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Review Paper

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ANALYSIS OF THE BONUS SYSTEM IN INSURANCE USING MARKOV CHAINS

АНАЛИЗА БОНУС СИСТЕМА У ОСИГУРАЊУ ПРИМЈЕНОМ МАРКОВЉЕВИХ ЛАНАЦА

Summary: *One of the ways in the insurance premium calculation procedure is to determine the premium based on experience. A significant role belongs to the so-called bonus system in practical examples of determining the number of insurance premiums. The procedure for applying this system is based on the previous compensation claims analysis, in the operationalization of which the application of Markov chains occupies a pivotal place. The goal is to show and point out the importance of Markov processes and one of the segments of their application in an appropriate way, which in this case is the insurance field. This paper deals with the bonus system used in motor vehicle insurance. Insurance users fall into three discount categories, and an analysis of the number of future premiums was performed depending on the damages incurred and the probability of their reporting. It has been shown that there may be a disproportion between the current amounts of premiums to be paid and the amounts of reported claims. This situation arises due to the small number of discount categories and slight differences in discount levels. The paper also discusses the possibility of analyzing future premiums depending on the damage done and the probability of its reporting in particular cases of an insured individual.*
Keywords: *bonus system, transition probability, Markov chains, lognormal distribution.*
JEL Classification: ***C02***

Резиме: *Један од начина у поступку прорачуна премије осигурања је одређивање премије на основу искуства. У практичним примјерима одређивања висине премија осигурања значајна улога припада тзв. бонус систему. Поступак примјене овог система заснован је на анализи претходних одштетних захтјева, а у чијој операционализацији кључно мјесто заузима примјена Марковљевих ланаца. Циљ је да се на прикладан начин прикаже и укаже на значај Марковљевих процеса и на један од сегмената њихове примјене, а то је у овом случају област осигурања. У овом раду је обрађено кориштење бонус система код осигурања моторних возила. Корисници осигурања сврстани су у три категорије попуста и извршена је анализа висине будућих премија у зависности од насталих штета и вјероватноће њиховог пријављивања. Показало се да може доћи и до диспропорције између актуелних висина премија за плаћање и висина пријављених штета. Ова ситуација настаје због малог броја категорија попуста и незнатно малих разлика у нивоима попуста. У раду се говори и о могућности анализе висине будућих премија у зависности од висине учињене штета и вјероватноће њеног пријављивања на појединачним случајевима сваког осигураника.*
Кључне ријечи: бонус систем, прелазна вјероватноћа, Марковљеви ланци, логнормална дистрибуција.
ЈЕЛ касификација: ***C02***

INTRODUCTION

One of the possible ways of determining the insurance premium is the procedure of resolving it based on experience. The research and analysis focus on the conditions for transitioning from one discount category to another, as well as the achieved effect on the insured. The research aims to demonstrate the possibility of applying Markov chains in the analysis of the use of the bonus system in insurance. Earlier research on this topic is related to the paper Quantitative models of risk management in life insurance, in the doctoral dissertation of Professor Mira Pešić Andrijić, as well as in Application of Markov processes in finance (Stojić, Babić and Petrović 2019). This system in motor vehicle insurance, for premium determining by previous claims, is one of the applications of Markov chains. This method was developed by the Russian scientist Andrei Markov (1856-1922), who laid down its fundamental principles. This theory is specific because it represents a synthesis of Probability Theory and Matrix Calculus Theory. The insured's past loss is considered in the final charge when assessing the premium in this way.

When determining the amount that the insured should pay, the insurer bases its decision on the number of compensation claims of the insured in the past. This method is known as the bonus system, and it actually represents the return of a part of the premium that is a consequence of the achieved result. This system works by getting a discount based on the premium, directly related to the number of years without a claim of the insured. The insured pays a reduced premium, and this method is widely used in motor vehicle insurance.

1. BONUS SYSTEM

Elements of the bonus system are discount category and rules for moving from one to another. Under categories we indicate the number of years without a claim. Under moving from one category to another, we imply whether a claim is returned without a discount or the insured moved to a lower discount category. We will consider situations that illustrate this method:

Case 1

Let's examine the three-category bonus system:

Table 1 Three-category bonus system

Category	Discount (%)
0	0
1	25
2	40

Source: processed by the author

Category "0" includes insured persons who pay the total premium; category "1" includes insured persons who pay 75% of the total amount, while the last category comprises the policies of those insured persons who pay 60% of the premium amount. If the insurance holders do not report a claim during the year, they move to a higher discount category, and if there are several reported claims, they move to a lower category.

