# SEMANTIC NUMERATION SYSTEMS AS INFORMATION TOOLS FOR FUZZY DATA PROCESSING

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**Contribution to the State of the Art**

https://doi.org/10.7251/JIT2201014C UDC: 004.6.056.55:519.852

**Abstract**: We describe the concept of semantic numeration systems (SNS) as a certain class of context-based numeration methods. The main attention is paid to the key elements of semantic numeration systems - cardinal semantic operators. A classification of semantic numeration systems is given. The concept of fuzzy cardinal semantic transformation as a basis for creating fuzzy semantic numeration systems is advanced. Both fuzziness of the initial data - cardinals of abstract entities - and fuzziness of the parameters of the cardinal semantic operators are considered. The principle of formation of the fuzzy common carry in the cardinal semantic operators with multiple inputs is formulated.

**Keywords:** Cardinal Abstract Entity, Cardinal Semantic Operator, Semantic Numeration System, Fuzzy Cardinal Semantic Transformation.

## INTRODUCTION

Any data processing in modern information systems is ultimately based on the use of one or another numeration system. A numeration system is a symbolic method of representing numbers using signs. Modern numeration systems are usually divided into three classes: positional (Place-value), non-positional and mixed. Despite a significant variety of works in the field of positional numeration systems [1-3], we can say that the overwhelming majority of them do not go beyond the traditionally linear representation of numbers.

“Place-value” is a representation of an abstract number by a system of characters (digits) - the contents of places - which are named in a certain way (by numbers, symbols or words). The semantics of the traditional place-value representation can be expressed as follows: *n* units of some abstract entity *i* are given the meaning (~>) of a unit of another abstract entity *j*: $n \cdot 1_i = n_i \sim> 1_j$, and further, recursively: $m \cdot 1_j = m_j \sim> 1_k$. Such representation, in which a certain amount of *one* entity is associated with a unit of *another* entity, can be called a linear or (1-1)-representation.

Assume that there are such abstract entities *i*, whose *n* units $n_i$ are given the meaning of both the unit of an abstract entity *j* ($1_j$) and the unit of an abstract entity *k* ($1_k$) simultaneously: $n_i \sim> (1_j, 1_k)$. Consider another situation: to form a unit of an abstract entity *k*, exactly *n* units of an abstract entity *i* and *m* units of an abstract entity *j* are required: $(n_i, m_j) \sim> 1_k$.

Aren’t linear numeration systems the methods of constraining efficient data/information processing by reducing the semantic content of the data to its numerical value?

The author's previous works [4, 5] were an attempt to construct semantic numeration systems without cardinal semantic operators, but only on the basis of cardinal abstract entities connected in some topology. Abstract entities were given an active role both in the formation of the carry and in the formation of a structure of connectivity (entanglement) with other abstract entities.

## PRELIMINARIES [6, 7]

Understanding *the entity* as something distinguished in being and having some meaning, we will introduce several definitions that concretize the application of this concept in the area covered.

*An abstract entity* (Æ) is an entity of arbitrary nature, provided with an identifier name that allows it to be distinguished from other entities. For example, a number, a car, a galaxy. Name as entity identifier can be either elementary (i) - one-element (letter, digit, word, symbol), or complex (composite, multi-element), corresponding to abstract coordinates (<i |) of the entity $Æ_{<i\,|}$ in some semantic variety. For example, "the zero bit of a number in binary notation."

Manifold (Multeity) is the next concept that is important for further presentation. From the many different definitions of this concept, we synthesize the following. *Multeity* - a manifestation of something uniform in essence in various kinds and forms, as well as a quality or condition of being multiple or consisting of many parts.

Since in what follows we will deal with the transformation of meanings, we will define the corresponding specific type of multeity as semantic. *Semantic multeity* is an abstract space with no more than a countable set of abstract entities, semantically united by the unity of the goal description (context). A semantic multeity will be called *open* if the number of abstract entities in it is countable (at least potentially), *closed* if it contains a finite number of abstract entities and *bounded* if the number of abstract entities in it is finite and unchanged.

Differences between semantic multeities and manifolds in mathematics are:

- semantic *heterogeneity* of entities that make up semantic diversity;

- *goal-setting*: a semantic multeity initially includes only those abstract entities that are used (potentially can be used) to solve a specific problem;

- *openness*: the possibility of new abstract entities generation as a result of transformations. In the end, a semantic multeity can form a semantic universe that includes any conceivable abstract entity.

*Cardinal Semantic Multeity* (CSM) is a semantic multeity, each element of which is equipped with a cardinal characteristic - the multiplicity of a given abstract entity is represented in multeity. From a set-theoretic point of view, a cardinal semantic multeity is a multiset, the carrier of which is contextually conditioned. The elements of cardinal semantic multeity will be called *cardinal abstract entities*.

