DOI: 10.7251/QOL1401053P **UDC:** 636.52:637.5

Professional paper

POSSIBILITY OF USING FOURIER'S DIFFERENTIAL EQUATION IN COOLING PROCESS OF MEAT STEAKS

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Summary: In this paper, based on derived differential equation for the heat transfer and a suitable boundary condition (equation of heat exchange), the appropriate model of cooling of meat stakes in the form of a flat plate is set up. By using theory of similarity, Fourier and Biot criterion is defined, which allowed setting criterial equation of temperature field which included an unnamed temperature and geometric criterion. For solving the obtained criterial equation, existing diagrams of temperature functions were utilized, which were specified by analytical method for the core, the surface and the interior of the observed model. The proposed model is used for analytical solving of a number of practical problems in the cooling process of meat steaks in a gaseous environment. Particular focus has been on the temperature of the surface and core of steak as well as ambient temperature and cooling time. Also, it was shown that proposed model can be used to define temperatures or temperature field along the thickness of steak, depending on the distance from the central plane. Special possibility of applying the model is for the case of preventing freezing steaks, when their temperature is maintained within specified limits.

Key words: Furier's equation, partial differential equations, unsteady heat distribution, Fourier and Biot criterion, temperature function, cooling of meat steaks

Introduction

The theoretical basis of heating and cooling belongs to the field of thermodynamics. The observation of these processes is of great practical interest in many fields of technology, especially in the field of heat treatment technology metals [1, 2]. In this general case, the temperature changes with time, while the observed body is occasionally heated or cooled [2 - 4].

The determination of temperatures on the surface and the core of the observed body, as well as describing the temperature field depending on the coordinates is of particular importance [1, 4]. Time of heating and cooling is also important parameter. All these tasks are solved for the case of simple geometric bodies, based on the application of Fourier differential equation for the transfer of heat, whereby the solution comes through analytical procedure, with the use of appropriate diagrams which are constructed for this purpose [4 - 6].

Taking into account the above, the idea of work was set up in the sense to apply the Fourier differential equation of heat transfer for analytical solving of particular problems encountered in meat processing and cooling of steaks of certain geometric shapes. It should be noted that the approach presented in this paper is not common in the literature.

Derivation of the heat transfer differential equation

For controlling the temperature during heating and cooling, technical requirements for monitoring of developments in the core of the observed body are usually difficult [7, 8]. It is therefore an important theoretical and empirical function for the case of heating and cooling, in the form of:

$$t=f\,(x,\,y,\,z,\,\tau) \tag{1}$$

Here x, y, z are coordinates of arbitrary point which is cooled, and τ is time. During heating and cooling, the temperature at any point changes with time, so this process is non-stationary.

Thermodynamic theory of heating and cooling in terms of pure heat transfer where there is no transmission of mass [8-11] is observed here. In this, the homogeneous and isotropic body is observed where the heat conductivity, specific density and specific heat capacity are independent of temperature and pressure. Also, it is assumed that during the cooling process there is no phase change of the material [2, 9, 10].

In the observed body, the geometric position of all points that have the same temperature at the same point of time forms an isothermal surface [7, 11]. Measure of rate at which temperature changes in one field during movement at any observed direction l is:

$$\frac{\Delta t}{\Delta l} \tag{2}$$

Rate of temperature change will be:

$$\frac{\Delta t}{\Delta n} \tag{3}$$

Where n is the normal line to the surface.

The temperature gradient is represented by special vector:

$$\left|grad\ t\right| = lim_{\Delta n \to 0}\left|\frac{\Delta t}{\Delta l}\right| = \frac{\partial t}{\partial n} \tag{4}$$

which has a special significance in the temperature field theory [2, 5, 11].

Projections of this vector to Descartes system are:

$$(grad\ t)_x = \frac{\partial t}{\partial x}$$
$$(grad\ t)_y = \frac{\partial t}{\partial y}$$
$$(grad\ t)_z = \frac{\partial t}{\partial z} \tag{5}$$

Flow of heat will occur only if the temperature gradient is different from zero [1, 6, 7]

Vector of heat flow is proportional to temperature gradient

$$\vec{q} = -\lambda \cdot grad\ t \tag{6}$$

which represents Fourier's law [5, 12].

Here λ is the coefficient of thermal conductivity.

Components of the vector of heat flow, due to (6) are: [13 - 17]

$$q_x = -\lambda \cdot \frac{\partial t}{\partial x}$$
$$q_y = -\lambda \cdot \frac{\partial t}{\partial y} \tag{7}$$
$$q_z = -\lambda \cdot \frac{\partial t}{\partial z}$$

Complete heat flow taken per unit area dF and unit of time $d\tau$ will be, Figure 1:

$$dQ_x = -\lambda \cdot \frac{\partial t}{\partial x} \cdot dF \cdot d\tau$$

$$dQ_y = -\lambda \cdot \frac{\partial t}{\partial y} \cdot dF \cdot d\tau \qquad (8)$$

$$dQ_z = -\lambda \cdot \frac{\partial t}{\partial z} \cdot dF \cdot d\tau$$

Figure 1. Scheme for derivation of the heat transfer differential equation

At the direction dx heat flow will be changed for

$$\frac{\partial q_x}{\partial x} \cdot dx \qquad \frac{\partial q_y}{\partial y} \cdot dy \qquad \frac{\partial q_z}{\partial z} \cdot dz \qquad (9)$$

Due to values of qx, qy, qz, according to (8) it shall be:

$$dQ_x = \frac{\partial q_x}{\partial x} \cdot dx \cdot dy \cdot dz \cdot d\tau = -\lambda \cdot \frac{\partial^2 t}{\partial x^2} \cdot dV \cdot d\tau$$

$$dQ_y = \frac{\partial q_y}{\partial y} \cdot dx \cdot dy \cdot dz \cdot d\tau = -\lambda \cdot \frac{\partial^2 t}{\partial y^2} \cdot dV \cdot d\tau \qquad (10)$$

$$dQ_z = \frac{\partial q_z}{\partial z} \cdot dx \cdot dy \cdot dz \cdot d\tau = -\lambda \cdot \frac{\partial^2 t}{\partial z^2} \cdot dV \cdot d\tau$$

where dV is the volume of observed parallelepiped.

