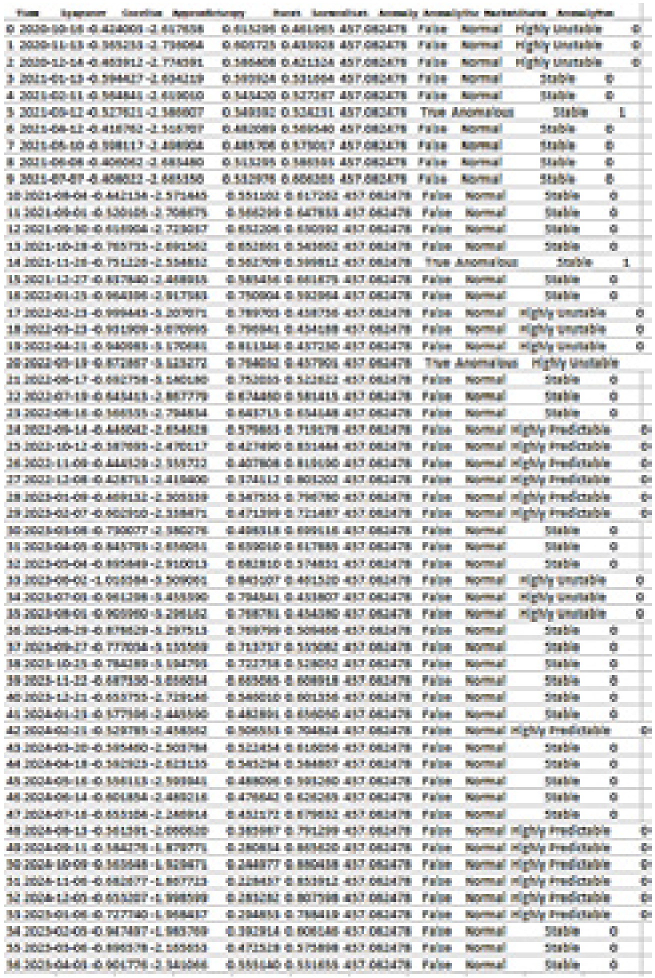


Figure 21. *Summary of Quantitative Data for Intel Inc.*

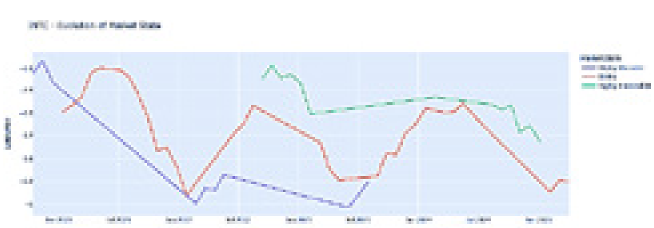


Figure 22. *Evaluation of Market State*

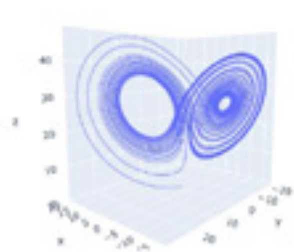


Figure 23. *Lorenz Attractor Reference*

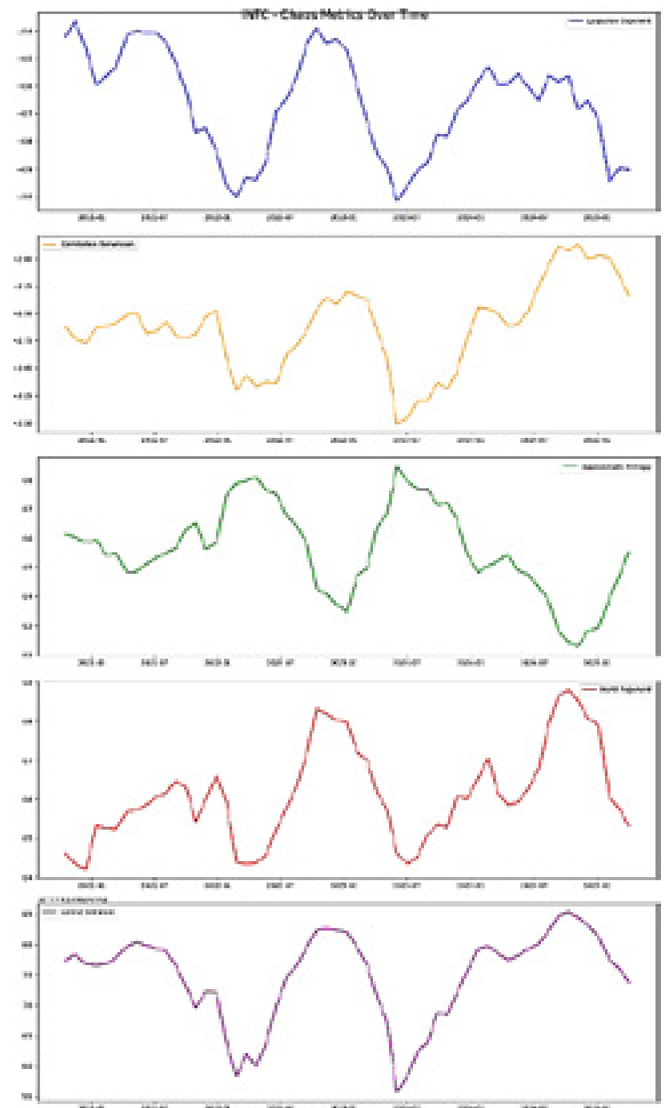


Figure 24. *Chaos Metrics Over Time*

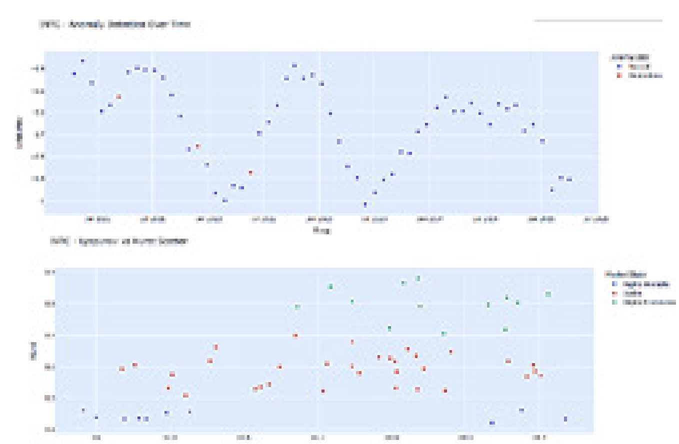


Figure 25. *Lyapunov vs Hurst Scatter and Anomaly Detection over Time for Intel Inc.*

The following images show the results for AMD (Figure 26, Figure 27, Figure 28, Figure 29 and Figure 30).