*Cardinal Abstract Entity* (CÆ) is an abstract entity with a cardinal characteristic $CÆ_i = (i; N_i)$, where *i* is the name of the cardinal abstract entity, $N_i = \text{Card}\ (CÆ_i) = \#\ (1_i,\dots,1_i)$, $N_i \in \mathbf{N}$. We will assume that the named unit $1_i$ is a quantum of the meaning for the abstract entity $Æ_i$.

## THE CONCEPT OF A CARDINAL SEMANTIC OPERATOR

This paper can serve as a certain theoretical supplement advantage to the work [8], in which the concept of an operator is given the status of conceptual in theory and practice.

It seems that there is no strict uniform definition of a *semantic operator* yet. Each application area - logic, linguistics, and programming - interprets this concept in its own way. However, it is possible to single out a certain semantic invariant that allows you to define the action of the semantic operator (SO) as a change/transformation of the meaning of a certain entity ($Æ_i$) or a set of entities ($Æ_i$, ..., $Æ_k$) into another meaning ($Æ_j$): $Æ_j = SO(Æ_i)$ or

$Æ_j = SO(Æ_i, …, Æ_k)$.

Let us introduce the following basic concept - the concept of *a cardinal semantic operator*. In essence, the action of the cardinal semantic operator is to give a certain number of units $n_i$ of the cardinal abstract entity $CÆ_i$ the meaning of unit $1_j$ of the cardinal abstract entity $CÆ_j$, $(i \neq j)$: $n_i \rightsquigarrow 1_j$. In principle, other options are also possible, for example, when the $n_i$ of an abstract entity $CÆ_i$ is assigned not one, but simultaneously several different semantic units of respectively different CÆs: $n_i \rightsquigarrow (1_j,..., 1_k)$. Or, for the generation of the unit of meaning $1_j$ of the abstract entity $CÆ_j$, the corresponding *n*-s of several other CÆs are simultaneously needed: $(n_i,…, n_k) \rightsquigarrow 1_j$.

A *Cardinal Semantic Operator* is a multivalued mapping of the cardinal semantic multeity on itself, which associates a set of entity operands from the multeity with a set of entity images from the same multeity, transforming their cardinals using the operations defined by the operator signature: $Signt(CSO) = (K, Form, |n>_w, |r>_v)$, where K is a kind of operator, Form is a type of operator, |n> is a radix vector, and |r> is a conversion vector. The pair (W, V) is a valence of the Cardinal Semantic Operator.

Depending on whether the cardinal value of CÆ-operands changes under the action of the cardinal semantic operator, the latter can be representatives of one of three families - *transforming* operators (.), which change the value of the cardinals of both CÆ-operands and CÆ-images; *preserving* operators [.], changing the value of cardinals of CÆ-images and not affecting the value of cardinals of CÆ-operands; and *complex* operators (.], acting on some CÆ-operands as transforming, and on others - as preserving. This approach determines the possibility of the existence of three classes of semantic numeration systems - transforming, preserving and complex.

The kind of the cardinal semantic operator indicates the content of the transformations (definition of carry and remainder), which are performed both with the cardinals of CÆ-operands and with the cardinals of CÆ-images.

For example, the following kinds of CSOs are possible:

- radix-multiplicity (**↑#**): carry $p_i = \lfloor N_i/n_i \rfloor$, remainder rem $N_i = N_i - p_i n_i$;
- radix excess value (**↑Δ**): $p_i = N_i - n_i$; rem $N_i = 0$;
- radix excess fact (**↑•**): $p_i = 1 \Leftrightarrow N_i > n_i$; rem $N_i = f(N_i, n_i)$;
- arbitrary function (**f**): $p_i = f(N_i, n_i)$, rem $N_i = g(N_i, p_i, n_i)$.

In this paper, we consider the transforming cardinal semantic operators of the radix-multiplicity kind.

We will call the number of CÆ-operands of a cardinal semantic operator its *input valence* (W), $W = \#(CÆ_i,…, CÆ_j) = \dim(|n>)$, and the number of CÆ-images - its *output valence* (V), $V = \#(CÆ_k,…, CÆ_l) = \dim(|q>)$.

Strictly speaking, the output valence of the transforming CSO is determined by the sum of the actual output valence and the valence of "feedbacks", returning remainders to CÆ-operands. Then the full valence (–arity) of the transforming CSO will be (W, V + W). However, doubling the output valence in practice can lead to confusion, so we will usually neglect the valence of the return of reminders and write the (full) valence of the transforming CSO as (W, V).