The overall increment of heat flow through the elementary parallelepiped is:

$$dQ = dQ_x + dQ_y + dQ_z \qquad (11)$$

that is, according to (10):

$$dQ = -\lambda \cdot (\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}) \cdot dV \cdot d\tau \qquad (12)$$

Heat dQ in its entirety performs only changes in the temperature of the body, that is, heating or cooling of the body, so considering that, it will be:

$$dQ = -dV \cdot \rho \cdot c \cdot \frac{\partial t}{\partial \tau} \cdot d\tau \qquad (13)$$

where:

ρ– density of the material of body

c – specific heat capacity of the body

By equating relations (12) and (13), the Fourier's differential equation is obtained

$$\frac{\partial t}{\partial \tau} = a \cdot (\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}) \qquad (14)$$

where the coefficient of thermal conductivity is:

$$a = \frac{\lambda}{c \cdot \rho} \qquad (15)$$

and is considered to be constant.

The left side of equation (14) defines the rate of change of temperature with time, and the right side spatial distribution of temperatures.

As seen, Fourier's differential equation is derived on the basis of a very small number of assumptions. Beside this equation, knowledge of supplementary conditions which uniquely define the phenomenon is required. These conditions can be time and space, which must satisfy the differential equation (14) [4, 9, 18]. Particularly significant and simple initial time condition is when for τ = o, t = t_0 = *const.*
Equation (14) is possible to write and simplify through the operator ∇ [17, 18]:

$$\frac{\partial t}{\partial \tau} = a \cdot \nabla^2 t \tag{16}$$

Differential equation of heat exchange

The amount of heat that is brought or taken from the surface of the body is:

$$q = \alpha \cdot t_p \tag{17}$$

which is a boundary condition, of special importance. Here is:
α – coefficient of heat transfer (convection)
t_p – surface temperature

The amount of heat that is brought to the surface boundary of the body is equal to the amount of heat which is taken

$$-\lambda \cdot \left(\frac{\partial t}{\partial n}\right)_p = \alpha \cdot t_p \tag{18}$$

In this, t_p must be distinguished from ambient temperature and from zero. The index *p* refers to the surface of the body. This boundary condition represents a differential equation of heat exchange, [2, 4, 7].

The task is defined closer with the geometry of the body. Analytical solutions exist only for simple bodies: an infinite plate of finite thickness, a cylinder of infinite length and a ball, [12, 15, 17, 18].

When the Fourier's equation can not be solved, the problems are solved by the theory of similarity. The same is true for complicated geometrical cases [3, 8, 11].

The task is more closely defined also with the geometric shape of the body.

In the cooling process, in practice, temperature distribution at any time over the whole cross section of the body is almost never requested, but a knowledge of the temperature on the surface of the core of piece (in the middle of the wall of piece) and the time required for cooling is most often necessary [5, 18].

Solving the problem of cooling starts from the Fourier differential equation and differential equation of heat exchange (boundary condition of the third-order), (14) and (18):

$$\frac{\partial t}{\partial \tau} = a \cdot \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right)$$

$$-\lambda \cdot \left(\frac{\partial t}{\partial n}\right)_p = \alpha \cdot t_p \tag{19}$$

Application of similarity theory in problem solving

Fourier's equation can be written as

$$\frac{\partial t}{\partial \tau} = a \cdot \sum_{i=1}^{i=3} \frac{\partial^2 t}{\partial x_i^2} \tag{20}$$

where $x_1 = x$ $x_2 = y$ $x_3 = z$

If in these equations, according to the origin of the theory of similarity, indexes and symbols of differentiation and summarizing are omitted, the equation will be obtained [4, 17]:

$$\frac{t}{\tau} = a \cdot \frac{t}{x^2}$$
$$\frac{\lambda \cdot t}{n} = \alpha \cdot t \qquad (21)$$

Here n is an arbitrary dimension of the body, so the size of the first equation X can be taken instead of n.

If each equation, according to the theory of dimensional analysis, is divided by suitable number, so that on one side of the equation remains 1, we get definitions of criteria

$$1 = \frac{a \cdot \tau}{X^2} \qquad (22)$$
$$1 = \frac{\alpha}{\lambda} \cdot X$$

Right sides of equations (22) are unnamed numbers. The first of criteria is derived from Fourier's equation and is called the Fourier criterion

$$F_0 = \frac{a \cdot \tau}{X^2} \qquad (23)$$

The second equation defines the Biot criterion

$$B_i = \frac{\alpha}{\lambda} \cdot X \qquad (24)$$

In this, coefficient of the heat transfer from the plate surface into the surrounding medium is constant.

In addition to these criteria, the final function should also introduce a criterion $\frac{x}{X}$ which is also unnamed number and indicates the place, because x is the distance of the observed point from the central plane, while X is a characteristically known length.

Unnamed temperature criterion, that represents the relationship between the required temperature at a given point of time in a given point and a known temperature that is given by the requirement of the task $\frac{\theta}{\theta_0}$, also has to be introduced [4, 16].

Meat steak in the form of a plate as a geometric model

In the analysis that follows, let us observe the steak meat Figure 2, wherein the average diameter is substantially greater than its thickness, $d_{sr} >> \delta$, which represents a thin flat plate from the geometric point of view. Also, we will assume that this geometric body is homogeneous and isotropic with its characteristics, whereby the heat conductivity, specific density and specific heat capacity are independent from the temperature. As a type of meat, the steak would fit the most to it.

Figure 2. Meat steak observed as a thin flat plate

Observed flat plate, with thickness of $\delta = 2X$ has some temperature t higher than the ambient temperature t_0 that represents the temperature of the cooling chamber, i.e. cold storage where the steak cools (Figure 3).