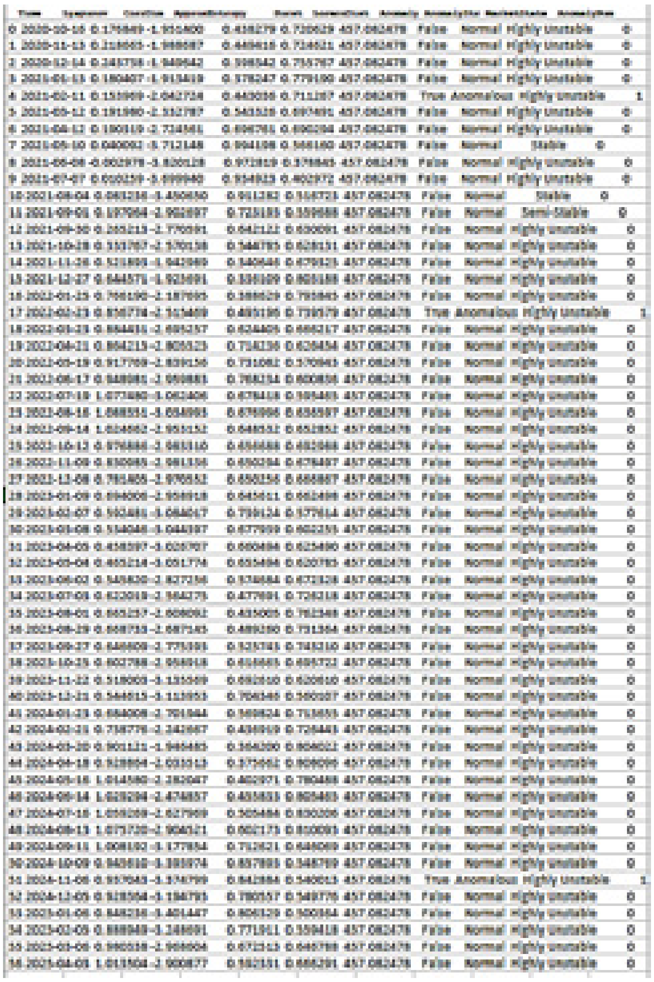


Figure 26. *Summary of Quantitative Data for AMD Inc.*

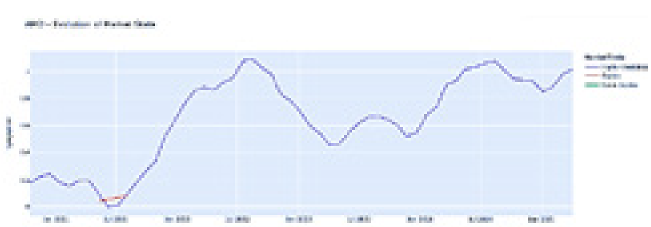


Figure 27. *Evaluation of Market State*

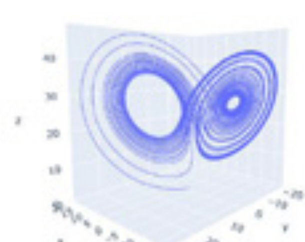


Figure 28. *Lorenz Attractor Reference*

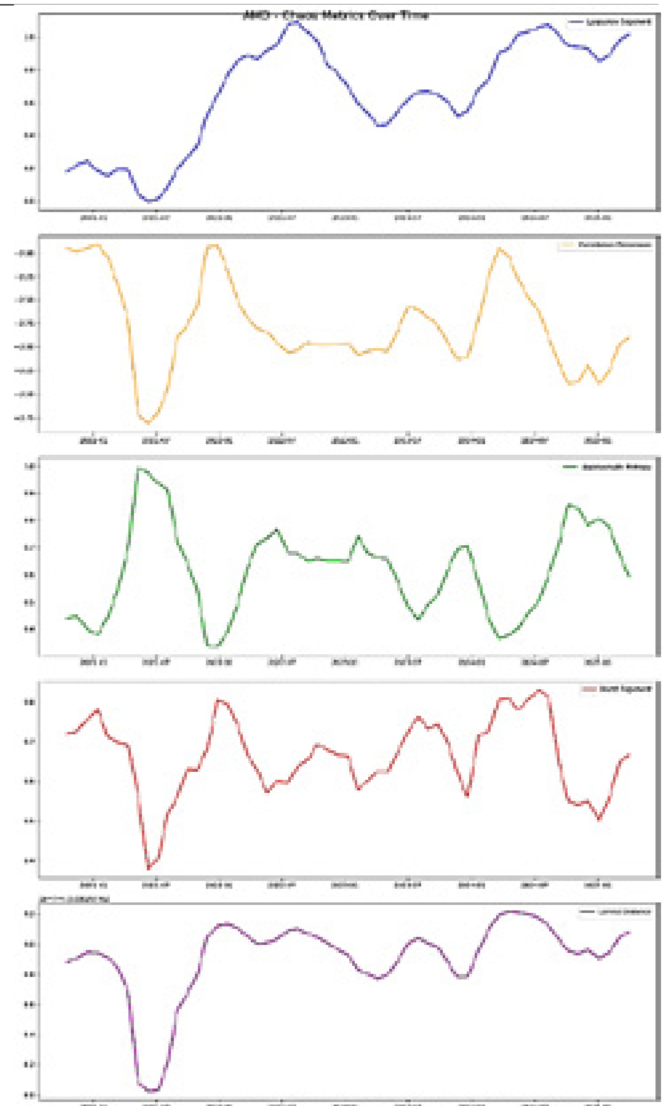


Figure 29. *Chaos Metrics Over Time*

Figure 30. *Lyapunov vs Hurst Scatter and Anomaly Detection over Time for AMD Inc.*

The following images show the results for IBM. (Figure 31, Figure 32, Figure 33, Figure 34 and Figure 35).