*Radix-vector* $|n>_w = (n_i,…, n_j)^T$ consists of the particular radices (bases) $n_i,…, n_j$, relative to which the particular i,…, j-carries $p_i,…, p_j$ are formed. They further participate in the formation of the *common carry p* (if it's necessary).

*The conversion vector* $|r>_v = (r_{\cdot k},…, r_{\cdot l})^T$ consists of the components that determine the "scale factors" of the transformation of the common carry into the components of the *transformant* $q_k,…, q_l$, which change the values of the cardinals of the CÆ-images. This means that, for example, the carry *p* formed according to a given rule will be associated not with $1_j$ of the cardinal abstract entity $CÆ_j$, but with $r_{.j}$ of such units. We will call $r_{.j}$ the *rate of conversion* (j-conversion) of the carry. The introduction of conversion rates allows you to create numeration systems with rational bases.

The transformant $|q>_v = (q_k,..., q_l)^T$ is a direct result of the action of the CSO on the CÆ-operands. However, we will consider the transformation complete only after the "recalculation" of the cardinals of all CÆ-images of the operator in accordance with the obtained values of the corresponding components of the transformant $|q>_v$.

Specific values of valency, components of the radix vector $|n>_w$ and conversion vector $|r>_v$ are determined by the *specification* of the cardinal semantic operator.

The case when the sets of CÆ-operands and CÆ-images of an operator are singleton corresponds to the generally accepted positional numeration systems.

## FORMS OF CARDINAL SEMANTIC OPERATORS

Let us define the *main forms* of cardinal semantic operators of the radix-multiplicity kind.

*L-operator* (Line-operator): ($\uparrow$#, L, $n_i$, $r_{ij}$) – a cardinal semantic operator of valency (W, V) = (1, 1), which assigns (gives the meaning of) $r_{ij}$ units of the transformant $q_j$, added to the cardinal $N_j$ of the abstract entity $CÆ_j$, to each $n_i$ of the cardinal abstract entity $CÆ_i$.

A schematic representation of the L-operator is shown in Fig.1.

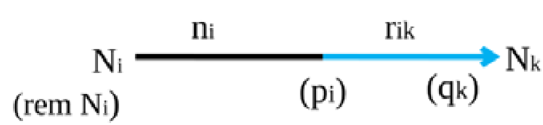



***Figure 1.***

When an L-operator acts on a $CÆ_i$-operand, the following operations are performed:

(i) $p_i = \lfloor N_i/n_i \rfloor$ – calculation of radix-multiplicity, that is, i-carry value;

(ii) Rem: $N_i` = \text{rem } N_i = N_i - p_i n_i = N_i \bmod n_i$ – finding the remainder in $CÆ_i$;

(iii) $q_j = p_i \cdot r_{ij}$ – calculation of the j-transformant value;

(iv) $N_j` = N_j + q_j$ – finding the change of the $CÆ_j$-image cardinal.

The L-operator of signature ($\uparrow$#, L, $n_i$, $1_j$) is basic for many traditional positional numeration systems. In particular, for the decimal numeration system: ($\uparrow$#, L, 10, 1).

*D-operator* (Distribution operator): ($\uparrow$#, D, $n_i$, ($r_{ij}$, ..., $r_{ik}$)) – a cardinal semantic operator of valency (1, v), which assigns the following units to each $n_i$ of the cardinal abstract entity CÆi *v* transformants: $r_{ij}$ units of j-transformants $q_j$ for a cardinal abstract entity CÆj,..., and $r_{ik}$ units of k-transformants $q_k$ for the cardinal abstract entity $CÆ_k$.

A schematic representation of the D-operators $D_2$ is shown in Fig.2.

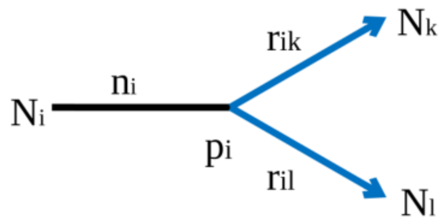



***Figure 2.***

When a D-operator acts on a $CÆ_i$-operand, the following operations are performed:

(i) $p_i = \lfloor N_i/n_i \rfloor$ – determining radix-multiplicity (i-carry value);

(ii) $N_i\text{`} = \text{rem } N_i = N_i - p_i\, n_i = N_i \bmod n_i$ – finding the remainder in $CÆ_i$;

(iii) $q_j = p_i \cdot r_{ij}$, $q_k = p_i \cdot r_{ik}$ – calculation of partial transformants;

(iv) $N_j\text{`} = N_j + q_j$, $N_k\text{`} = N_k + q_k$ – finding the change of the CÆ-images ($CÆ_j$, $CÆ_k$) cardinals.

Note that the L-operator is a variant of the degenerate D-operator in which all $r_{ij}$, except one, are equal to zero.