The temperature on the surface of the steak is t_z, while the temperature in the central transverse plane is t_m (core). The temperature in an arbitrary plane at a distance x is t_x.

Obviously, the criterion $\frac{x}{X}$ will have a value of 0 in the core of piece ($x=0$) and 1 for the surface of piece ($x=X$), since x is measured from the central plane (y axis), Figure 3.

Here, X represents the half of thickness of the plate (Figure 3). Obviously, cooling and thus the temperature on both sides of beef are symmetrical.

Figure 3. Model of meat steak as a flat plate at non-stationary heat conduction

Criterial equation of temperature field and its analytical solution

On the basis of equation (14) and for

$$\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0 \tag{25}$$

Due to the symmetry of cooling both sides of the plate and $\theta = t - t_0$, will be

$$\frac{\partial \theta}{\partial \tau} = a \cdot \frac{\partial^2 \theta}{\partial x^2} \tag{26}$$

By integrating the differential equation (26) in the form of endless row of transcendental functions:

$$\theta = \sum_{n=1}^{\infty} \frac{2 \cdot \sin \varepsilon_n}{\varepsilon_n + \sin \varepsilon_n \cdot \cos \varepsilon_n} \cdot \cos \varepsilon_n \frac{x}{s} \cdot \exp(-\varepsilon_n^2 \cdot F_0) \tag{27}$$

where ε_n are the roots of the characteristic equation

$$\log \varepsilon_n = \frac{\varepsilon_n}{B} \tag{28}$$

it is obtained temperature difference θ between the flat plate and the environment after the time of cooling τ at a distance x from the middle of cross-sectional plane of the plate:

$$\theta = \theta_c \cdot \phi(\frac{a \cdot \tau}{X^2}; \frac{x}{X}; \frac{\alpha}{\lambda} X) \tag{29}$$

Here, ϕ represents mentioned transcendental function in the form of infinite order (unnamed heat transfer function). The values of ε_n, which are multiple, can be found in the relevant tables [2, 8, 13, 14]. Relation (29) can be written as:

$$\theta = \theta_c \cdot \phi(Б\ ; B\ ; \frac{x}{X}) \tag{30}$$

or

$$\theta = \theta_c \cdot \phi(\frac{a\tau}{X^2}; \frac{\alpha}{\lambda} X; \frac{x}{X}) \tag{31}$$

which represents a criterial equation of the temperature field of the observed problem.

Here is $\theta_c = t_c - t_0$ (32)

If we mark the temperature in the middle cross section of plane with t_m, and with t_z surface temperature of the plate t_z, it will be analogous to above:

$\theta_m = t_m - t_0$ (33)

$\theta_z = t_z - t_0$ (34)

In this, it should be noted that the initial temperature of the steak and the ambient temperature are constant.

The temperature criterion, according to the previous one, in the general case is:

$$\frac{\theta}{\theta_c} = \frac{t - t_0}{t_c - t_0} \tag{35}$$

ie for the median plane and the surface:

$$\frac{\theta_m}{\theta_c} = \frac{t_m - t_0}{t_c - t_0} \qquad \frac{\theta_z}{\theta_c} = \frac{t_z - t_0}{t_c - t_0} \tag{36}$$

For t_m it will be $x=0$ and for t_z it will be $x=X$, so according to (31) it is:

$$\theta_m = \theta_c \cdot \phi_m \cdot (\text{Б}\ ;\ B\ )$$

$$\theta_z = \theta_c \cdot \phi_z \cdot (\text{Б}\ ;\ B\ ) \tag{37}$$

That is

$$\theta_m = \theta_c \cdot \phi_m \left(\frac{a \cdot \tau}{X^2}; \frac{\alpha}{\lambda} X\right)$$

$$\theta_z = \theta_c \cdot \phi_z \left(\frac{a \cdot \tau}{X^2}; \frac{\alpha}{\lambda} X\right) \tag{38}$$

Thus, the criteria that determine the unnamed temperature in the core and on the surface are

$$\phi_m = \frac{\theta_m}{\theta_c} \qquad \phi_z = \frac{\theta_z}{\theta_c} \tag{39}$$

For scheduling the temperature of the body in the form of plates for the core and the surface of the body, for a certain time τ = const, values of elementary functions F with Figure 4 and Figure 5., can be used [1, 15, 16, 18].

Fig 4. The temperature function $\frac{\theta_m}{\theta_c}$ for the middle plane of an unlimited plate (t_m – temperature in the middle plane)

Obviously, for temperature function for the nucleus (Figure 4) is valid:

$$\frac{\theta_m}{\theta_c} = f_m(B\ ,\text{Б}\ ) \tag{40}$$

where:

t_m – the temperature in the middle plane (core)

t_0 – ambient temperature

t_c – initial temperature

The diagram is constructed for a wider interval, F_0= 0.1 ÷500.

Figure 5. Temperature function $\frac{\theta_z}{\theta_c}$ for the surface of unlimited plate (t_z – temperature of the surface)

For the surface of the plate is valid (Figure 5):

$$\frac{\theta_z}{\theta_c} = f_z(Bi;Fo) \tag{41}$$

where is also

t_z – temperature on the surface

t_0 – ambient temperature

t_c – initial temperature

The diagram is constructed for a wider interval, F_0= 0.00025 ÷500.