2. METHODOLOGY

A stochastic process is a time-dependent random event model. If the random variable describes some random event, then the stochastic process is a family of random variables X_t, for each time instant t. We will mark the space of all elementary outcomes with S, and on the example of the observed insurance company that offers three levels of discounts, we will denote the set $S=(0, 1, 2)$.

A stochastic process is said to have the Markov property if its future state can be predicted solely using the current one without using past data (Rolski et al. 1998).

The mathematical notation of the aforementioned results is as follows:

$$P\left[x_t \in {}^{C}/_{x_{s1}} = x_1, \quad x_{s2} = x_2, \ldots, x_{sn} = x_n, x_s = x\right] =$$

$$P\left[x_t \in {}^{C}/_{x_s} = x\right] \; \forall S_1 < S_2 < \cdots < S_n < S < t$$

and all states $x_1, x_2, \ldots, x_n$, $x \in S$
and all subsets $C \leq S$

A Markov process with discrete state space and discrete time is called a Markov chain. The discount status of the insured forms a Markov chain. Information about the state of the system in a certain period, which is the initial period for predicting the state of the system, can be represented by a state vector of the following form (Backović, Vuleta and Popović 2011):

$$S(t) = S_1(t), S_2(t), \ldots S_n(t)$$

The transition of the system from one state to another in two consecutive periods can be represented by the corresponding transition probabilities that form a matrix of transition probabilities. When we have three discount levels, the states set is S= (0, 1, 2).

The transition matrix is a square matrix of the format *n x n*, where *n* is the number of states in the set S, i.e. the set of all elementary outcomes. The condition that must be met is the so-called condition of normality (Rolski et al., 1998), so the following must be valid:

$$\sum_{j \in s} p_{ij} = 1, \forall i$$

This relation actually states that the sum of the elements of each type must be one.

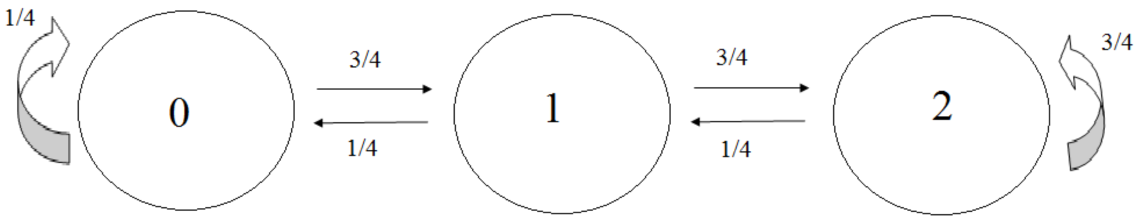
In this way, a matrix of transfers of insured persons from category i to category j is formed from year to year:

$$p = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \cdots & p_{0n} \\ p_{10} & p_{11} & p_{12} & \cdots & p_{1n} \\ p_{n0} & p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}$$

where p_{ij} represents the probability of the insured moving from category i to category j.

If we assume that the probability that the insured will not file a claim within one year is ¾, then the Markov chain that describes this process shown through the transition graph looks like this:

Picture 1 Transition graph number 1.



Notice that for probabilities p>0, the drawn arrow leaves the possibility of a direct transition from one state to another. The value ¾ is the probability that the insured will not have a claim and the possibility of moving to a higher discount level.

In the same way, the insured will descend from a particular discount level to a lower level with the opposite probability p= 1- ¾ = ¼.

In our particular three-state example, we have:

$$p = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & 0 & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

The transition matrix allows simple calculation of maximum discount probabilities. In the considered case, the policyholder who is at the zero discount level will reach the maximum discount amount with a probability:

$$p_{02} = p_{01} \text{ x } p_{12} = ¾ \text{ x } ¾ = 9/16$$

Note that Markov chains and the transition matrix can be used to estimate the expected number of policyholders at different discount levels in any year. It would be done with the help of the transitional matrix and the number of insured persons from the previous year if it is the current year.

3. ANALYSIS OF PREMIUM AMOUNTS

The bonus system, with all its advantages, still has certain disadvantages. Namely, there may be a situation where the insurance holders do not want to report the damage and then find their solution to the problem. The fact is that the insured will not report the damage if its amount is less than the amount of the future increased premium. In this case, the insurer cannot draw correct conclusions about the dangers that carry a particular risk. Therefore, application of the bonus system adjusts the premium to the individual risk because practice indicates that policyholders who are granted a bonus in advance have significantly fewer adverse events than other policyholders (Marović, Avdalović, 2003).

The paper considered the results of the applied method conditioned by different decisions of the insured on reporting the damage. The reason for not reporting the damage is the amount of the future increased premium. It is a kind of individual analysis of the possible gain or loss carried out by the insured himself (we imply analysis).