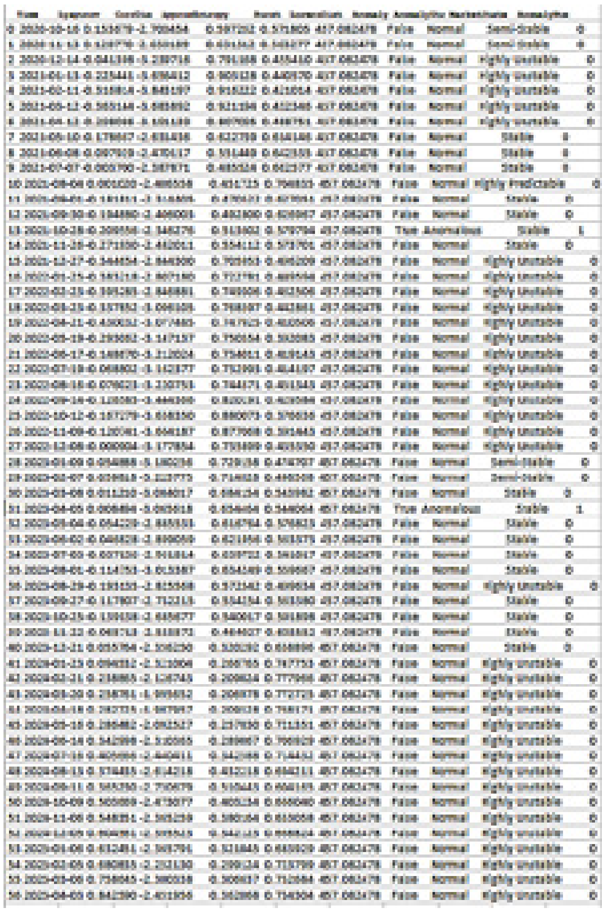


Figure 31. *Summary of Quantitative Data for IBM Inc*

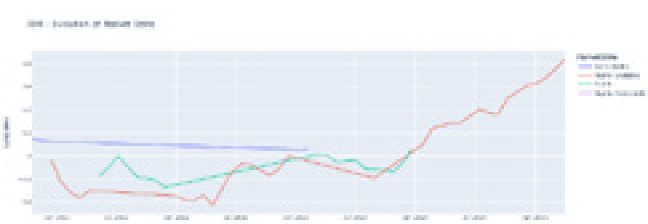


Figure 32. *Evaluation of Market State*

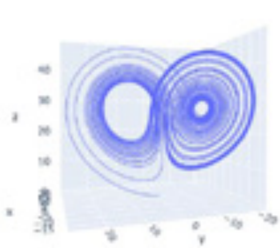


Figure 33. *Lorenz Attractor Reference*

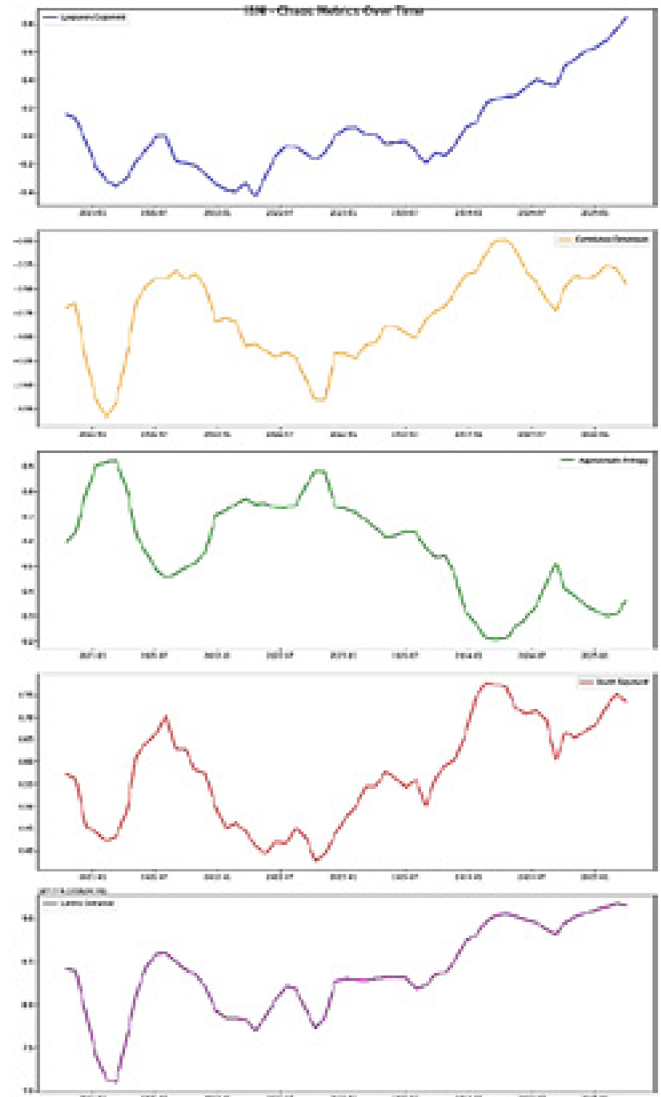


Figure 34. *Chaos Metrics Over Time*

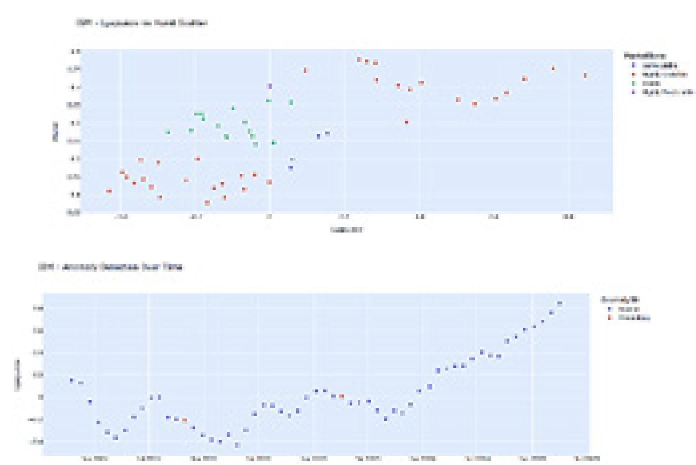


Figure 35. *Lyapunov vs Hurst Scatter and Anomaly Detection over Time for IBM Inc*

Discussion

The combination of chaotic metrics (such as the Lyapunov exponent, correlation dimension, approximate entropy, Hurst exponent, and Lorenz distance) with an artificial immune system enables efficient market classification based on various dynamic states. This methodology not only uncovers the chaotic characteristics of the market but also allows for market classification by identifying stable and unstable patterns. In this work, the algorithms are applied in two modes: one without false alarms, and another where false alarms are introduced to test the system's robustness under conditions of high volatility. The discussion below thoroughly examines the classification results by company, clearly indicating the use of both algorithms. For a summarized overview of the key metrics for each company, please refer to **Table 2**. Time series analysis for Apple indicates the presence of chaotic yet predictable patterns in market behavior. The Lyapunov exponent, with values ranging between 0.48 and 0.56, confirms the system's divergence and the presence of chaos, which is characteristic of dynamic and nonlinear systems. However, the Approximate Entropy, ranging from 0.33 to 0.40, shows a relatively low level of entropy, suggesting that price movement patterns are somewhat predictable. A high Hurst exponent (~0.79–0.86) further suggests the existence of long-term dependence and stable trends—when the price rises, there is a high probability that the upward trend will persist. The correlation dimension, although negative (likely due to a scaling error), indicates a high level of complexity in market behavior. The Lorenz distance, with a constant value around 457, points to a stable attractor distribution, and the absence of anomalies confirms that the market chaos