For determination of the line of temperature distribution inside the board, a diagram can be used according to Figure 6. Here is the obvious:

$$\phi_x = \frac{\theta_x}{\theta_m} = \frac{t_x - t_0}{t_m - t_0} \tag{42}$$

Figure 6. Temperature function $\frac{\theta_x}{\theta_m}$ for the inside of the plate

Here is
t_x – the temperature at an arbitrary place (for x coordinate)
t_0 – temperature in the middle (core)
t_c – ambient temperature

Possibilities of application of derived relations

Determination of the temperature on the surface and in the middle of sectional plane of meat steak

Given:
δ=3 cm = 0.03 m ρ=1200 kg/m^3
c_p=4.1 kJ/kg°C λ=0.45 W/m°C α=20 W/m^2°C

- initial temperature of the steak t_c= 25°C
- ambient temperature (cold storage) t_0= -5°C
- duration time of cooling τ=60 min

The temperature at the surface and in the core of the steak is requested.

$$a = \frac{\lambda}{c \cdot \rho} = \frac{0.45}{4100 \cdot 1200} = 9 \cdot 10^{-8} \text{m}^2/\text{s} \qquad X = \frac{\delta}{2} = 0.015 \text{ m}$$

Solution:

$$F_0 = \frac{a}{X^2} \cdot \tau = \frac{9 \cdot 10^{-8}}{0.015^2} \cdot 3600 = 1.44$$

$$B_i = \frac{\alpha}{\lambda} \cdot X = \frac{20}{0.45} \cdot 0.015 = 0.67$$

$$\phi_z = \frac{\theta_z}{\theta_c} = \frac{t_z - t_0}{t_c - t_0} = \frac{t_z - (-5)}{25 - (-5)} = 0.32 \quad \text{(fig.7a)} \quad \rightarrow \quad t_z = 4.6°\text{C}$$

a) Temperature of the surface t_z
b) Temperature in the middle of crossing plane t_m (core)

$$\phi_m = \frac{\theta_m}{\theta_c} = \frac{t_m - t_0}{t_c - t_0} = \frac{t_m - (-5)}{25 - (-5)} = 0.55 \text{ (fig.7b)} \rightarrow t_m = 11.5°\text{C}$$

F_0=1.44 Bi=0.67

Figure 7. Determination of the temperature on the surface and in the middle of plane of meat steak

Determination of cooling time

Given:
δ=25 mm=0.025 m ρ=1200 kg/m^3
c_p=4100 J/kg°C λ=0.45 W/m°C α=15 W/m^2°C

- initial temperature of the steak: t_c= 20 °C

- ambient temperature (cold storage) $t_0 = -10$ °C
- requested temperature of the surface of the steak $t_z = 1$ °C

Cooling time for given conditions and temperature in the core, in this case is requested

Solution:

$$a = \frac{\lambda}{c \cdot \rho} = 9 \cdot 10^{-8} \text{m}^2/\text{s} \qquad X = \frac{\delta}{2} = 0.0125 \text{ m}$$

$$\phi_z = \frac{t_z - t_0}{t_c - t_0} = \frac{1-(-10)}{20-(-10)} = 0.37$$

$$B_i = \frac{\alpha}{\lambda} \cdot X = \frac{15}{0.45} \cdot 0.0125 = 0.42$$

$$F_0 = \frac{a}{X^2} \cdot \tau = \frac{9 \cdot 10^{-8}}{0.0125^2} \cdot \tau = 1.5 \qquad \text{(fig.8a)} \qquad \rightarrow \tau = 2600 \text{ s} = 0.72 \text{ h}$$

a) Cooling time

b) The temperature in the middle sectional plane t_m

$B_i = 0.42$

$$\phi_m = \frac{t_m - t_0}{t_c - t_0} = \frac{t_m - (-10)}{20-(-10)} = 0.68 \qquad \text{(fig.8b)} \qquad \rightarrow t_m = 10.4 \text{ °C}$$

$F_0 = 1.5$

Figure 8. Determination of cooling time of meat steak

Time required for the surface to cool down to ambient temperature

Given:

X=20mm = 0.02 m α=15 W/m²°C λ = 0.45 W/m°C

c_p = 4100 J/kg°C ρ =1200 kg/m³

- ambient temperature (cold storage) t_0 = 0°C
- initial temperature t_c = 25°C

a) Determine the time required for the surface of steak to cool down to ambient temperature t_0

b) What time is required in previous case for the core to also get cooled down to ambient temperature?

a) $t_z = t_0$

$$B_i = \frac{\alpha}{\lambda} \cdot X = \frac{30}{0.45} \cdot 0.02 = 1.33 \qquad a = \frac{\lambda}{c \cdot \rho} = 9 \cdot 10^{-8} \text{m}^2/\text{s}$$

$$\phi_z = \frac{t_z - t_0}{t_c - t_0} = \frac{t_0 - t_0}{25 - t_0} = 0$$

$$F_0 = \frac{a}{X^2} \cdot \tau = \frac{9 \cdot 10^{-8}}{0.015^2} \cdot \tau = 5.5 \qquad \text{(fig.9a)} \qquad \rightarrow \tau = 12222 \text{ s} = 3.395 \text{ h}$$

b) $t_m = t_0$

$$B_i = \frac{\alpha}{\lambda} \cdot X = 1.33$$

$$\phi_m = \frac{t_m - t_0}{t_c - t_0} = \frac{t_0 - t_0}{25 - t_0} = 0$$

$$F_0 = \frac{a}{X^2} \cdot \tau = \frac{9 \cdot 10^{-8}}{0.02^2} \cdot \tau = 7 \qquad \text{(fig.9b)} \qquad \rightarrow \tau = 15556 \text{ s} = 4.32 \text{ h}$$

Solution:

a) $t_z = t_0$

Figure 9. Determination of time required for the surface and core of meat steak to cool down to ambient temperature

Determination of temperature t_x at arbitrary distance from the central plane (x)

Given:

δ=3 cm = 0.03 m ρ=1200 kg/m³

c_p=4100 J/kg°C λ=0.45 W/m°C α=20 W/m²°C

- ambient temperature (cool storage) t_0= -5 °C

$$\frac{x}{X}=\frac{\delta/4}{\delta/2}=0.5$$

$$B_i=\frac{\alpha}{\lambda}\cdot X=\frac{\alpha}{\lambda}\cdot\frac{\delta}{2}=\frac{20}{0.45}\cdot\frac{0.03}{2}=0.67$$

$$\phi_x=\frac{\theta_x}{\theta_m}=\frac{t_x-t_0}{t_m-t_0}=\frac{t_x-(-5)}{11.5-(-0.5)}=0.9 \qquad \text{fig.10} \rightarrow \qquad t_x = 9.8\ °\text{C}.$$

- temperature of the core t_m = 11.5°C

Determine the temperature at a distance $x=\frac{\delta}{4}$ from the central axis.