*F-operator* (Fusion operator): (↑#, F, ($n_i$, ..., $n_j$), $r_{.k}$) - a cardinal semantic operator of valency (W, V) = (w, 1), which assigns $r_{.k}$ units of the transformant $q_k$ to each *w*-tuple ($n_i$,... ,$n_j$) of CÆ-operands for the cardinal abstract entity $CÆ_k$.

A schematic representation of the F-operators ${}_2F$ and ${}_wF$ is shown in Fig.3.

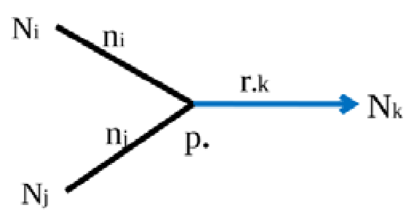



***Figure 3.***

When a ${}_2F$-operator acts on $CÆ_{i,j}$-operands, the following operations are performed:

(i) $p_i = \lfloor N_i/n_i \rfloor$, $p_j = \lfloor N_j/n_j \rfloor$ – calculation of partial carries;

(ii) $p = \min\{p_i, p_j\}$ – calculation of common carry;

(iii) $N_i\text{`} = N_i - p{\cdot}n_i$, $N_j\text{`} = N_j - p{\cdot}n_j$ – calculation of the remainders in $CÆ_i$, $CÆ_j$;

(iv) $q_k = p{\cdot}r_{.k}$ – calculation of the transformant;

(v) $N_k\text{`} = N_k + q_k$ – finding the change of the $CÆ_k$-image cardinal.

Since the partial carries $p_i$, ..., $p_j$ will be different, in the general case, the common carry must be determined from the condition of the existence of non-negative remainders in all CÆ-operands. This condition will be satisfied if we choose the minimal partial carry as the common carry *p*: $p_{.} = \min\{p_i, ..., p_j\}$.

*M-operator* (Multi-operator): (↑#, M, ($n_i$,... ,$n_j$), ($r_{.k}$,..., $r_{.l}$)) - a cardinal semantic operator of valency (W, V) = (w, v), which assigns *v*-tuple conversion coefficients ($r_{.k}$,..., $r_{.l}$) of transformants to the *w*-tuple ($n_i$,... ,$n_j$) for CÆ-operands: $r_{.k}$ units of k-transformant $q_k$ for the cardinal abstract entity $CÆ_k$,..., and $r_{.l}$ units of l-transformant $q_l$ for the cardinal abstract entity $CÆ_l$.

A schematic representation of the M-operator ${}_2M_2$ is shown in Fig. 4.

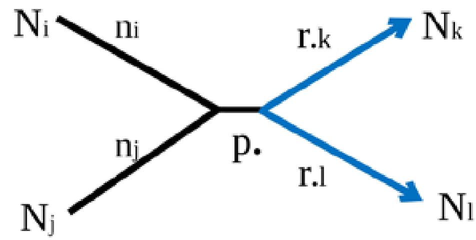



***Figure 4.***

When the ${}_2M_2$-operator acts on $CÆ_{i,j}$-operands, the following operations are performed:

(i) $p_i = \lfloor N_i/n_i \rfloor$, $p_j = \lfloor N_j/n_j \rfloor$ – calculation of partial carries;

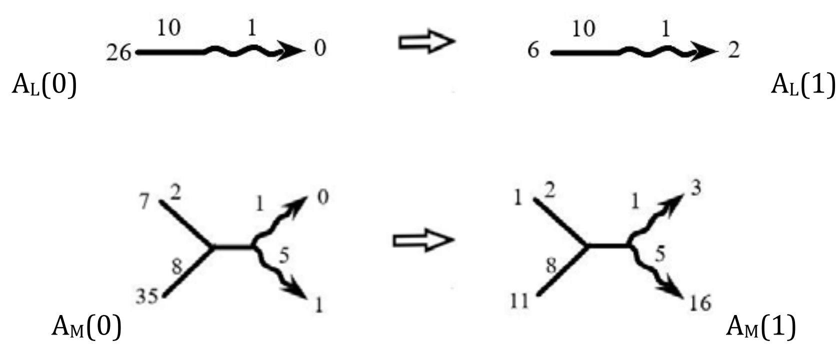
(ii) $p = \min\{p_i, p_j\}$ – calculation of common carry;

(iii) $N_i` = N_i - p \cdot n_i$, $N_j` = N_j - p \cdot n_j$ – calculation of the remainders in $CÆ_i$, $CÆ_j$;

(iv) $q_k = p \cdot r_{\cdot k}$ , $q_l = p \cdot r_{\cdot l}$ – calculation of the partial transformants;

(v) $N_k` = N_k + q_k$, $N_l` = N_l + q_l$ – finding the changes of the $CÆ_{k,l}$-image cardinals.