is unfolding within expected bounds. Time series analysis for Microsoft reveals more pronounced chaotic characteristics compared to Apple. The Lyapunov exponent ranges from 0.68 to 1.0, indicating a higher degree of chaos and greater system divergence. The correlation dimension, with values between -2.5 and -2.9, also suggests a high level of complexity, although the negative values are likely due to a scaling error. Approximate Entropy, in the range of 0.57 to 0.72, reflects greater unpredictability of patterns compared to Apple, meaning that Microsoft's market behavior is harder to model. The Hurst exponent remains relatively high (0.69–0.76), confirming the presence of long-term dependencies, although somewhat less pronounced than in Apple's case. The Lorenz distance indicates lower attractor stability compared to Apple, further contributing to the depiction of a more dynamic and potentially more volatile market. Nevertheless, despite the stronger chaotic behavior, no anomalies were detected, indicating that Microsoft's market behavior—though complex and unpredictable—still occurs within expected bounds. Microsoft exhibits stronger chaotic characteristics and lower predictability compared to Apple, while maintaining fundamental structural stability. The Lyapunov exponent is negative (ranging from -0.18 to -0.00), indicating that the system is not divergent and does not exhibit chaotic characteristics—instead, the behavior is stable and predictable. The Hurst exponent, ranging from 0.5 to 0.6, suggests behavior close to a random walk, with no pronounced long-term dependence. Approximate Entropy falls within a moderate range (0.45–0.61), indicating a medium level of unpredictability—higher than Apple's, but lower than Microsoft's. The correlation dimension points to a complex structure, similar to the previous companies, suggesting multilayered dynamics despite the absence of chaos. The Lorenz distance remains stable, supporting the existence of a consistent attractor structure over time. No anomalies were recorded, further confirming the consistency of market behavior. Time series analysis for Google shows more stable dynamic behavior compared to Apple and Microsoft. In conclusion, Google stands out as a system with stable and