The solution:

Using different distances $x \in \{0, \delta/2\}$, it is possible by the above procedure to determine the function of the temperature distribution along the thickness of steak, depending on the distance x.

Figure 10. Determination of temperature of meat steak at a distance

$x=\frac{\delta}{4}$ *from the central plane*

Determination of the ambient temperature (cool storage)

Given:

δ=2.5 cm ρ=1200 kg/m³ c_p= 4100 J/kg°C

λ=0.45 W/m°C α=25 W/m²°C

- Time of cooling τ=1.5h = 5400 s
- temperature of the surface t_z= 1 °C
- initial temperature of the steak t_c=20 °C

$$B_i=\frac{\alpha}{\lambda}\cdot X=\frac{25}{0.45}\cdot 0.0125=0.69 \qquad a=\frac{\lambda}{c\cdot\rho}=9\cdot 10^{-8}\,\text{m}^2/\text{s}$$

$$x=\frac{\delta}{2}=\frac{2.5}{2}=1.25\text{cm}=0.0125\text{m}$$

$$F_0=\frac{a}{X^2}\cdot\tau=\frac{9\cdot 10^{-8}}{0.0125^2}\cdot 5400=3.1$$

$$\phi_z=\frac{\theta_z}{\theta_c}=\frac{t_z-t_0}{t_c-t_0}=\frac{1-t_0}{20-t_0}=0.15 \qquad (\text{fig.11a}) \quad \rightarrow \quad t_0=-2.4°\text{C}.$$

b) Bi=0.69

$$F_0=\frac{a}{X^2}\cdot\tau=3.1$$

$$\phi_m=\frac{\theta_m}{\theta_c}=\frac{t_m-t_0}{t_c-t_0}=\frac{t_m-(-2.4)}{20-(-2.4)}=0.3 \qquad (\text{fig. 11.b}) \quad \rightarrow \quad t_m=4.3\ °\text{C}$$

a) Determine the ambient temperature (cool storage)

b) What is the temperature of the core for the previous conditions?

Solution:

Figure 11. Determination of ambient temperature (cool storage) in cooling of meat cuts

Determination of heat transfer coefficient in cooling meat cuts in order to avoid frosting

Given:

δ=3 cm = 0.03 m ρ=1200 kg/m^3

c_p=4100 J/kg°C λ=0.45 W/m°C $a = 9.03\cdot10^{-8}$ m^2/s

- ambient temperature (cool storage) t_0= -15 °C
- temperature in middle plane (coor) t_m= +4 °C

$$\phi_x = \frac{t_x - t_0}{t_m - t_0} = \frac{-1-(-15)}{4-(-15)} = 0.74$$

$$B_i = \frac{\alpha}{\lambda}\cdot X = \alpha\cdot\frac{0.015}{0.45} = 0.65 \qquad \text{(fig.12)} \qquad \rightarrow \qquad \alpha = 19.5\,\mathrm{W/m^2{}^\circ C}$$

- temperature at the surface of steak: t_z= -1 °C

We will determine the heat transfer coefficient α (which can be regulated by the speed of rotation of the fan in the cooler), so that the temperature of steak is held within the given limits (-1°C ÷ 4°C).

Solution:

This represents the required heat transfer coefficient, which can be adjusted by changing the speed of rotation of the fan in the cool storage. In this way it will avoid frosting of meat steaks, due to their temperature that will be maintained within the given limits (-1 °C ÷ +4°C).

Previous relations are valid at [17, 18]:

$$F_0 = \frac{a}{X^2}\cdot\tau > 0.2 \qquad (43)$$

$$\tau > 0.2\cdot 0.015^2\cdot\frac{4100\cdot1200}{0.45}$$

From here it is:

$$\tau > 0.2\cdot\frac{X^2}{a} \qquad \rightarrow \qquad \tau > 0.2\cdot X^2\cdot\frac{c\rho}{\lambda} \qquad (44)$$

that is:

$\tau > 492$ s

$\tau > 8.2$ min

Figure 12. Determination of heat transfer coefficients in cooling of meat cuts in order to avoid frosting

Conclusion

Fourier partial differential equation of heat flow is derived from a very small number of assumptions, which, as shown, allows the possibility of its wide application in technique and technology. It has been shown that it may be efficiently used in the process of cooling of meat steaks with a thickness substantially smaller than the average diameter.

A complex system of partial differential equations in the present model is solved efficiently using the similarity theory, by introducing appropriate dimensionless criteria. In the direct problem solving, there would be some mathematical difficulties, that is, complications of the solution.

Considering that the cooling problems are solved analytically using existing diagrams that define appropriate temperature functions, special attention must be given to approximation of curve diagrams to get the most accurate results.

Also, the accuracy of the results depends on the coefficient of thermal conductivity, which takes into account the physical characteristics of the body, i.e. the steak in this case.

Heat is exchanged with the environment depending on the time of cooling, also may be determined on the basis of this model, as in this case, in the literature there are corresponding diagrams.

The model presented in the paper can be used for verification of the analytical results in practical examples of cooling process of meat steaks.

At the same time, it would be particularly interesting to experimentally determine the temperature field, depending on the cooling time of the observed body and compare the results with the analytical solution.

In the case of more complex shapes than observed, a solution can be reached by using appropriate numerical methods.