Examples of cardinal semantic operators execution.




It is easy to see that any of the cardinal semantic operators considered above is a special case of the M-operator: $L \sim {}_1M_1$, $D_v \sim {}_1M_v$, ${}_wF \sim {}_wM_1$. However, for the construction of specific numeration systems and the analysis of cardinal semantic transforms in them, it is often more convenient to use such reduced forms of the M-operator.

These four cardinal semantic operators form the operator basis of any semantic numeration system.

## Numeration Space. Cardinal Abstract Object

To represent complex multistage semantic transformations, mono-operator transformations, as usual, are not enough. Let us introduce the concept of a *numeration space*, the elements of which are the numeration methods. By the *method of numeration* we mean a contextually conditioned method of transforming semantic units from a cardinal semantic multeity using cardinal semantic operators. Let us formalize the last statement with the concept of a *cardinal abstract object*.

*Cardinal Abstract Object* (CAO) is a collection of cardinal abstract entities connected in a certain topology by cardinal semantic operators. The signature of $CAO_I$:

$$\mathrm{Signt(CAO)} = (\mathbf{I}; \mathbf{CSM}; \mathbf{CSO}; \mathbf{STop}),$$

where **I** is the set of names denoting (naming) methods of numeration, **CSM** is the cardinal semantic multeity, **CSO** is the set of cardinal semantic operators, **STop** are the possible topologies of the semantic connectivity of cardinal abstract entities by cardinal semantic operators.

*Numeration Space* (NS) is an abstract space, the elements of which are cardinal abstract objects. A concrete $CAO_I$ implements a specific method of numeration *I* in a numeration space.

The cardinal abstract entities included in the CAO that have an output, but do not have an input will be called *initial*, with input and output - *intermediate*, only with input - *final*, without input and output - *detached*. The main accepted assumption for the numeration methods (i.e. CAO) considered in this work is that any initial or intermediate cardinal abstract entity has a single output (associated with only one "perceiving" semantic operator) for an arbitrary (finite) number of inputs (transformants of other semantically consistent operators).

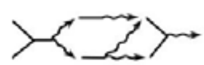
The concretizations of the CAO name, the composition of the cardinal semantic multeity, the type of operators and the topology of connectivity are determined by the *specification* of the cardinal abstract object.

## TOPOLOGY OF SEMANTIC CONNECTIVITY

The topology of semantic connectivity (STop) is determined by a given semantics of cardinal transformations and consists in connecting cardinal abstract entities from a cardinal semantic multeity by cardinal semantic operators of a given form.

The topology of semantic connectivity can be specified:

- descriptive/textually. For example, "serial connection of L-operators". Suitable for simple topology;
- in diagram form. For example:



- analytically: in the form of operator formulas of various types. For example, for the above graphical representation from left to right, top to bottom: $({}_2M_2|L, D_2|_2F)$;
- in tabular form.

The topology of semantic connectivity can be both "one operator"-type and "many operators"-type, both regular (for example, a 2-lattice, tree), and irregular, periodic or non-periodic, as well as cyclic. Linear topology can be specified recursively. For example, $(L)^m = L(L)^{m-1}$.

Thus, we can say that the positional numeration system in the traditional sense is a set of linearly connected cardinal abstract entities with bit semantics.

## CARDINAL SEMANTIC TRANSFORMATION. MULTINUMBERS AND MULTICARDINALS

Let us agree to call a CSO "allowed" if the values of the cardinals of all its operands ensure the execution of the given operator.

*Cardinal Semantic Transformation* (CST) consists in executing, for a given $CAO_I$, all "allowed" cardinal semantic operators. A CST' *step* will mean a single execution for a given $CAO_I$ of all "allowed" cardinal semantic operators. The minimal sequence of CST' steps, leading to steady values of all cardinals in $CAO_I$, will be called a *complete* CST, and the number of such steps will be called the *length* of CST.

A single implementation of the transformation will be called its *step*. Cardinal-semantic transformations can be both mono-operator and multi-operator. A mono-operator transformation is always one-step. Multi-operator transformations are usually multi-step.

The multiset of cardinals of all CÆs from $CAO_I$ after an arbitrary step τ of a cardinal semantic transformations will be called the *multicardinal* of $CAO_I$ of the step τ and denoted $<A_I(\tau)>$ ($<A_I(\tau)> = [N_i(\tau), N_j(\tau), \ldots, N_k(\tau)]$).