relatively predictable patterns, lacking chaos and exhibiting less long-term dependence compared to Apple and Microsoft.The Lyapunov exponent for NVIDIA has extremely negative values (~-1.5 to -1.6), indicating an exceptionally stable system with no signs of divergence. The Hurst exponent ranges from 0.6 to 0.7, suggesting the presence of mild, mostly upward trends in the time series. Approximate Entropy, ranging from 0.30 to 0.44, indicates a relatively low level of unpredictability, meaning that behavioral patterns are clearly present and can be modeled with relative ease. NVIDIA demonstrates a high degree of stability with moderate trends and low entropy, making it a system with well-defined and predictable dynamics. The Lyapunov exponent for Intel ranges between -0.42 and -0.59, indicating stable system behavior without signs of divergence, though not as extremely stable as in NVIDIA's case. Approximate Entropy, ranging from 0.54 to 0.61, indicates a moderate level of entropy, meaning that Intel exhibits a moderate degree of predictability—patterns are present but not fully clearly defined. Approximate Entropy for AMD, ranging from 0.37 to 0.44, indicates a moderate level of predictability—behavioral patterns are present but not fully stable. The Lyapunov exponent, ranging from 0.17 to 0.24, shows a slightly divergent system with low but positive values, indicating a certain degree of chaotic behavior. In conclusion, AMD's market behavior is characterized by a balance between predictable patterns and mild instability, making it a moderate yet dynamic system. For IBM, Approximate Entropy shows a significant increase—from 0.59 to 0.91—which clearly indicates growing unpredictability in market behavior patterns. At the same time, the Lyapunov exponent shifts from positive (0.15) to negative values (-0.31), signaling a transition of the system from a mildly chaotic state toward more stable dynamics. This combination points to a complex change: while the system's structure is stabilizing in terms of divergence, its local patterns are becoming increasingly irregular and harder to predict. IBM is in a specific transitional phase—structurally moving toward stability, while simultaneously experiencing an increase in internal chaos.