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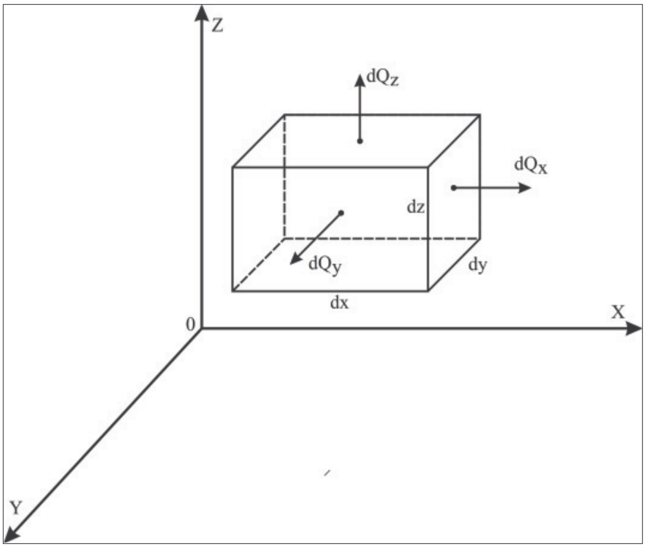
Recived: 04.02.2014.
Accepted: 16.06.2013.



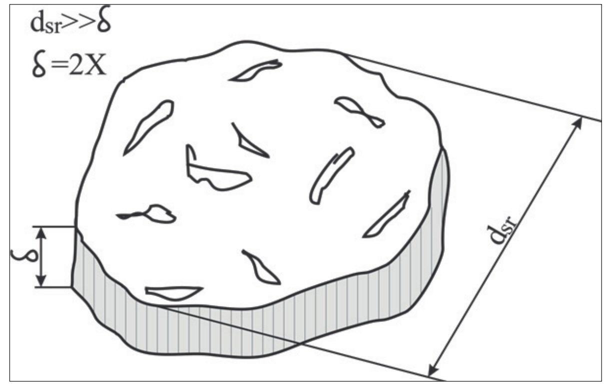
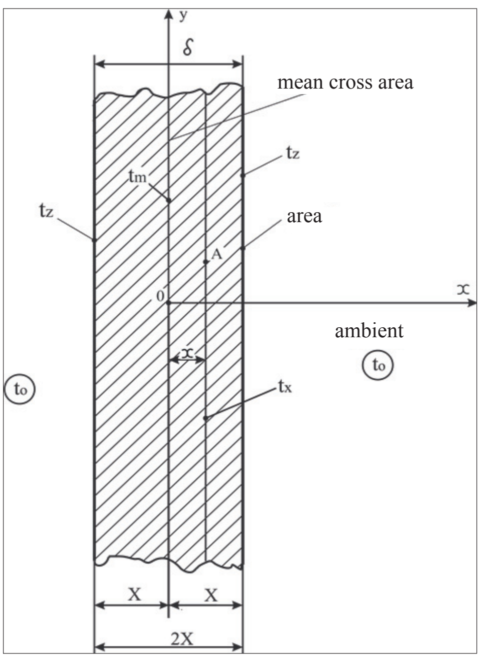


Fig. 2. Meat steak observed as thin flat plate

Fig. 1. Scheme for derivation of the heat transfer differential equation



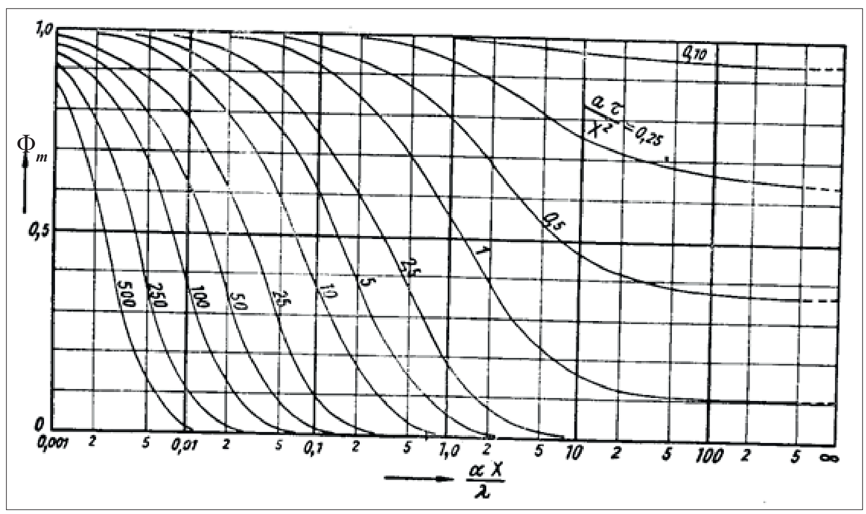


Fig. 4. The temperature function $\frac{\theta_m}{\theta_c}$ for the middle plane of an unlimited plate (t_m – temperature in the middle plane)

Fig. 3. Model of meat steak as flat plate at non-stationary heat conduction

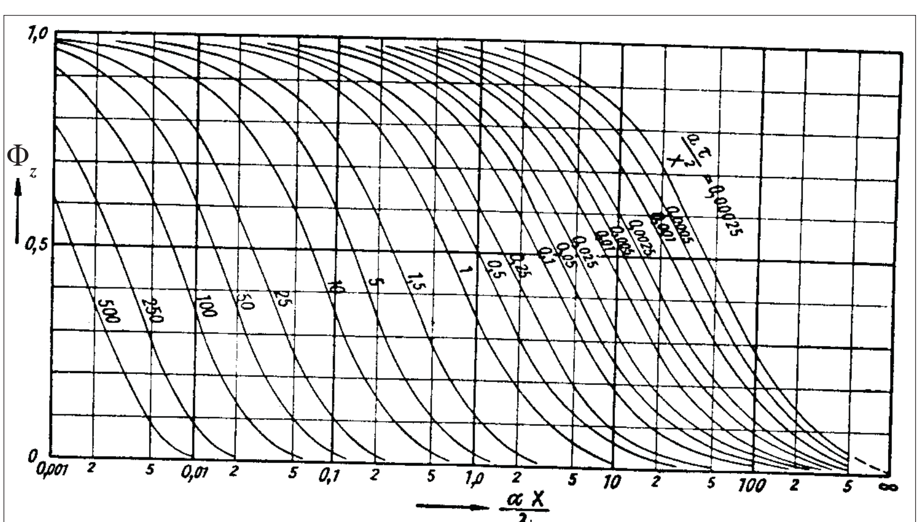


Fig. 5. Temperature function $\frac{\theta_z}{\theta_c}$ for the surface of unlimited plate *(t_z – temperature of the surface)*