The multicardinal meaningfully characterizes only the “card-fullness” of $CAO_I$ after each step of the cardinal semantic transformation, but in no way reflects the semantic aspect of the CST. The multicardinal of $CAO_I$ before the first step of the transformation (CST) will be called the *initial* multicardinal $<A_I(0)>$, after a certain transformation step $\tau$ - the *intermediate* multicardinal $<A_I(\tau)>$, upon completion of transformations – the *final* multicardinal $<A_I(\omega)>$.

The holistic structural-cardinal representation of $CAO_I$ after the $\tau$-th step of the cardinal semantic transformation will be called the I-*multinumber* of the step $\tau$ (multinumber) and denoted by $A_I(\tau)$. By analogy with the multicardinal, before the first step of CST we will call a multinumber the *initial* multinumber $A_I(0)$, after a certain step ($\tau$) of the transformation – the *intermediate* $A_I(\tau)$, upon completion of CST – the *final* multinumber $A_I(\omega)$.

We will assume that the multicardinal determines precisely the *meaning* of the CAO after the $\tau$-th step of the cardinal semantic transformation, and the multinumber is its *sense*. Informally, a multinumber is a structured multicardinal, and a multicardinal is a de-structured multinumber.

Thus, a certain I-method of numeration ($CAO_I$) is a contextually determined complete cardinal semantic transformation of both multinumbers in the numeration space (NS) and the corresponding multicardinals in the cardinal semantic multeity (CSM).

Any number represented in one or another traditional positional numeration system is a multinumber, despite the absence of cardinal semantic operators in the numbers. This is due to both the linearity of the traditionally used CSOs, and the insignificant radix variability (negotiated separately), which allows you to write numbers into a string without distorting the meaning of the representation.

## SEMANTIC NUMERATION SYSTEMS

Informally, the semantic numeration system (SNS) can be defined as a collection of homogeneous numeration methods.

By the *semantic numeration system* in the numeration space NS we mean its subspace $SNS_\varphi$ with the given properties determined by the classification features. Here $\varphi$ is the name-identifier of the semantic numeration system, due to a set of classification features.

We propose the following *classification of Semantic Numeration Systems*:

(1) by influence on the operand cardinal: transforming or preserving;

(2) by type of uncertainty: deterministic, stochastic and fuzzy;

(3) by kind of transformation: radix-multiplicity, radix-excess value; radix-excess fact;
arbitrary function;

(4) by kinds of number systems used in the numeration system: natural, integer or
rational numbers;

(5) by controllability: autonomous or controlled;

(6) by variability of the operator parameters (radices and conversion rates): homogeneous (the same for all operators) or heterogeneous (different for different operators);

(7) by topology of operators’ connectivity: linear (with not only L-operators), tree-like,
lattice, cyclic, amorphous or of a special form. Regular structures can be either isotropic or anisotropic, and the latter can be homogeneous or heterogeneous.

Thus, most of the generally accepted "numeration systems", for example, binary or decimal, will hardly need to be renamed. Within the framework of the above classification, they are particular (with n=2 and n=10, respectively) methods of numeration of the *transforming*, *deterministic*, *radix-multiplicity, natural, autonomous, linear*, *homogeneous* semantic numeration system.

## FUZZY CARDINAL SEMANTIC TRANSFORMATIONS

In practice, one of the most crucial and common aspects of numerical data is its uncertainty. At present, mathematical theories that are considered to treat uncertainty are interval analysis [9], probability theory and the theory of fuzzy sets [10].

This state of affairs necessitates the development of tools for representing and processing uncertainty numerical information, in particular, in the field of semantic numeration systems. Since cardinal semantic transformations in semantic operators are basic for any SNS, we will focus on one-step transformations.

In principle, the following variances of the initial fuzziness are possible:

(i) "Fuzzy input, crisp operator". In this case, at least, one CÆ-operand has a fuzzy cardinal: card($CÆ_i$) = $\widetilde{N}_i$.

(ii) "Crisp input, fuzzy operator". All the input cardinals card($N_i$), ..., card($N_j$) are crisp. Some of the parameters (n, r) of CSO, or all of them, are fuzzy.

(iii) "Fuzzy input, fuzzy operator".

If a cardinal semantic operator is fuzzy, there are three possible ways of its performance: 1) when some radices are fuzzy with crisp conversion rates; 2) when radices are crisp with some fuzzy conversion rates; 3) when both the radices and the conversion rates are fuzzy.

Let $\tilde{A}$ be a continuous triangular fuzzy number and its membership function $\mu_{\tilde{A}}(x)$ is defined as [10]:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < \underline{a}, \\ (x - a)/(a - a), a & \le x \le a, \\ (\bar{a} - x)/(\bar{a} - a), a & < x \le \bar{a}, \\ 0, & x > \bar{a}, \end{cases}$$

where $x$ is an element of a support X, ($x \in$ X).