Table 2. *Summary of Chaotic Metrics by Company*

Company	Lyapunov Exponent	Approx. Entropy	Hurst Exponent	Corr. Dimension	Lorenz Distance
AAPL	0.48 – 0.56	0.33 – 0.40	0.79 – 0.86	(error)	~457
MSFT	0.68 – 1.00	0.57 – 0.72	0.69 – 0.76	-2.5 to -2.9	Lower than Apple
GOOGL	-0.18 – -0.00	0.45 – 0.61	0.50 – 0.60	High complexity	Stable
NVDA	-1.5 – -1.6	0.30 – 0.44	0.60 – 0.70	N/A	Stable
INTC	-0.42 – -0.59	0.54 – 0.61	N/A	N/A	Stable
AMD	0.17 – 0.24	0.37 – 0.44	N/A	N/A	N/A
IBM	0.15 to -0.31	0.59 – 0.91	N/A	N/A	Stable

Microsoft showed the highest stability in terms of long-term predictability, indicated by its negative Lyapunov exponents and relatively low entropy values, suggesting a more consistent and predictable market behavior. Apple, on the other hand, demonstrated the best balance between growth and predictability, with chaotic traits combined with long-term stability and low entropy, indicating the potential for both stable trends and growth opportunities. NVIDIA and Google exhibited negative Lyapunov exponents and low to moderate entropy, reflecting their relatively stable and predictable dynamics, though their market behavior was somewhat less dynamic compared to companies like Apple and Microsoft. AMD, with more pronounced chaotic characteristics and lower predictability, was better suited for short-term and active trading strategies. Intel, offering moderate stability without significant fluctuations, represents a more conservative option with relatively predictable behavior. IBM, however, showed a sharp increase in entropy, signaling growing unpredictability despite indications of structural stability, suggesting that it may not be ideal for long-term positions.