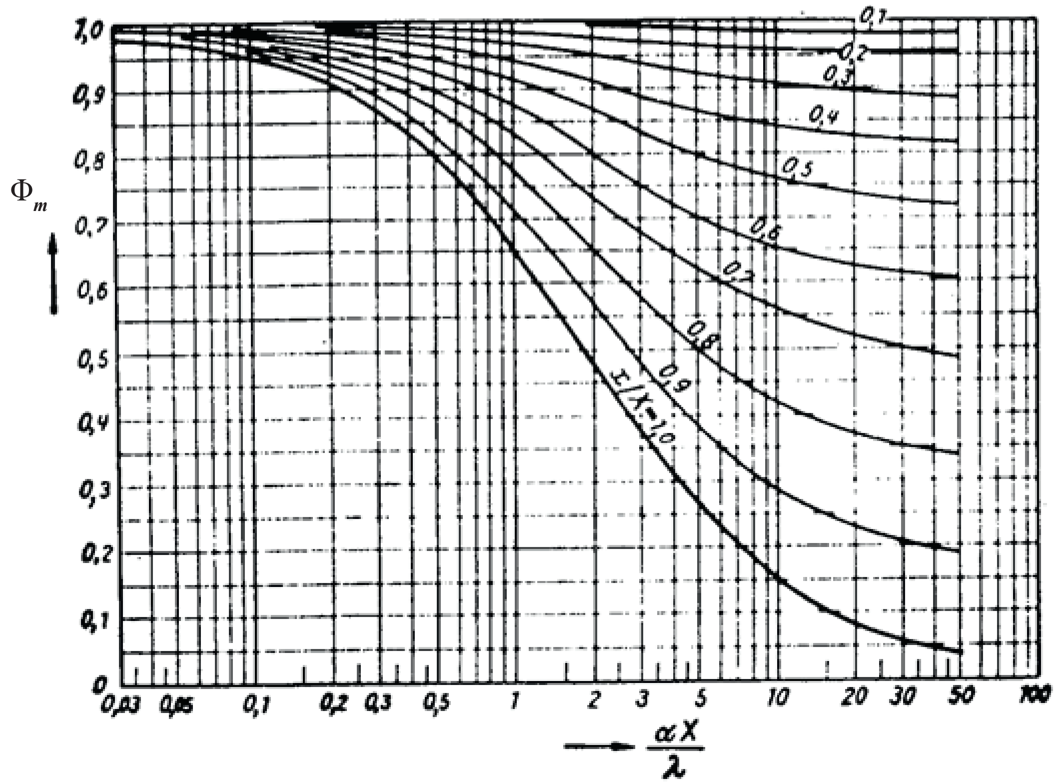


Fig. 6. Temperature function $\dfrac{\theta_x}{\theta_m}$ for the inside of the plate

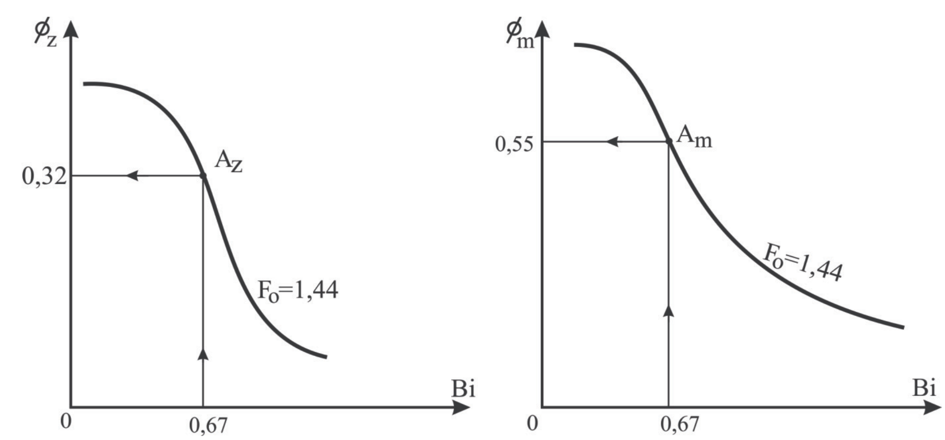


Figure 7. Determination of the temperature on the surface and in the middle of plane of meat steak