So each triangular fuzzy number $\tilde{A}$ may be defined as a triple $\tilde{A} = (a, a, \bar{a})$, where $a$ is a lower bound, $a$ is a mode, and $\bar{a}$ is an upper bound of the fuzzy number $\tilde{A}$.

All arithmetic operations on triangular fuzzy numbers are performed according to the rules of interval arithmetic [9, 10]. A feature of the cardinal semantic transformation in the radix-multiplicity kind of operators is the presence of the floor function $\lfloor * \rfloor$. So, we must introduce a rule for finding the floor function of both a fuzzy argument and a complex argument-operation on fuzzy numbers.

We assume that *the floor function of a triangular fuzzy number is a triangular fuzzy number, each component of which is the floor function of the corresponding component of the original number*.

Then,

$$\lfloor \tilde{A} \rfloor = \lfloor (\underline{a}, a, \bar{a}) \rfloor = (\lfloor \underline{a} \rfloor, \lfloor a \rfloor, \lfloor \bar{a} \rfloor), \forall \tilde{A} = (a, a, \bar{a}).$$

Respectively, for the floor function of the complex argument-operation on fuzzy numbers, for example, $\tilde{A} = (a, a, \bar{a}), \tilde{n} = (n, n, \bar{n})$ we can get the following expression:

$$\lfloor \tilde{A}/\tilde{n} \rfloor = \lfloor (\underline{a}, a, \bar{a}) / (n, n, \bar{n}) \rfloor = ( \lfloor \underline{a}/\bar{n} \rfloor, \lfloor a/n \rfloor, \lfloor \bar{a}/\underline{n} \rfloor ).$$

Similarly, expressions for more a complex argument-operation on fuzzy numbers can be obtained.

Thus, *fuzzy cardinal semantic transformation* is a cardinal semantic transformation performed according to the rules of fuzzy set arithmetic, fuzzy floor function and fuzzy common carry formation.

## THE PRINCIPLE OF FUZZY COMMON CARRY FORMATION IN $_W$F AND $_W$M$_V$ OPERATORS

Unlike the comparison of two fuzzy numbers and selection of a fuzzy number that is less than the other, in cardinal semantic operators $_W$F and $_W$M$_V$ the fuzzy common carry has to be 'synthesized' based on a set of fuzzy partial carries $(\tilde{p}_i, \ldots, \tilde{p}_j)$. Forming, but not choosing!

The concept of common carry calculation lies in its formation from the partial carries of each CÆ-operand of CSO, with guarantee that the cardinals of the remainders are zeroes or natural numbers in each CÆ of operands. In case that partial carries are represented as triangular fuzzy numbers, the following *"min-method"* of forming the fuzzy common carry is proposed.

The fuzzy common carry $\tilde{p}_. = (p_., p_., \bar{p}_.)$ is formed from fuzzy partial carries $(\tilde{p}_i, \ldots, \tilde{p}_j)$ so that its lower bound, mode and upper bound are defined as the minimum value of the corresponding bounds and the modes of the fuzzy partial carries:

$$\underline{p}_. = \min(\underline{p}_i, \ldots, \underline{p}_j),$$
$$p_. = \min(p_i, \ldots, p_j),$$
$$\bar{p}_. = \min(\bar{p}_i, \ldots, \bar{p}_j).$$

Linear ordering of the components: $\underline{a} \leq a \leq \bar{a}$ for any $\tilde{A} = (\underline{a}, a, \overline{a})$ ensures the uniqueness of the solution.

An example of cardinal semantic transformation of fuzzy initial data by crisp $_3$F-operator is shown in fig. 5-7.

Let the initial cardinals be represented as triangular fuzzy numbers $\widetilde{N}_i = (9, 31, 45), \widetilde{N}_j = (38, 73, 80)$, $\widetilde{N}_k$ = (29, 50, 101). The crisp $_3$F-operator has radices $n_i$ = 5, $n_j$ = 10, $n_k$ = 10 and the conversion rate $r_{.l}$ = 2 (fig.5).

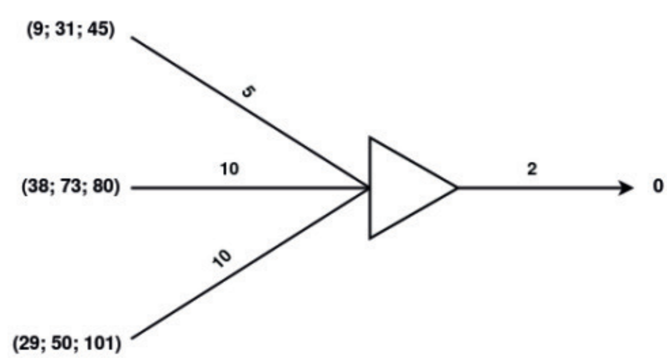



***Figure 5.***

Fuzzy partial carries obtained by the method $\tilde{p}_i = \lfloor \widetilde{N}_i / n_i \rfloor$, ..., and the fuzzy common carry $\tilde{p}_.$ = (1, 5, 8) formed by the "min-method" are shown in fig.6.