Conclusion and Future Work

This work presents a comprehensive system for analyzing chaotic patterns in financial markets, combining classical chaos theory metrics with artificial immune system algorithms for anomaly detection and market classification. The system not only detects chaotic behaviors but also classifies market states into categories such as "chaotic," "stable," or "predictable," based on the calculated metrics. By utilizing indicators such as the Lyapunov exponent, correlation dimension, approximate entropy, Hurst exponent, and the distance from a reference Lorenz trajectory, the system enables both quantitative and qualitative assessment of market stability, predictability, and dynamic transitions between different market states over time. This classification framework provides a deeper understanding of market behavior, highlighting periods of instability and offering insights for market prediction and risk assessment. The analysis reveals clear differences in the dynamic behavior of the companies under consideration. While Apple and Microsoft exhibit more pronounced chaotic characteristics—marked by high Lyapunov and Hurst exponents indicating long-term dependencies—companies like NVIDIA and Google demonstrate more stable and predictable behavioral patterns. Particularly notable is IBM, which seems to be in a transitional phase—shifting from mild chaos towards greater structural stability, while also experiencing an increase in short-term unpredictability.

From an investment strategy perspective, the results enable a practical classification of market options. If maximum stability is the goal, NVIDIA and Google stand out as the most reliable choices due to their negative Lyapunov exponents and low to moderate entropy values, indicating consistent and predictable dynamics. For those seeking a balance between growth and predictability, Apple emerges as the optimal option—exhibiting chaotic traits along with stable long-term trends and low entropy. Microsoft and AMD, with more pronounced chaotic behavior and lower predictability, are better suited for active trading and short-term strategies. Intel offers a more conservative option—stable and moderately predictable, without significant fluctuations. After results analysis we can conclude that IBM is not recommended for long-term positions due to a sharp increase in entropy, which points to growing unpredictability despite signs of structural stabilization. The proposed algorithm, a combination of artificial immune systems and chaos theory metrics, proved effective in detecting anomalous behavior and dynamic shifts without generating false alarms, further confirming the robustness of the proposed system. Interactive visualizations enable intuitive interpretation of complex results and contribute to a better understanding of the nonlinear processes that characterize modern financial markets. This approach represents a step toward the development of advanced tools for early instability detection and potential crisis forecasting, with potential applications in financial engineering, risk management, and strategic investment planning. Future research will focus on refining the classification system by incorporating additional market factors and expanding the scope to include more diverse financial instruments, such as commodities and cryptocurrencies. Further improvements can be made to the anomaly detection algorithms, enhancing their sensitivity to subtle market shifts without increasing the risk of false positives. Additionally, exploring the integration of machine learning techniques to complement the chaos-based analysis could offer deeper insights into market behavior, improving both the accuracy and reliability of predictions. The system could also be expanded to support real-time market monitoring and decision-making, enabling proactive responses to emerging market conditions.

References

[1] Axtell, R. L., & Farmer, J. D. (2025). Agent-based modeling in economics and finance: Past, present, and future. Journal of Economic Literature, 63(1), 197-287.

[2] Sussillo, D., & Abbott, L. F. (2009). Generating coherent patterns of activity from chaotic neural networks. Neuron, 63(4), 544-557.

[3] Borin, D. (2024). Hurst exponent: A method for characterizing dynamical traps. Physical Review E, 110(6), 064227.

[4] Ferreira, R. B., Vieira, V. M., Gleria, I., & Lyra, M. L. (2009). Correlation and complexity analysis of well logs via Lyapunov, Hurst, Lempel–Ziv and neural network algorithms. Physica A: Statistical Mechanics and its Applications, 388(5), 747-754.

[5] Uthamacumaran, A. (2021). A review of dynamical systems approaches for the detection of chaotic attractors in cancer networks. Patterns, 2(4).

[6] Sparrow, C. (1982). The lorenz equations (Vol. 109). New York: Springer.

[7] Tamijetchelvy, R., & Sankaranarayanan, P. (2013). An Optimized Multikeying Chaotic Encryption for Real Time Applications. International Journal of Advanced Computer Research, 3(4), 151.

[8] Tucker, W. (1999). The Lorenz attractor exists. Comptes Rendus de l'Académie des Sciences-Series I-Mathematics, 328(12), 1197-1202.

[9] Chomiak, T., & Hu, B. (2024). Time-series forecasting through recurrent topology. Communications Engineering, 3(1), 9.