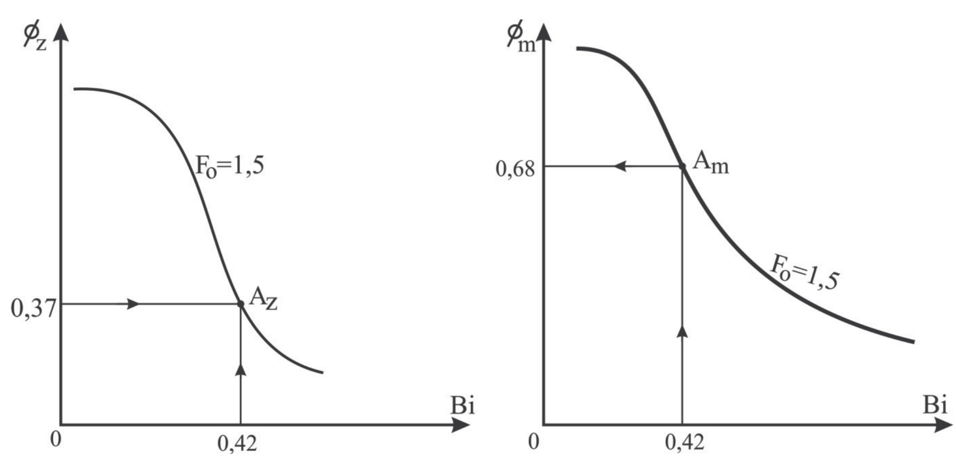


Figure 8. Determination of cooling time of meat steak

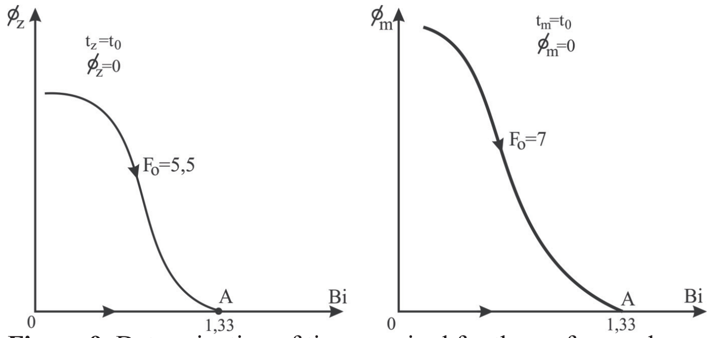


Figure 9. Determination of time required for the surface and core of meat steak to cool to ambient temperature

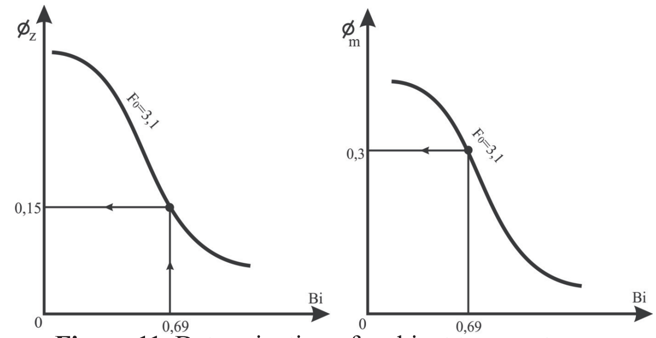


Figure 11. Determination of ambient temperature (cool storage) in cooling of meat cuts

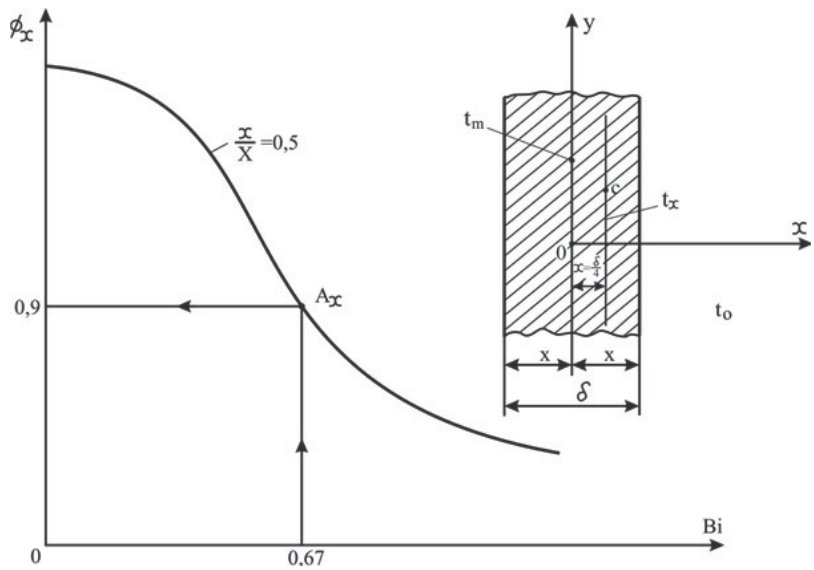


Figure 10. Determination of temperature of meat steak at a distance $x = \frac{\delta}{4}$ from the central plane

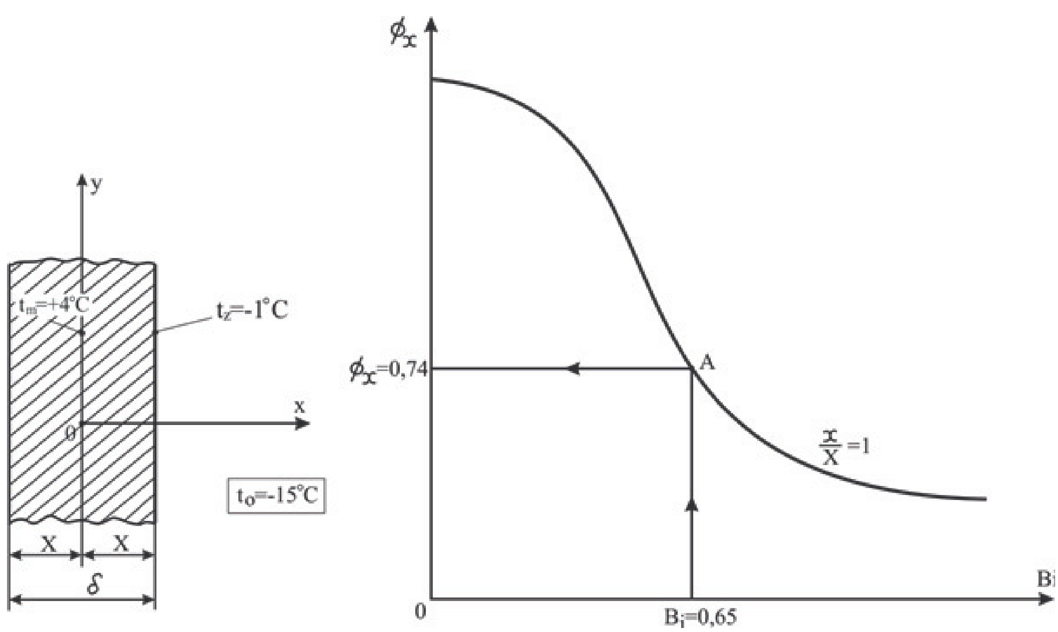


Fig. 12. Determination of heat transfer coefficients in cooling of meat cuts for the case of avoiding frosting