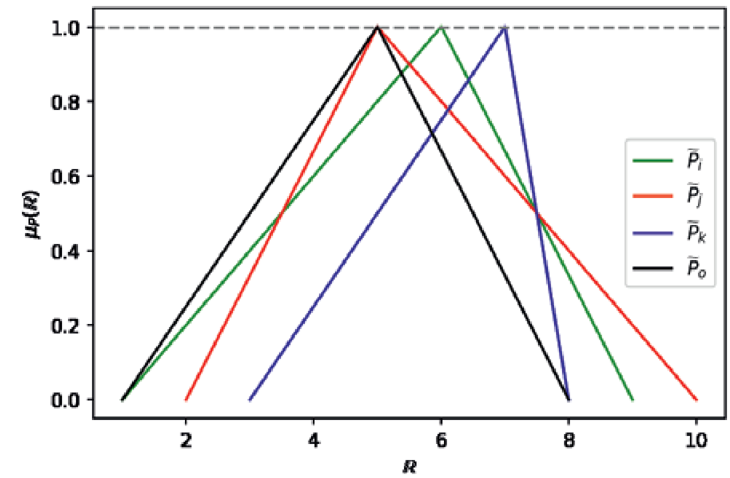



*Figure 6.*

The result of the fuzzy cardinal semantic transformation (fig.7) consists of CÆ$_l$-image fuzzy cardinal $\widetilde{N}_l = (2, 10, 16)$ and the corresponding fuzzy remainders.

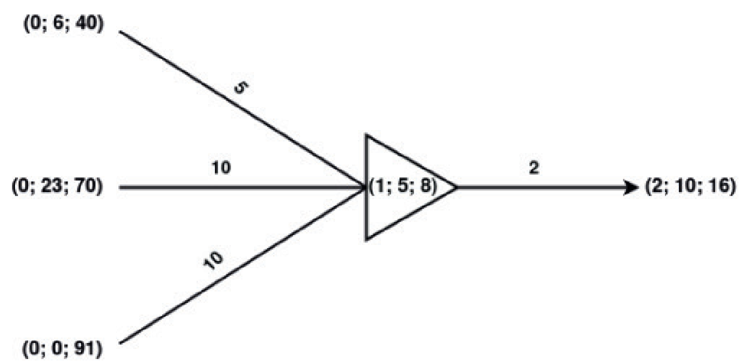



*Figure 7.*

## CONCLUSION

The undoubted advantages of semantic numeration systems in comparison with existing positional ones are:

- a broader view on numeration systems as a way of representing not only numbers, but also sets, multisets, etc.;
- a variety of structures for representing numerical data within the framework of a single numeration method from positional to position-structured;
- an emphasis on the difference between meaning and sense in numeration systems. Sense as the entanglement of meanings and structure.

The extension of SNS to fuzzy semantic numeration systems makes it possible to take into account in the representation of numerical data such types of uncertainty as interval and fuzzy. This makes fuzzy semantic numeration systems indispensable in applied artificial intelligence systems.

It seems important to research the possibility of creating semantic numeration systems with a probabilistic type of uncertainty. For applications, it is necessary to explore a number of "mix directions" such as combination of different types of uncertainty in one CSO (CAO), for example, fuzzy and probabilistic, as well as the possibility of combining discrete and continuous fuzzy numbers in one cardinal semantic operator.

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Received: March 2, 2022
Accepted: May 19, 2022

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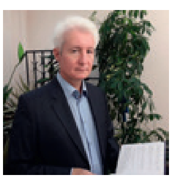

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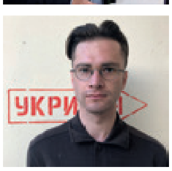


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## For citation

Alexander Yu. Chunikhin, Vadym Zhytniuk, Semantic Numeration Systems as Information Tools for Fuzzy Data Processing, *JITA – Journal of Information Technology and Applications, Banja Luka*, Pan-Europien University APEIRON, Banja Luka, Republika Srpska, Bosna i Hercegovina, JITA 12(2022) 1:14-25, (UDC: 004.6.056.55:519.852), (DOI: 10.7251/JIT2201014C), Volume 12, Number 1, Banja Luka, June (1-64), ISSN 2232-9625 (print), ISSN 2233-0194 (online), UDC 004