[10] Marwan, N., & Braun, T. (2023). Power spectral estimate for discrete data. Chaos: An Interdisciplinary Journal of Nonlinear Science, 33(5).

[11] Su, X., Wang, Y., Duan, S., & Ma, J. (2014). Detecting chaos from agricultural product price time series. Entropy, 16(12), 6415-6433.

[12] Lehnertz, K. (2024). Time-series-analysis-based detection of critical transitions in real-world non-autonomous systems. Chaos: An Interdisciplinary Journal of Nonlinear Science, 34(7).

[13] Pincus, S. (2008). Approximate entropy as an irregularity measure for financial data. Econometric Reviews, 27(4-6), 329-362.

[14] Carvalho Júnior, O. A., Guimarães, R. F., Gillespie, A. R., Silva, N. C., & Gomes, R. A. (2011). A new approach to change vector analysis using distance and similarity measures. Remote Sensing, 3(11), 2473-2493.

[15] Gnanasekaran, A. (2023). Non-Linear Time Series Analysis Methods and Their Application to Fluctuation Data From Plasma Experiments.

[16] Marczewski, A. W. (2010). Application of mixed order rate equations to adsorption of methylene blue on mesoporous carbons. Applied Surface Science, 256(17), 5145-5152.

[17] Kinsner, W. (2007). A unified approach to fractal dimensions. International Journal of Cognitive Informatics and Natural Intelligence (IJCINI), 1(4), 26-46.

[18] Delliaux, S., Delaforge, A., Deharo, J. C., & Chaumet, G. (2019). Mental workload alters heart rate variability, lowering non-linear dynamics. Frontiers in physiology, 10, 565.

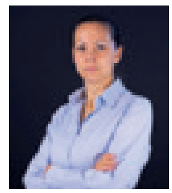
[19] Dasgupta, D., Yu, S., & Nino, F. (2011). Recent advances in

artificial immune systems: models and applications. Applied Soft Computing, 11(2), 1574-1587.
[20] Ran, R. (2019). yfinance: Yahoo! Finance market data downloader [Python package]. GitHub. https://github.com/ranaroussi/yfinance, Retrieved April 14, 2025,
[21] Yahoo Finance, *Historical market data*. Yahoo. Retrieved April 10, 2025, from https://finance.yahoo.com/
[22] Sawaya, Antonio. (2010). Financial time series analysis : Chaos and neurodynamics approach. Borlänge.

Received: May 4, 2025
Accepted: May 25, 2025

About the authors

Msc Marko Živanović is a doctoral student at the Faculty of Technical Sciences at the University of Čačak, specializing in Information Technology. He has completed both vocational and academic undergraduate and master's studies. His research interests include artificial intelligence, machine learning, and soft computing. Marko has written several scientific papers and works on industry projects. He is currently employed at the Faculty of Information Technology at Metropolitan University in Belgrade and also teaches part-time at a modern high school.



PhD Emilija Kisić is an Assistant Professor at Belgrade Metropolitan University, teaching at the Faculty of Information Technologies. She completed her undergraduate and master's studies at the School of Electrical Engineering, University of Belgrade, Signals and Systems department. She earned her PhD in 2016 from the School of Electrical Engineering, University of Belgrade, Signals and Systems department, specializing in the study module Systems Control and Signal Processing. Her doctoral dissertation focused on the field of statistical signal processing, predictive maintenance, and fault detection, titled "Application of T2 Control Charts and Hidden Markov Models in Predictive Maintenance of Technical Systems." Her research interests include artificial intelligence, machine learning, data analysis and statistical signal processing. She is the author and co-author of numerous papers published in national and international journals and presented at both domestic and international conferences and events of national or international importance.

For citation

Marko M. Živanović, Emilija Kisić, A Hybrid Model Based on Chaos Theory and Artificial Immune Systems for the Analysis and Classification of Stock Market Anomalies, *JITA – Journal of Information Technology and Applications, Banja Luka*, Pan-Europien University APEIRON, Banja Luka, Republika Srpska, Bosna i Hercegovina, JITA 15(2025)1:15-29, (UDC: 330.342.14:[34:124.1), (DOI: 10.7251/JIT2501015Z), Volume 15, Number 1, Banja Luka, June (1-80), ISSN 2232-9625 (print), ISSN 2233-0194 (online), UDC 004