If the insured does not report the damage in the first year (or in subsequent years), his prospective premiums will amount to:

375 monetary units (MUs)	300 MUs	300 MUs

If the damage is reported in the first year, prospective premiums will be:

500 MUs	375 MUs	300 MUs

In this case, the difference in premiums the insured would pay amounts to 200 monetary units. The difference in premiums for all discount categories can be calculated using this methodology. E.g. the insured who is in the 25% discount category, i.e. in the first discount category, and suffered damage, can consider two cases:

I If one does not report the damage, the prospective premiums will amount to:
- 375 MUs (being in the first category of discount)
- 300 MUs (assuming that one did not have a compensation claim in the previous year)
- 300 MUs (using the same no-damage assumption)

II If the damage is reported, the insured will pay three subsequent premiums:
- 375 MUs (being at the beginning of the first discount level)
- 500 MUs (for filing a claim and reverts to the zero discount level)
- 375 MUs (assuming that one had no damage in the previous period, thus returning to the first discount level).

The difference in premiums for the observed three-year period, which depends on different decisions of the insured, will be:

$$(375+500+375) - (375+300+300) = 275$$

These differences are a consequence of observing the future period up to the year in which the maximum discount is achieved. There is a significant difference between the probability of damage occurring and the probability of reporting it. The conclusion here is evident: the insured will report the damage only if the damage exceeds 275 monetary units.

It is also possible to generalize this conclusion. Let us mark with X the random variable defined as X - the charge of damage, and with Y the difference in the amounts of future premiums due to different decisions of the insured about reporting the damage. Assuming a known distribution of the random variable x, one can then determine the probabilities of reporting claims at the appropriate discount levels, as follows:

$$P\ (\text{damage/accident report}) = P\ (X>Y)$$

It is assumed that the random variable X has a lognormal distribution with parameters μ and σ^2. The random variable X has a lognormal distribution with parameters μ and σ^2, if the random variable $Y=lnX$ has normal distribution, i.e. if this applies:

$$Y = \ln X: N\ (\mu, \sigma^2)$$

Therefore, the insured can make a kind of analysis depending on, e.g. the damage caused and subsequent premium amounts, as well as whether one realizes a loss or a profit. Of course, the damage charge determines the decision about reporting it and the current discount category to which the insured belongs.

We mentioned that there is a possibility of determining the probability that the insured will report the damage. It is done using the distribution of the random variable X that describes the amount of damage. In our example, we have:

X – damage charge

Y – the maximum amount of damage below which the insured will not report damage

Assume that $\mu = 4$ i $\sigma^2 = 4$ and it holds:

$$\ln X: N\ (\mu, \sigma^2) = N\ (4, 4)$$

The insured bases the decision on the damage report on the X-Y difference, and the required odds will be equal to:

$$P(X>Y) = P\ (\ln X > \ln Y) = 1 - P(\ln X \leq \ln Y) = 1 - \theta\left(\frac{\ln Y - \mu}{\sigma}\right)$$

In our three-category discount example, considering that we observed the case of the insured in the first discount category, the situation is as follows:

$$P(X>Y) = 1 - \theta\left(\frac{\ln 275 - 4}{2}\right) = 1 - \theta\left(\frac{5,62 - 4}{2}\right) = 1 - \theta\,(0,81) = 1 - 0,7910 = 0,2090$$

The probability that the insured will report the damage, depending on the category to which they belong, can be quite different. It indicates constant monitoring and assessment of the frequency and amount of future damages when applying the bonus system. Only with such an approach premiums that provide sufficient funds needed to pay the damages incurred can be formed.

CONCLUSION

Insurance companies try to manage their assets rationally, and in this context, there is a need for a sufficient amount of money to pay out possible damages. By using the bonus method, it is possible to determine the premium automatically. Namely, the insured who claims the damage less often will pay less than the one who reports the damage more than once.

However, there may be disproportions between the amount of premiums paid by the insured and the reported claims. The reason is the small number of discount categories and the slight differences between the discount levels. The efficient use of the bonus system can be hindered by the low probability of claims occurring, which leads to the majority of policyholders finding themselves at the maximum discount level. We have also seen that the probabilities of reporting claims, depending on their amount and the amount of future premiums, can be determined using the relationship between the lognormal and normal distributions.

The use of the bonus system is justified due to competitiveness when determining the insurance premium and contribution to its more correct calculation. Likewise, this system is also accepted in practice due to the reduction in the number of low compensation claims, which leads to lower overhead costs for the insured and, ultimately, lower costs for processing compensation claims.

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